Computer algebra independent integration tests

5-Inverse-trig-functions/5.3-Inverse-tangent/5.3.5-u-a+b-arctan-c+d-x-^p

Nasser M. Abbasi

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [70]. This is test number [151].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (70)	% 0. (0)
Mathematica	% 95.71 (67)	% 4.29 (3)
Maple	% 98.57 (69)	% 1.43 (1)
Maxima	% 45.71 (32)	% 54.29 (38)
Fricas	% 40. (28)	% 60. (42)
Sympy	% 32.86 (23)	% 67.14 (47)
Giac	% 40. (28)	% 60. (42)

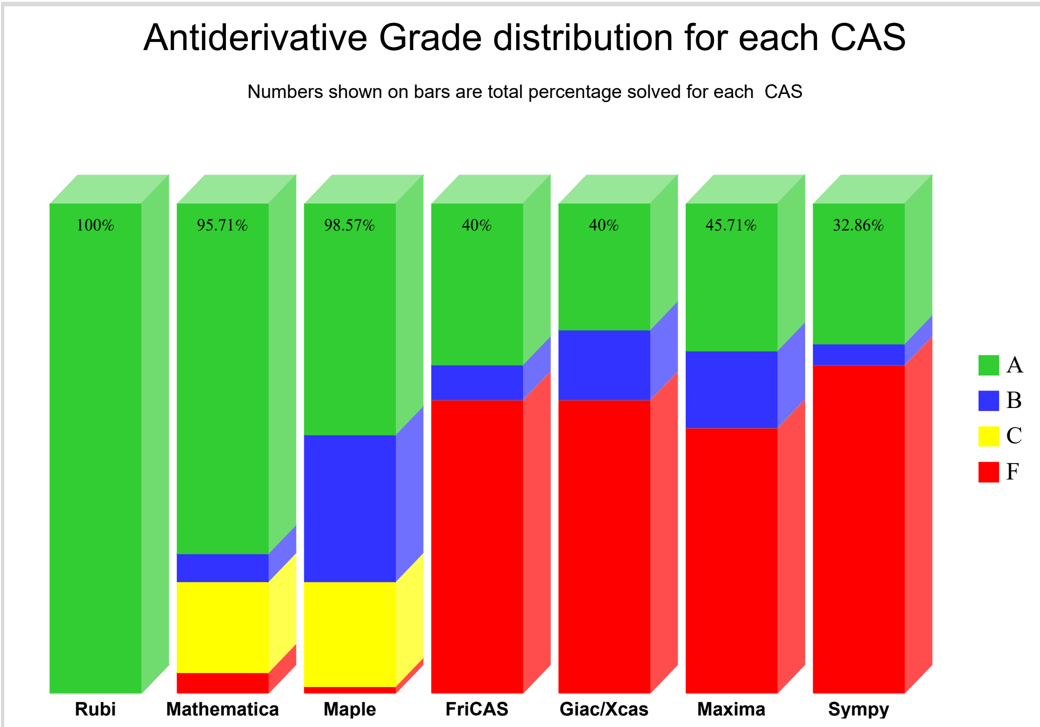
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

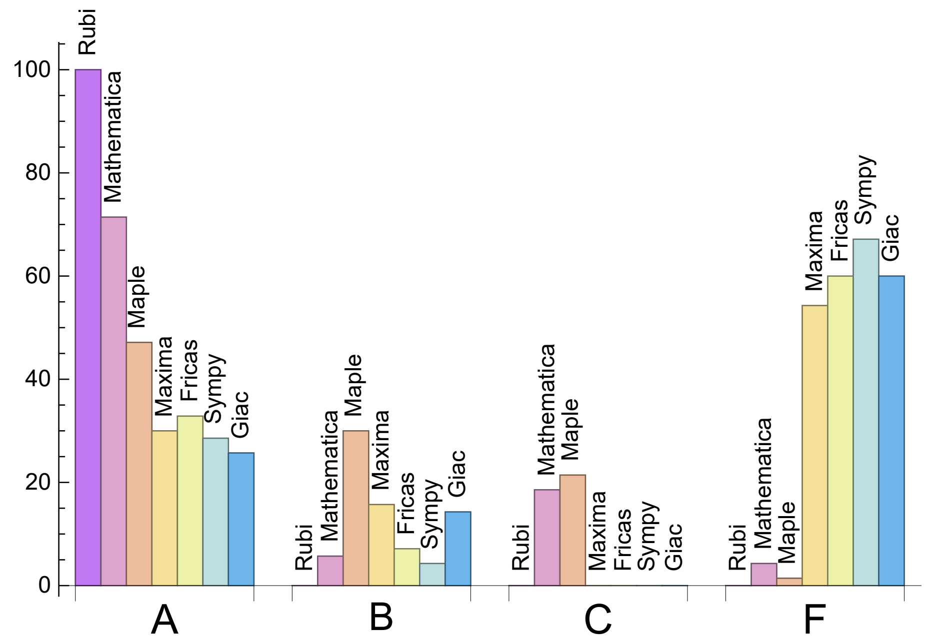
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	71.43	5.71	18.57	4.29
Maple	47.14	30.	21.43	1.43
Maxima	30.	15.71	0.	54.29
Fricas	32.86	7.14	0.	60.
Sympy	28.57	4.29	0.	67.14
Giac	25.71	14.29	0.	60.

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.32	228.04	0.9	159.5	1.
Mathematica	1.23	266.03	1.45	163.	0.99
Maple	0.44	1882.19	4.99	225.	1.83
Maxima	1.59	240.47	2.18	164.	1.7
Fricas	1.74	289.43	2.54	193.5	2.68
Sympy	10.38	397.17	4.7	178.	2.3
Giac	1.16	311.54	2.37	126.	1.63

1.4 list of integrals that has no closed form antiderivative

{23, 42, 43, 65, 66, 69, 70}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {65, 66, 69, 70}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {8, 11, 13, 15, 16, 18, 19, 20, 28, 31, 32, 33, 35, 36, 37, 38, 48, 54, 55, 56, 57, 62, 63, 64, 65, 66, 67, 68, 69, 70}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command AboluteTiming[] was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command Usage was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's time.time() call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is ValueError. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

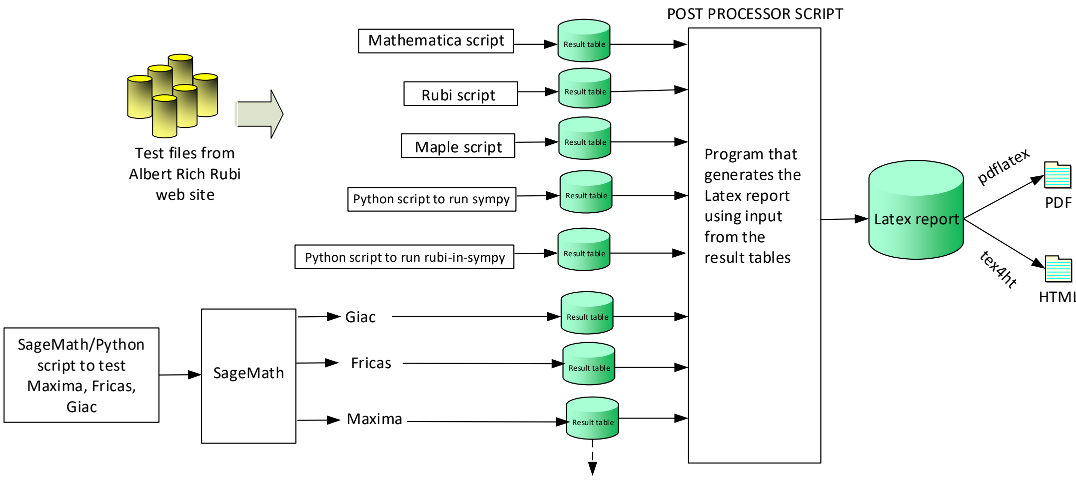
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 27, 28, 32, 33, 35, 37, 38, 41, 42, 43, 47, 48, 52, 53, 54, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70 }

B grade: { 31, 36, 55, 56 }

C grade: { 6, 24, 25, 26, 29, 30, 44, 45, 46, 49, 50, 51, 57 }

F grade: { 34, 39, 40}

2.1.3 Maple

A grade: { 5, 6, 12, 14, 23, 25, 26, 27, 28, 29, 30, 33, 35, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 54, 55, 63, 64, 65, 66, 67, 68, 69, 70 }

B grade: { 1, 2, 3, 4, 7, 8, 9, 11, 13, 16, 19, 21, 22, 24, 31, 32, 38, 53, 60, 61, 62 }

C grade: { 10, 15, 17, 18, 20, 34, 36, 37, 39, 40, 52, 56, 57, 58, 59 }

F grade: { 41 }

2.1.4 Maxima

A grade: { 5, 24, 25, 26, 27, 29, 30, 44, 45, 46, 47, 48, 49, 50, 51, 55, 60, 65, 66, 69, 70

B grade: { 1, 2, 3, 6, 7, 9, 12, 14, 21, 22, 54 }

C grade: { }

F grade: { 4, 8, 10, 11, 13, 15, 16, 17, 18, 19, 20, 23, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 52, 53, 56, 57, 58, 59, 61, 62, 63, 64, 67, 68 }

2.1.5 FriCAS

A grade: { 3, 5, 6, 9, 12, 24, 25, 26, 27, 29, 42, 43, 44, 45, 46, 47, 49, 50, 51, 65, 66, 69, 70 }

B grade: { 1, 2, 7, 14, 30 }

C grade: { }

F grade: { 4, 8, 10, 11, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 67, 68 }

2.1.6 Sympy

A grade: { 1, 2, 3, 5, 6, 7, 9, 12, 23, 24, 25, 26, 27, 44, 45, 46, 47, 65, 66, 69 }

B grade: { 49, 50, 51 }

C grade: { }

F grade: { 4, 8, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 67, 68, 70 }

2.1.7 Giac

A grade: { 5, 6, 23, 27, 29, 42, 43, 44, 45, 46, 47, 49, 50, 51, 65, 66, 69, 70 }

B grade: { 1, 2, 3, 7, 9, 14, 24, 25, 26, 30 }

C grade: { }

F grade: { 4, 8, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 67, 68 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	56	225	500	315	231	317
normalized size	1	1.	0.78	3.12	6.94	4.38	3.21	4.4
time (sec)	N/A	0.251	0.06	0.035	1.518	1.577	4.44	1.151

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	54	161	321	278	178	216
normalized size	1	1.	0.81	2.4	4.79	4.15	2.66	3.22
time (sec)	N/A	0.055	0.017	0.037	1.514	1.699	2.473	1.169

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	40	92	162	139	95	178
normalized size	1	1.	0.83	1.92	3.38	2.9	1.98	3.71
time (sec)	N/A	0.03	0.012	0.036	1.499	1.548	1.567	1.139

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	52	132	0	0	0	0
normalized size	1	1.	0.83	2.1	0.	0.	0.	0.
time (sec)	N/A	0.058	0.022	0.046	0.	0.	0.	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	50	73	124	184	2814	100
normalized size	1	1.	0.82	1.2	2.03	3.02	46.13	1.64
time (sec)	N/A	0.047	0.018	0.041	0.988	1.679	125.41	1.087

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	51	71	162	162	314	113
normalized size	1	1.	0.81	1.13	2.57	2.57	4.98	1.79
time (sec)	N/A	0.045	0.013	0.041	1.483	1.688	5.35	1.126

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	216	543	806	703	575	784
normalized size	1	1.	1.38	3.46	5.13	4.48	3.66	4.99
time (sec)	N/A	0.221	0.098	0.046	5.514	1.797	11.764	1.375

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	183	183	163	593	0	0	0	0
normalized size	1	1.	0.89	3.24	0.	0.	0.	0.
time (sec)	N/A	0.221	0.362	0.123	0.	0.	0.	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	107	220	294	335	240	294
normalized size	1	1.	1.13	2.32	3.09	3.53	2.53	3.09
time (sec)	N/A	0.119	0.058	0.046	5.202	1.723	3.045	1.238

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	170	1433	0	0	0	0
normalized size	1	1.	0.93	7.83	0.	0.	0.	0.
time (sec)	N/A	0.339	0.063	0.588	0.	0.	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	119	119	135	471	0	0	0	0
normalized size	1	1.	1.13	3.96	0.	0.	0.	0.
time (sec)	N/A	0.187	0.191	0.125	0.	0.	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	194	182	362	470	986	0
normalized size	1	1.	1.66	1.56	3.09	4.02	8.43	0.
time (sec)	N/A	0.153	0.108	0.053	1.621	1.879	6.516	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	163	547	0	0	0	0
normalized size	1	1.	0.84	2.82	0.	0.	0.	0.
time (sec)	N/A	0.257	0.606	0.125	0.	0.	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	245	242	721	937	0	1902
normalized size	1	1.	1.44	1.42	4.24	5.51	0.	11.19
time (sec)	N/A	0.227	0.271	0.054	1.879	2.094	0.	1.494

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	271	271	349	3242	0	0	0	0
normalized size	1	1.	1.29	11.96	0.	0.	0.	0.
time (sec)	N/A	0.439	0.53	0.947	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	164	164	196	567	0	0	0	0
normalized size	1	1.	1.2	3.46	0.	0.	0.	0.
time (sec)	N/A	0.243	0.267	0.132	0.	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	252	2894	0	0	0	0
normalized size	1	1.	0.9	10.37	0.	0.	0.	0.
time (sec)	N/A	0.458	0.092	0.377	0.	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	163	163	263	2696	0	0	0	0
normalized size	1	1.	1.61	16.54	0.	0.	0.	0.
time (sec)	N/A	0.3	0.449	0.4	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	180	180	225	631	0	0	0	0
normalized size	1	1.	1.25	3.51	0.	0.	0.	0.
time (sec)	N/A	0.319	0.226	0.138	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	287	287	360	7083	0	0	0	0
normalized size	1	1.	1.25	24.68	0.	0.	0.	0.
time (sec)	N/A	0.5	0.824	0.864	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	68	59	0	0	0
normalized size	1	1.	1.	2.19	1.9	0.	0.	0.
time (sec)	N/A	0.038	0.003	0.046	1.622	0.	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	34	98	166	0	0	0
normalized size	1	1.	0.83	2.39	4.05	0.	0.	0.
time (sec)	N/A	0.045	0.007	0.046	1.682	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	5.408	0.472	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	157	494	467	667	627	961
normalized size	1	1.	0.67	2.12	2.	2.86	2.69	4.12
time (sec)	N/A	0.381	0.253	0.042	1.543	1.624	10.232	3.444

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	118	283	297	432	357	562
normalized size	1	1.	0.76	1.83	1.92	2.79	2.3	3.63
time (sec)	N/A	0.19	0.139	0.041	1.532	1.466	4.998	1.516

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	163	146	157	232	177	269
normalized size	1	1.	1.68	1.51	1.62	2.39	1.82	2.77
time (sec)	N/A	0.112	0.055	0.039	1.483	1.422	2.468	1.205

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	49	42	49	119	51	49
normalized size	1	1.	1.29	1.11	1.29	3.13	1.34	1.29
time (sec)	N/A	0.018	0.013	0.033	0.973	1.515	0.566	1.082

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	162	162	160	224	0	0	0	0
normalized size	1	1.	0.99	1.38	0.	0.	0.	0.
time (sec)	N/A	0.15	0.104	0.072	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	121	205	239	435	0	393
normalized size	1	1.	0.8	1.36	1.58	2.88	0.	2.6
time (sec)	N/A	0.121	0.157	0.045	1.533	3.038	0.	1.094

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	175	438	552	1415	0	1723
normalized size	1	1.	0.77	1.93	2.43	6.23	0.	7.59
time (sec)	N/A	0.303	0.555	0.046	1.562	11.938	0.	6.762

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	382	382	801	1622	0	0	0	0
normalized size	1	1.	2.1	4.25	0.	0.	0.	0.
time (sec)	N/A	0.574	3.655	0.133	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	222	222	264	748	0	0	0	0
normalized size	1	1.	1.19	3.37	0.	0.	0.	0.
time (sec)	N/A	0.373	0.379	0.119	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	102	102	109	180	0	0	0	0
normalized size	1	1.	1.07	1.76	0.	0.	0.	0.
time (sec)	N/A	0.107	0.073	0.105	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	261	261	0	2149	0	0	0	0
normalized size	1	1.	0.	8.23	0.	0.	0.	0.
time (sec)	N/A	0.165	5.445	1.339	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	568	568	419	1087	0	0	0	0
normalized size	1	1.	0.74	1.91	0.	0.	0.	0.
time (sec)	N/A	1.351	6.315	0.132	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	564	564	1844	6682	0	0	0	0
normalized size	1	1.	3.27	11.85	0.	0.	0.	0.
time (sec)	N/A	0.937	8.962	2.95	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	337	337	592	16362	0	0	0	0
normalized size	1	1.	1.76	48.55	0.	0.	0.	0.
time (sec)	N/A	0.632	0.679	1.261	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	143	143	212	359	0	0	0	0
normalized size	1	1.	1.48	2.51	0.	0.	0.	0.
time (sec)	N/A	0.214	0.115	0.155	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	372	372	0	4389	0	0	0	0
normalized size	1	1.	0.	11.8	0.	0.	0.	0.
time (sec)	N/A	0.203	71.529	0.765	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1233	1233	0	4764	0	0	0	0
normalized size	1	1.	0.	3.86	0.	0.	0.	0.
time (sec)	N/A	2.311	70.268	1.013	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	162	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.246	0.288	1.284	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	4.445	1.105	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.405	1.097	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	95	132	140	203	155	139
normalized size	1	1.	0.9	1.25	1.32	1.92	1.46	1.31
time (sec)	N/A	0.111	0.07	0.039	1.504	1.685	2.368	1.108

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	114	95	115	161	117	111
normalized size	1	1.	1.44	1.2	1.46	2.04	1.48	1.41
time (sec)	N/A	0.092	0.048	0.036	1.503	1.749	1.432	1.093

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	90	66	92	123	78	81
normalized size	1	1.	1.5	1.1	1.53	2.05	1.3	1.35
time (sec)	N/A	0.055	0.031	0.035	1.514	1.693	0.965	1.1

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	39	36	42	97	46	42
normalized size	1	1.	1.18	1.09	1.27	2.94	1.39	1.27
time (sec)	N/A	0.011	0.014	0.035	1.008	1.748	0.559	1.118

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	120	171	103	181	0	0	0
normalized size	1	1.	1.42	0.86	1.51	0.	0.	0.
time (sec)	N/A	0.106	0.008	0.047	1.711	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	67	63	104	151	323	95
normalized size	1	1.	1.08	1.02	1.68	2.44	5.21	1.53
time (sec)	N/A	0.039	0.056	0.043	1.544	1.716	10.951	1.112

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	92	105	151	225	644	155
normalized size	1	1.	0.96	1.09	1.57	2.34	6.71	1.61
time (sec)	N/A	0.083	0.096	0.043	1.526	1.892	16.572	1.085

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	128	162	223	321	1127	239
normalized size	1	1.	0.99	1.26	1.73	2.49	8.74	1.85
time (sec)	N/A	0.114	0.136	0.043	1.543	1.653	26.956	1.101

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	863	863	701	631	0	0	0	0
normalized size	1	1.	0.81	0.73	0.	0.	0.	0.
time (sec)	N/A	1.207	0.805	0.636	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	543	543	409	2192	0	0	0	0
normalized size	1	1.	0.75	4.04	0.	0.	0.	0.
time (sec)	N/A	0.607	0.329	0.713	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	152	152	231	198	383	0	0	0
normalized size	1	1.	1.52	1.3	2.52	0.	0.	0.
time (sec)	N/A	0.138	0.02	0.054	1.974	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	244	244	771	317	383	0	0	0
normalized size	1	1.	3.16	1.3	1.57	0.	0.	0.
time (sec)	N/A	0.239	11.075	0.06	2.018	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	668	668	1536	53434	0	0	0	0
normalized size	1	1.	2.3	79.99	0.	0.	0.	0.
time (sec)	N/A	0.855	21.489	2.177	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	933	933	933	682	0	0	0	0
normalized size	1	1.	1.	0.73	0.	0.	0.	0.
time (sec)	N/A	1.367	7.074	0.658	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	673	673	604	344	0	0	0	0
normalized size	1	1.	0.9	0.51	0.	0.	0.	0.
time (sec)	N/A	0.898	0.518	0.338	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	770	770	666	377	0	0	0	0
normalized size	1	1.	0.86	0.49	0.	0.	0.	0.
time (sec)	N/A	0.982	0.719	0.318	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	283	833	443	0	0	0
normalized size	1	1.	1.03	3.04	1.62	0.	0.	0.
time (sec)	N/A	0.271	0.053	0.444	1.835	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	543	543	409	2192	0	0	0	0
normalized size	1	1.	0.75	4.04	0.	0.	0.	0.
time (sec)	N/A	0.6	0.354	0.702	0.	0.	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	367	367	443	4743	0	0	0	0
normalized size	1	1.	1.21	12.92	0.	0.	0.	0.
time (sec)	N/A	0.673	0.413	1.036	0.	0.	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	132	132	97	143	0	0	0	0
normalized size	1	1.	0.73	1.08	0.	0.	0.	0.
time (sec)	N/A	0.096	0.101	0.397	0.	0.	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	216	216	125	176	0	0	0	0
normalized size	1	1.	0.58	0.81	0.	0.	0.	0.
time (sec)	N/A	0.162	0.064	0.439	0.	0.	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	22	0	163	0	0	0	0	0
normalized size	1	0.	7.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.363	1.044	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	24	0	165	0	0	0	0	0
normalized size	1	0.	6.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	0.073	0.996	0.	0.	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	187	187	145	187	0	0	0	0
normalized size	1	1.	0.78	1.	0.	0.	0.	0.
time (sec)	N/A	0.216	0.657	0.619	0.	0.	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	281	281	189	222	0	0	0	0
normalized size	1	1.	0.67	0.79	0.	0.	0.	0.
time (sec)	N/A	0.331	0.133	1.125	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	29	0	181	0	0	0	0	0
normalized size	1	0.	6.24	0.	0.	0.	0.	0.
time (sec)	N/A	0.131	1.433	1.329	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	31	0	225	0	0	0	0	0
normalized size	1	0.	7.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.179	0.312	1.3	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [35] had the largest ratio of [1.25]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	5	1.	21	0.238
2	A	6	5	1.	21	0.238
3	A	5	5	1.	19	0.263
4	A	5	4	1.	21	0.19
5	A	7	7	1.	21	0.333
6	A	5	5	1.	21	0.238
7	A	13	9	1.	23	0.391
8	A	11	10	1.	23	0.435
9	A	8	7	1.	21	0.333
10	A	8	7	1.	23	0.304
11	A	6	6	1.	23	0.261
12	A	10	9	1.	23	0.391
13	A	10	9	1.	23	0.391
14	A	15	10	1.	23	0.435
15	A	14	11	1.	23	0.478

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
16	A	10	10	1.	21	0.476
17	A	10	8	1.	23	0.348
18	A	7	8	1.	23	0.348
19	A	9	8	1.	23	0.348
20	A	16	13	1.	23	0.565
21	A	5	4	1.	12	0.333
22	A	5	4	1.	19	0.21
23	A	0	0	0.	0	0.
24	A	7	6	1.	18	0.333
25	A	7	6	1.	18	0.333
26	A	7	6	1.	16	0.375
27	A	4	3	1.	10	0.3
28	A	5	5	1.	18	0.278
29	A	8	8	1.	18	0.444
30	A	9	8	1.	18	0.444
31	A	16	13	1.	20	0.65
32	A	13	10	1.	18	0.556
33	A	6	6	1.	12	0.5
34	A	2	2	1.	20	0.1
35	A	25	25	1.	20	1.25
36	A	21	14	1.	20	0.7
37	A	15	11	1.	18	0.611
38	A	6	7	1.	12	0.583
39	A	2	2	1.	20	0.1
40	A	35	22	1.	20	1.1
41	A	6	4	1.	18	0.222
42	A	0	0	0.	0	0.
43	A	0	0	0.	0	0.
44	A	7	6	1.	10	0.6
45	A	7	6	1.	10	0.6
46	A	7	6	1.	8	0.75
47	A	3	3	1.	6	0.5

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
48	A	5	5	1.	10	0.5
49	A	7	7	1.	10	0.7
50	A	8	7	1.	10	0.7
51	A	8	7	1.	10	0.7
52	A	23	5	1.	16	0.312
53	A	17	5	1.	16	0.312
54	A	5	5	1.	14	0.357
55	A	15	7	1.	16	0.438
56	A	25	7	1.	16	0.438
57	A	31	7	1.	16	0.438
58	A	31	13	1.	18	0.722
59	A	37	16	1.	18	0.889
60	A	17	5	1.	14	0.357
61	A	17	5	1.	16	0.312
62	A	12	8	1.	19	0.421
63	A	2	2	1.	28	0.071
64	A	3	3	1.	33	0.091
65	A	0	0	0.	0	0.
66	A	0	0	0.	0	0.
67	A	4	4	1.	35	0.114
68	A	5	5	1.	40	0.125
69	A	0	0	0.	0	0.
70	A	0	0	0.	0	0.

Chapter 3

Listing of integrals

3.1 $\int (ce+dex)^3 \left(a+b\tan^{-1}(c+dx)\right) dx$

Optimal. Leaf size=72

$$\frac{e^3(c+dx)^4\left(a+b\tan^{-1}(c+dx)\right)}{4d}-\frac{be^3(c+dx)^3}{12d}-\frac{be^3\tan^{-1}(c+dx)}{4d}+\frac{1}{4}be^3x$$

```
[Out] (b*e^3*x)/4 - (b*e^3*(c + d*x)^3)/(12*d) - (b*e^3*ArcTan[c + d*x])/(4*d) +
(e^3*(c + d*x)^4*(a + b*ArcTan[c + d*x]))/(4*d)
```

Rubi [A] time = 0.250649, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5043, 12, 4852, 302, 203}

$$\frac{e^3(c+dx)^4\left(a+b\tan^{-1}(c+dx)\right)}{4d}-\frac{be^3(c+dx)^3}{12d}-\frac{be^3\tan^{-1}(c+dx)}{4d}+\frac{1}{4}be^3x$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] (b*e^3*x)/4 - (b*e^3*(c + d*x)^3)/(12*d) - (b*e^3*ArcTan[c + d*x])/(4*d) +
(e^3*(c + d*x)^4*(a + b*ArcTan[c + d*x]))/(4*d)
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int (ce+dex)^3\left(a+b\tan^{-1}(c+dx)\right)dx &= \frac{\text{Subst}\left(\int e^3x^3\left(a+b\tan^{-1}(x)\right)dx,x,c+dx\right)}{d}\\ &= \frac{e^3\,\text{Subst}\left(\int x^3\left(a+b\tan^{-1}(x)\right)dx,x,c+dx\right)}{d}\\ &= \frac{e^3(c+dx)^4\left(a+b\tan^{-1}(c+dx)\right)}{4d}-\frac{\left(be^3\right)\text{Subst}\left(\int \frac{x^4}{1+x^2}dx,x,c+dx\right)}{4d}\\ &= \frac{e^3(c+dx)^4\left(a+b\tan^{-1}(c+dx)\right)}{4d}-\frac{\left(be^3\right)\text{Subst}\left(\int \left(-1+x^2+\frac{1}{1+x^2}\right)dx,x,c+\right.}{4d}\\ &= \frac{1}{4}be^3x-\frac{be^3(c+dx)^3}{12d}+\frac{e^3(c+dx)^4\left(a+b\tan^{-1}(c+dx)\right)}{4d}-\frac{\left(be^3\right)\text{Subst}\left(\int \frac{}{1+}\right.}{4d}\\ &= \frac{1}{4}be^3x-\frac{be^3(c+dx)^3}{12d}-\frac{be^3\tan^{-1}(c+dx)}{4d}+\frac{e^3(c+dx)^4\left(a+b\tan^{-1}(c+dx)\right)}{4d}\end{aligned}$$

Mathematica [A] time = 0.0595514, size = 56, normalized size = 0.78

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4\left(a+b\tan^{-1}(c+dx)\right)-\frac{1}{4}b\left(\frac{1}{3}(c+dx)^3+\tan^{-1}(c+dx)-dx\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] (e^3*(-(b*(-(d*x) + (c + d*x)^3/3 + ArcTan[c + d*x]))/4 + ((c + d*x)^4*(a +
 b*ArcTan[c + d*x]))/4))/d
```

Maple [B] time = 0.035, size = 225, normalized size = 3.1

$$\frac{d^3x^4ae^3}{4}+d^2x^3ace^3+\frac{3\,dx^2ac^2e^3}{2}+xac^3e^3+\frac{ac^4e^3}{4\,d}+\frac{d^3\arctan(dx+c)\,x^4be^3}{4}+d^2\arctan(dx+c)\,x^3bce^3+\frac{3\,d\,\text{arctar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arctan(d*x+c)),x)
```

```
[Out] 1/4*d^3*x^4*a*e^3+d^2*x^3*a*c*e^3+3/2*d*x^2*a*c^2*e^3+x*a*c^3*e^3+1/4/d*a*c
^4*e^3+1/4*d^3*arctan(d*x+c)*x^4*b*e^3+d^2*arctan(d*x+c)*x^3*b*c*e^3+3/2*d*
```

```
arctan(d*x+c)*x^2*b*c^2*e^3+arctan(d*x+c)*x*b*c^3*e^3+1/4/d*arctan(d*x+c)*b
*c^4*e^3-1/12*d^2*x^3*b*e^3-1/4*d*x^2*b*c*e^3-1/4*x*b*c^2*e^3-1/12/d*b*c^3*
e^3+1/4*b*e^3*x+1/4/d*b*c*e^3-1/4*b*e^3*arctan(d*x+c)/d
```

Maxima [B] time = 1.51803, size = 500, normalized size = 6.94

$$\frac{1}{4}ad^3e^3x^4 + acd^2e^3x^3 + \frac{3}{2}ac^2de^3x^2 + \frac{3}{2}\left(x^2\arctan(dx+c) - d\left(\frac{x}{d^2} + \frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3} - \frac{c\log(d^2x^2+2cdx+c^2}{d^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*a*d^3*e^3*x^4 + a*c*d^2*e^3*x^3 + 3/2*a*c^2*d*e^3*x^2 + 3/2*(x^2*arctan
(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^
2 + 2*c*d*x + c^2 + 1)/d^3))*b*c^2*d*e^3 + 1/2*(2*x^3*arctan(d*x + c) - d*(
(d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2 -
1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*b*c*d^2*e^3 + 1/12*(3*x^4*arctan(
d*x + c) - d*((d^2*x^3 - 3*c*d*x^2 + 3*(3*c^2 - 1)*x)/d^4 + 3*(c^4 - 6*c^2
+ 1)*arctan((d^2*x + c*d)/d)/d^5 - 6*(c^3 - c)*log(d^2*x^2 + 2*c*d*x + c^2
+ 1)/d^5))*b*d^3*e^3 + a*c^3*e^3*x + 1/2*(2*(d*x + c)*arctan(d*x + c) - log
((d*x + c)^2 + 1))*b*c^3*e^3/d
```

Fricas [B] time = 1.5769, size = 315, normalized size = 4.38

$$\frac{3\,ad^4e^3x^4 + (12\,ac-b)d^3e^3x^3 + 3\left(6\,ac^2-bc\right)d^2e^3x^2 + 3\left(4\,ac^3-bc^2+b\right)de^3x + 3\left(bd^4e^3x^4 + 4\,bcd^3e^3x^3 + 6\,bc^2d^2e^3x^2 + \right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*d^4*e^3*x^4 + (12*a*c - b)*d^3*e^3*x^3 + 3*(6*a*c^2 - b*c)*d^2*e^
3*x^2 + 3*(4*a*c^3 - b*c^2 + b)*d*e^3*x + 3*(b*d^4*e^3*x^4 + 4*b*c*d^3*e^3*
x^3 + 6*b*c^2*d^2*e^3*x^2 + 4*b*c^3*d*e^3*x + (b*c^4 - b)*e^3)*arctan(d*x +
 c))/d
```

Sympy [A] time = 4.43974, size = 231, normalized size = 3.21

$$\begin{cases} ac^3e^3x + \frac{3ac^2de^3x^2}{2} + acd^2e^3x^3 + \frac{ad^3e^3x^4}{4} + \frac{bc^4e^3\,\mathrm{atan}\,(c+dx)}{4d} + bc^3e^3x\,\mathrm{atan}\,(c+dx) + \frac{3bc^2de^3x^2\,\mathrm{atan}\,(c+dx)}{2} - \frac{bc^2e^3x}{4} + bcd^2e^3x^3 \\ c^3e^3x\,(a + b\,\mathrm{atan}\,(c)) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*atan(d*x+c)),x)
```

```
[Out] Piecewise((a*c**3*e**3*x + 3*a*c**2*d*e**3*x**2/2 + a*c*d**2*e**3*x**3 + a*
d**3*e**3*x**4/4 + b*c**4*e**3*atan(c + d*x)/(4*d) + b*c**3*e**3*x*atan(c +
 d*x) + 3*b*c**2*d*e**3*x**2*atan(c + d*x)/2 - b*c**2*e**3*x/4 + b*c*d**2*e
**3*x**3*atan(c + d*x) - b*c*d*e**3*x**2/4 + b*d**3*e**3*x**4*atan(c + d*x)
/4 - b*d**2*e**3*x**3/12 + b*e**3*x/4 - b*e**3*atan(c + d*x)/(4*d), Ne(d, 0
)), (c**3*e**3*x*(a + b*atan(c)), True))
```

Giac [B] time = 1.15081, size = 317, normalized size = 4.4

$$\frac{6\,bd^4x^4\arctan\left(dx+c\right)e^3 + 6\,ad^4x^4e^3 + 24\,bcd^3x^3\arctan\left(dx+c\right)e^3 + 24\,acd^3x^3e^3 + 36\,bc^2d^2x^2\arctan\left(dx+c\right)e^3 + 3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(6*b*d^4*x^4*arctan(d*x + c)*e^3 + 6*a*d^4*x^4*e^3 + 24*b*c*d^3*x^3*ar
ctan(d*x + c)*e^3 + 24*a*c*d^3*x^3*e^3 + 36*b*c^2*d^2*x^2*arctan(d*x + c)*e
^3 + 36*a*c^2*d^2*x^2*e^3 - 2*b*d^3*x^3*e^3 + 24*b*c^3*d*x*arctan(d*x + c)*
e^3 + 3*pi*b*c^4*e^3*sgn(d*x + c) - 3*pi*b*c^4*e^3 + 24*a*c^3*d*x*e^3 - 6*b
*c*d^2*x^2*e^3 - 6*b*c^4*arctan(1/(d*x + c))*e^3 - 6*b*c^2*d*x*e^3 + 6*b*d*
x*e^3 - 3*pi*b*e^3*sgn(d*x + c) + 3*pi*b*e^3 + 6*b*arctan(1/(d*x + c))*e^3)
/d
```

3.2 $\int (ce+dex)^2 \left(a+b\tan^{-1}(c+dx)\right) dx$

Optimal. Leaf size=67

$$\frac{e^2(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)}{3d}-\frac{be^2(c+dx)^2}{6d}+\frac{be^2\log\left((c+dx)^2+1\right)}{6d}$$

```
[Out] -(b*e^2*(c + d*x)^2)/(6*d) + (e^2*(c + d*x)^3*(a + b*ArcTan[c + d*x]))/(3*d
) + (b*e^2*Log[1 + (c + d*x)^2])/(6*d)
```

Rubi [A] time = 0.0547487, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5043, 12, 4852, 266, 43}

$$\frac{e^2(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)}{3d}-\frac{be^2(c+dx)^2}{6d}+\frac{be^2\log\left((c+dx)^2+1\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] -(b*e^2*(c + d*x)^2)/(6*d) + (e^2*(c + d*x)^3*(a + b*ArcTan[c + d*x]))/(3*d
) + (b*e^2*Log[1 + (c + d*x)^2])/(6*d)
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
```

```
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int (ce+dex)^2 \left(a+b\tan^{-1}(c+dx)\right) dx &= \frac{\text{Subst}\left(\int e^2x^2 \left(a+b\tan^{-1}(x)\right) dx, x, c+dx\right)}{d} \\
&= \frac{e^2\,\text{Subst}\left(\int x^2 \left(a+b\tan^{-1}(x)\right) dx, x, c+dx\right)}{d} \\
&= \frac{e^2(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)}{3d} - \frac{\left(be^2\right)\text{Subst}\left(\int \frac{x^3}{1+x^2}\,dx, x, c+dx\right)}{3d} \\
&= \frac{e^2(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)}{3d} - \frac{\left(be^2\right)\text{Subst}\left(\int \frac{x}{1+x}\,dx, x, (c+dx)^2\right)}{6d} \\
&= \frac{e^2(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)}{3d} - \frac{\left(be^2\right)\text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right) dx, x, (c+dx)^2\right.}{6d} \\
&= -\frac{be^2(c+dx)^2}{6d} + \frac{e^2(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)}{3d} + \frac{be^2\log\left(1+(c+dx)^2\right)}{6d}
\end{aligned}
$$

Mathematica [A] time = 0.016689, size = 54, normalized size = 0.81

$$
\frac{e^2\left(\frac{1}{3}(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)-\frac{1}{6}b\left((c+dx)^2-\log\left((c+dx)^2+1\right)\right)\right)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] (e^2*(((c + d*x)^3*(a + b*ArcTan[c + d*x]))/3 - (b*((c + d*x)^2 - Log[1 + (
c + d*x)^2]))/6))/d
```

Maple [B] time = 0.037, size = 161, normalized size = 2.4

$$\frac{d^2x^3ae^2}{3} + dx^2ace^2 + xac^2e^2 + \frac{ac^3e^2}{3d} + \frac{d^2\arctan(dx+c)\,x^3be^2}{3} + d\arctan(dx+c)\,x^2bce^2 + \arctan(dx+c)\,xbc^2e^2 + \frac{b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arctan(d*x+c)),x)
```

```
[Out] 1/3*d^2*x^3*a*e^2+d*x^2*a*c*e^2+x*a*c^2*e^2+1/3/d*a*c^3*e^2+1/3*d^2*arctan(
d*x+c)*x^3*b*e^2+d*arctan(d*x+c)*x^2*b*c*e^2+arctan(d*x+c)*x*b*c^2*e^2+1/3/
d*arctan(d*x+c)*b*c^3*e^2-1/6*d*x^2*b*e^2-1/3*x*b*c*e^2-1/6/d*b*c^2*e^2+1/6
*b*e^2*ln(1+(d*x+c)^2)/d
```

Maxima [B] time = 1.51354, size = 321, normalized size = 4.79

$$\frac{1}{3}ad^2e^2x^3 + acde^2x^2 + \left(x^2\arctan(dx+c) - d\left(\frac{x}{d^2} + \frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3} - \frac{c\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^3}\right)\right)bcde^2 + \frac{1}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*a*d^2*e^2*x^3 + a*c*d*e^2*x^2 + (x^2*arctan(d*x + c) - d*(x/d^2 + (c^2
- 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))
*b*c*d*e^2 + 1/6*(2*x^3*arctan(d*x + c) - d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 -
 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2
 + 1)/d^4))*b*d^2*e^2 + a*c^2*e^2*x + 1/2*(2*(d*x + c)*arctan(d*x + c) - lo
g((d*x + c)^2 + 1))*b*c^2*e^2/d
```

Fricas [B] time = 1.69925, size = 278, normalized size = 4.15

$$\frac{2\,ad^3e^2x^3 + (6\,ac-b)d^2e^2x^2 + 2\left(3\,ac^2-bc\right)de^2x + be^2\log\left(d^2x^2+2\,cdx+c^2+1\right) + 2\left(bd^3e^2x^3 + 3\,bcd^2e^2x^2 + 3\,bc^2de^2x\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*d^3*e^2*x^3 + (6*a*c - b)*d^2*e^2*x^2 + 2*(3*a*c^2 - b*c)*d*e^2*x
+ b*e^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*(b*d^3*e^2*x^3 + 3*b*c*d^2*e^2
*x^2 + 3*b*c^2*d*e^2*x + b*c^3*e^2)*arctan(d*x + c))/d
```

Sympy [A] time = 2.47253, size = 178, normalized size = 2.66

$$\begin{cases} ac^2e^2x + acde^2x^2 + \frac{ad^2e^2x^3}{3} + \frac{bc^3e^2\operatorname{atan}(c+dx)}{3d} + bc^2e^2x\operatorname{atan}(c+dx) + bcde^2x^2\operatorname{atan}(c+dx) - \frac{bce^2x}{3} + \frac{bd^2e^2x^3\operatorname{atan}(c+dx)}{3} - \\ c^2e^2x(a + b\operatorname{atan}(c)) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*atan(d*x+c)),x)
```

```
[Out] Piecewise((a*c**2*e**2*x + a*c*d*e**2*x**2 + a*d**2*e**2*x**3/3 + b*c**3*e*
*2*atan(c + d*x)/(3*d) + b*c**2*e**2*x*atan(c + d*x) + b*c*d*e**2*x**2*atan
(c + d*x) - b*c*e**2*x/3 + b*d**2*e**2*x**3*atan(c + d*x)/3 - b*d*e**2*x**2
/6 + b*e**2*log(c**2/d**2 + 2*c*x/d + x**2 + d**(-2))/(6*d), Ne(d, 0)), (c*
*2*e**2*x*(a + b*atan(c)), True))
```

Giac [B] time = 1.16889, size = 216, normalized size = 3.22

$$\frac{2\,bd^3x^3\arctan(dx+c)\,e^2 + 2\,ad^3x^3e^2 + 6\,bcd^2x^2\arctan(dx+c)\,e^2 + 6\,acd^2x^2e^2 + 6\,bc^2dx\arctan(dx+c)\,e^2 - 2\,\pi bc^3e}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*d^3*x^3*arctan(d*x + c)*e^2 + 2*a*d^3*x^3*e^2 + 6*b*c*d^2*x^2*arct
an(d*x + c)*e^2 + 6*a*c*d^2*x^2*e^2 + 6*b*c^2*d*x*arctan(d*x + c)*e^2 - 2*p
i*b*c^3*e^2*sgn(d*x + c) + 6*a*c^2*d*x*e^2 - b*d^2*x^2*e^2 + 2*b*c^3*arctan
(d*x + c)*e^2 - 2*b*c*d*x*e^2 + b*e^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

3.3 $\int (ce+dex)\left(a+b\tan^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=48

$$\frac{e(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)}{2d}+\frac{be\tan^{-1}(c+dx)}{2d}-\frac{bex}{2}$$

```
[Out] -(b*e*x)/2 + (b*e*ArcTan[c + d*x])/(2*d) + (e*(c + d*x)^2*(a + b*ArcTan[c +
 d*x]))/(2*d)
```

Rubi [A] time = 0.0302586, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5043, 12, 4852, 321, 203}

$$\frac{e(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)}{2d}+\frac{be\tan^{-1}(c+dx)}{2d}-\frac{bex}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] -(b*e*x)/2 + (b*e*ArcTan[c + d*x])/(2*d) + (e*(c + d*x)^2*(a + b*ArcTan[c +
 d*x]))/(2*d)
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
```

```
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int (ce+dex)\left(a+b\tan^{-1}(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int ex\left(a+b\tan^{-1}(x)\right)\,dx,x,c+dx\right)}{d}\\
&= \frac{e\,\text{Subst}\left(\int x\left(a+b\tan^{-1}(x)\right)\,dx,x,c+dx\right)}{d}\\
&= \frac{e(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)}{2d}-\frac{(be)\,\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx,x,c+dx\right)}{2d}\\
&= -\frac{1}{2}bex+\frac{e(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)}{2d}+\frac{(be)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,c+dx\right)}{2d}\\
&= -\frac{1}{2}bex+\frac{be\tan^{-1}(c+dx)}{2d}+\frac{e(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)}{2d}
\end{aligned}$$

Mathematica [A] time = 0.0120132, size = 40, normalized size = 0.83

$$\frac{e\left((c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)+b\left(\tan^{-1}(c+dx)-dx\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] (e*(b*(-(d*x) + ArcTan[c + d*x]) + (c + d*x)^2*(a + b*ArcTan[c + d*x])))/(2
*d)
```

Maple [B] time = 0.036, size = 92, normalized size = 1.9

$$\frac{dx^2ae}{2} + xace + \frac{ac^2e}{2d} + \frac{d\arctan(dx+c)\,x^2be}{2} + \arctan(dx+c)\,xbce + \frac{\arctan(dx+c)\,bc^2e}{2d} - \frac{bex}{2} - \frac{bce}{2d} + \frac{be\arctan(d}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arctan(d*x+c)),x)
```

```
[Out] 1/2*d*x^2*a*e+x*a*c*e+1/2/d*a*c^2*e+1/2*d*arctan(d*x+c)*x^2*b*e+arctan(d*x+
c)*x*b*c*e+1/2/d*arctan(d*x+c)*b*c^2*e-1/2*b*e*x-1/2/d*b*c*e+1/2*b*e*arctan
(d*x+c)/d
```

Maxima [B] time = 1.49881, size = 162, normalized size = 3.38

$$\frac{1}{2}\,adex^2 + \frac{1}{2}\left(x^2\arctan(dx+c) - d\left(\frac{x}{d^2} + \frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3} - \frac{c\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^3}\right)\right)bde + acex + \frac{(2\,(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*e*x^2 + 1/2*(x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2
*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*b*d*e + a*c*e*x
 + 1/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*b*c*e/d
```

Fricas [A] time = 1.54816, size = 139, normalized size = 2.9

$$\frac{ad^2ex^2 + (2\,ac-b)dex + \left(bd^2ex^2 + 2\,bcdex + \left(bc^2+b\right)e\right)\arctan(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d^2*e*x^2 + (2*a*c - b)*d*e*x + (b*d^2*e*x^2 + 2*b*c*d*e*x + (b*c^2
+ b)*e)*arctan(d*x + c))/d
```

Sympy [A] time = 1.56704, size = 95, normalized size = 1.98

$$\begin{cases} acex + \frac{adex^2}{2} + \frac{bc^2e\operatorname{atan}(c+dx)}{2d} + bcex\operatorname{atan}(c+dx) + \frac{bdex^2\operatorname{atan}(c+dx)}{2} - \frac{bex}{2} + \frac{be\operatorname{atan}(c+dx)}{2d} & \text{for } d \neq 0 \\ cex(a + b\operatorname{atan}(c)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*atan(d*x+c)),x)
```

```
[Out] Piecewise((a*c*e*x + a*d*e*x**2/2 + b*c**2*e*atan(c + d*x)/(2*d) + b*c*e*x*
atan(c + d*x) + b*d*e*x**2*atan(c + d*x)/2 - b*e*x/2 + b*e*atan(c + d*x)/(2
*d), Ne(d, 0)), (c*e*x*(a + b*atan(c)), True))
```

Giac [B] time = 1.13934, size = 178, normalized size = 3.71

$$\frac{2\,bd^2x^2\arctan(dx+c)\,e + 2\,ad^2x^2e + 4\,bcdx\arctan(dx+c)\,e + \pi bc^2e\operatorname{sgn}(dx+c) - \pi bc^2e + 4\,acdxe - 2\,bc^2\arctan\Big(\ldots}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(2*b*d^2*x^2*arctan(d*x + c)*e + 2*a*d^2*x^2*e + 4*b*c*d*x*arctan(d*x +
 c)*e + pi*b*c^2*e*sgn(d*x + c) - pi*b*c^2*e + 4*a*c*d*x*e - 2*b*c^2*arctan
(1/(d*x + c))*e - 2*b*d*x*e + pi*b*e*sgn(d*x + c) - pi*b*e - 2*b*arctan(1/(
d*x + c))*e)/d
```

3.4 $\int \frac{a+b\tan^{-1}(c+dx)}{ce+dex}\,dx$

Optimal. Leaf size=63

$$\frac{ib\text{PolyLog}(2,-i(c+dx))}{2de}-\frac{ib\text{PolyLog}(2,i(c+dx))}{2de}+\frac{a\log(c+dx)}{de}$$

```
[Out] (a*Log[c + d*x])/(d*e) + ((I/2)*b*PolyLog[2, (-I)*(c + d*x)])/(d*e) - ((I/2
)*b*PolyLog[2, I*(c + d*x)])/(d*e)
```

Rubi [A] time = 0.0579499, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {5043, 12, 4848, 2391}

$$\frac{ib\text{PolyLog}(2,-i(c+dx))}{2de}-\frac{ib\text{PolyLog}(2,i(c+dx))}{2de}+\frac{a\log(c+dx)}{de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])/(c*e + d*e*x),x]
```

```
[Out] (a*Log[c + d*x])/(d*e) + ((I/2)*b*PolyLog[2, (-I)*(c + d*x)])/(d*e) - ((I/2
)*b*PolyLog[2, I*(c + d*x)])/(d*e)
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(c+dx)}{ce+dex}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{ex}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x}\,dx, x, c+dx\right)}{de}$$
$$= \frac{a\log(c+dx)}{de} + \frac{(ib)\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, c+dx\right)}{2de} - \frac{(ib)\,\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, c+dx\right)}{2de}$$
$$= \frac{a\log(c+dx)}{de} + \frac{ib\text{Li}_2(-i(c+dx))}{2de} - \frac{ib\text{Li}_2(i(c+dx))}{2de}$$

Mathematica [A] time = 0.0216952, size = 52, normalized size = 0.83

$$\frac{\frac{1}{2}ib\text{PolyLog}(2,-i(c+dx)) - \frac{1}{2}ib\text{PolyLog}(2,i(c+dx)) + a\log(c+dx)}{de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])/(c*e + d*e*x),x]
```

```
[Out] (a*Log[c + d*x] + (I/2)*b*PolyLog[2, (-I)*(c + d*x)] - (I/2)*b*PolyLog[2, I
*(c + d*x)])/(d*e)
```

Maple [B] time = 0.046, size = 132, normalized size = 2.1

$$\frac{a\ln(dx+c)}{de} + \frac{b\ln(dx+c)\arctan(dx+c)}{de} + \frac{\frac{i}{2}b\ln(dx+c)\ln(1+i(dx+c))}{de} - \frac{\frac{i}{2}b\ln(dx+c)\ln(1-i(dx+c))}{de} + \frac{\frac{i}{2}b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))/(d*e*x+c*e),x)
```

```
[Out] a*ln(d*x+c)/d/e+1/d*b/e*ln(d*x+c)*arctan(d*x+c)+1/2*I/d*b/e*ln(d*x+c)*ln(1+
I*(d*x+c))-1/2*I/d*b/e*ln(d*x+c)*ln(1-I*(d*x+c))+1/2*I/d*b/e*dilog(1+I*(d*x
+c))-1/2*I/d*b/e*dilog(1-I*(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2 b \int \frac{\arctan(dx+c)}{2(dex+ce)}\, dx + \frac{a \log(dex+ce)}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] 2*b*integrate(1/2*arctan(d*x + c)/(d*e*x + c*e), x) + a*log(d*e*x + c*e)/(d
*e)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \arctan(dx+c)+a}{dex+ce}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(d*x + c) + a)/(d*e*x + c*e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c+dx}\, dx + \int \frac{b \operatorname{atan}(c+dx)}{c+dx}\, dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))/(d*e*x+c*e),x)
```

```
[Out] (Integral(a/(c + d*x), x) + Integral(b*atan(c + d*x)/(c + d*x), x))/e
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\arctan(dx+c)+a}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(d*x + c) + a)/(d*e*x + c*e), x)
```

3.5 $\int \frac{a+b\tan^{-1}(c+dx)}{(ce+dex)^2}\,dx$

Optimal. Leaf size=61

$$-\frac{a+b\tan^{-1}(c+dx)}{de^2(c+dx)}+\frac{b\log(c+dx)}{de^2}-\frac{b\log\left((c+dx)^2+1\right)}{2de^2}$$

```
[Out] -((a + b*ArcTan[c + d*x])/(d*e^2*(c + d*x))) + (b*Log[c + d*x])/(d*e^2) - (
b*Log[1 + (c + d*x)^2])/(2*d*e^2)
```

Rubi [A] time = 0.0472608, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5043, 12, 4852, 266, 36, 29, 31}

$$-\frac{a+b\tan^{-1}(c+dx)}{de^2(c+dx)}+\frac{b\log(c+dx)}{de^2}-\frac{b\log\left((c+dx)^2+1\right)}{2de^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])/(c*e + d*e*x)^2,x]
```

```
[Out] -((a + b*ArcTan[c + d*x])/(d*e^2*(c + d*x))) + (b*Log[c + d*x])/(d*e^2) - (
b*Log[1 + (c + d*x)^2])/(2*d*e^2)
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
```

```
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tan^{-1}(c+dx)}{(ce+dex)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{e^2x^2}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x^2}\,dx, x, c+dx\right)}{de^2} \\
&= -\frac{a+b\tan^{-1}(c+dx)}{de^2(c+dx)} + \frac{b\,\text{Subst}\left(\int \frac{1}{x(1+x^2)}\,dx, x, c+dx\right)}{de^2} \\
&= -\frac{a+b\tan^{-1}(c+dx)}{de^2(c+dx)} + \frac{b\,\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, (c+dx)^2\right)}{2de^2} \\
&= -\frac{a+b\tan^{-1}(c+dx)}{de^2(c+dx)} + \frac{b\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, (c+dx)^2\right)}{2de^2} - \frac{b\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, (c+dx)^2\right)}{2de^2} \\
&= -\frac{a+b\tan^{-1}(c+dx)}{de^2(c+dx)} + \frac{b\log(c+dx)}{de^2} - \frac{b\log\left(1+(c+dx)^2\right)}{2de^2}
\end{aligned}
$$

Mathematica [A] time = 0.0181161, size = 50, normalized size = 0.82

$$
\frac{-\frac{a+b\tan^{-1}(c+dx)}{c+dx} + b\log(c+dx) - \frac{1}{2}b\log\left((c+dx)^2+1\right)}{de^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])/(c*e + d*e*x)^2,x]
```

```
[Out] (-((a + b*ArcTan[c + d*x])/(c + d*x)) + b*Log[c + d*x] - (b*Log[1 + (c + d*
x)^2])/2)/(d*e^2)
```

Maple [A] time = 0.041, size = 73, normalized size = 1.2

$$
-\frac{a}{de^2\,(dx+c)} - \frac{b\arctan(dx+c)}{de^2\,(dx+c)} - \frac{b\ln\left(1+(dx+c)^2\right)}{2\,de^2} + \frac{b\ln(dx+c)}{de^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))/(d*e*x+c*e)^2,x)
```

```
[Out] -1/d*a/e^2/(d*x+c)-1/d*b/e^2/(d*x+c)*arctan(d*x+c)-1/2*b*ln(1+(d*x+c)^2)/d/
e^2+b*ln(d*x+c)/d/e^2
```

Maxima [A] time = 0.988298, size = 124, normalized size = 2.03

$$-\frac{1}{2}\left(d\left(\frac{\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^2e^2}-\frac{2\,\log\left(dx+c\right)}{d^2e^2}\right)+\frac{2\,\arctan\left(dx+c\right)}{d^2e^2x+cde^2}\right)b-\frac{a}{d^2e^2x+cde^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(d*(log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*e^2) - 2*log(d*x + c)/(d^2*e
^2)) + 2*arctan(d*x + c)/(d^2*e^2*x + c*d*e^2))*b - a/(d^2*e^2*x + c*d*e^2)
```

Fricas [A] time = 1.67866, size = 184, normalized size = 3.02

$$-\frac{2\,b\arctan\left(dx+c\right)+\left(bdx+bc\right)\log\left(d^2x^2+2\,cdx+c^2+1\right)-2\,\left(bdx+bc\right)\log\left(dx+c\right)+2\,a}{2\,\left(d^2e^2x+cde^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*arctan(d*x + c) + (b*d*x + b*c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)
- 2*(b*d*x + b*c)*log(d*x + c) + 2*a)/(d^2*e^2*x + c*d*e^2)
```

Sympy [A] time = 125.41, size = 2814, normalized size = 46.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))/(d*e*x+c*e)**2,x)
```

```
[Out] Piecewise((x*(a + b*atan(c))/(c**2*e**2), Eq(d, 0)), (-9*a*d**2*x**2/(9*d**
4*e**2*x**3 - 9*sqrt(3)*I*d**3*e**2*x**2 - 2*sqrt(3)*I*d*e**2) + 6*sqrt(3)*
```

```
I*a*d*x/(9*d**4*e**2*x**3 - 9*sqrt(3)*I*d**3*e**2*x**2 - 2*sqrt(3)*I*d*e**2
) - 6*a/(9*d**4*e**2*x**3 - 9*sqrt(3)*I*d**3*e**2*x**2 - 2*sqrt(3)*I*d*e**2
) + 9*b*d**3*x**3*log(x - sqrt(3)*I/(3*d))/(9*d**4*e**2*x**3 - 9*sqrt(3)*I*
d**3*e**2*x**2 - 2*sqrt(3)*I*d*e**2) - 9*b*d**3*x**3*log(x - I/d - sqrt(3)*
I/(3*d))/(9*d**4*e**2*x**3 - 9*sqrt(3)*I*d**3*e**2*x**2 - 2*sqrt(3)*I*d*e**
2) + 9*I*b*d**3*x**3*atan(d*x - sqrt(3)*I/3)/(9*d**4*e**2*x**3 - 9*sqrt(3)*
I*d**3*e**2*x**2 - 2*sqrt(3)*I*d*e**2) - 9*sqrt(3)*I*b*d**2*x**2*log(x - sq
rt(3)*I/(3*d))/(9*d**4*e**2*x**3 - 9*sqrt(3)*I*d**3*e**2*x**2 - 2*sqrt(3)*I
*d*e**2) + 9*sqrt(3)*I*b*d**2*x**2*log(x - I/d - sqrt(3)*I/(3*d))/(9*d**4*e
**2*x**3 - 9*sqrt(3)*I*d**3*e**2*x**2 - 2*sqrt(3)*I*d*e**2) - 9*b*d**2*x**2
*atan(d*x - sqrt(3)*I/3)/(9*d**4*e**2*x**3 - 9*sqrt(3)*I*d**3*e**2*x**2 - 2
*sqrt(3)*I*d*e**2) + 9*sqrt(3)*b*d**2*x**2*atan(d*x - sqrt(3)*I/3)/(9*d**4*
e**2*x**3 - 9*sqrt(3)*I*d**3*e**2*x**2 - 2*sqrt(3)*I*d*e**2) + 6*sqrt(3)*I*
b*d*x*atan(d*x - sqrt(3)*I/3)/(9*d**4*e**2*x**3 - 9*sqrt(3)*I*d**3*e**2*x**
2 - 2*sqrt(3)*I*d*e**2) - 2*sqrt(3)*I*b*log(x - sqrt(3)*I/(3*d))/(9*d**4*e*
*2*x**3 - 9*sqrt(3)*I*d**3*e**2*x**2 - 2*sqrt(3)*I*d*e**2) + 2*sqrt(3)*I*b*
log(x - I/d - sqrt(3)*I/(3*d))/(9*d**4*e**2*x**3 - 9*sqrt(3)*I*d**3*e**2*x*
*2 - 2*sqrt(3)*I*d*e**2) - 6*b*atan(d*x - sqrt(3)*I/3)/(9*d**4*e**2*x**3 -
9*sqrt(3)*I*d**3*e**2*x**2 - 2*sqrt(3)*I*d*e**2) + 2*sqrt(3)*b*atan(d*x - s
qrt(3)*I/3)/(9*d**4*e**2*x**3 - 9*sqrt(3)*I*d**3*e**2*x**2 - 2*sqrt(3)*I*d*
e**2), Eq(c, -sqrt(3)*I/3)), (-3*sqrt(3)*I*a*d**3*x**3/(9*d**4*e**2*x**3 +
9*sqrt(3)*I*d**3*e**2*x**2 + 2*sqrt(3)*I*d*e**2) - 6*sqrt(3)*I*a*d*x/(9*d**
4*e**2*x**3 + 9*sqrt(3)*I*d**3*e**2*x**2 + 2*sqrt(3)*I*d*e**2) - 4*a/(9*d**
4*e**2*x**3 + 9*sqrt(3)*I*d**3*e**2*x**2 + 2*sqrt(3)*I*d*e**2) + 9*b*d**3*x
**3*log(x + sqrt(3)*I/(3*d))/(9*d**4*e**2*x**3 + 9*sqrt(3)*I*d**3*e**2*x**2
 + 2*sqrt(3)*I*d*e**2) - 9*b*d**3*x**3*log(x - I/d + sqrt(3)*I/(3*d))/(9*d*
*4*e**2*x**3 + 9*sqrt(3)*I*d**3*e**2*x**2 + 2*sqrt(3)*I*d*e**2) + 9*I*b*d**
3*x**3*atan(d*x + sqrt(3)*I/3)/(9*d**4*e**2*x**3 + 9*sqrt(3)*I*d**3*e**2*x*
*2 + 2*sqrt(3)*I*d*e**2) + 9*sqrt(3)*I*b*d**2*x**2*log(x + sqrt(3)*I/(3*d))
/(9*d**4*e**2*x**3 + 9*sqrt(3)*I*d**3*e**2*x**2 + 2*sqrt(3)*I*d*e**2) - 9*s
qrt(3)*I*b*d**2*x**2*log(x - I/d + sqrt(3)*I/(3*d))/(9*d**4*e**2*x**3 + 9*s
qrt(3)*I*d**3*e**2*x**2 + 2*sqrt(3)*I*d*e**2) - 9*sqrt(3)*b*d**2*x**2*atan(
d*x + sqrt(3)*I/3)/(9*d**4*e**2*x**3 + 9*sqrt(3)*I*d**3*e**2*x**2 + 2*sqrt(
3)*I*d*e**2) - 9*b*d**2*x**2*atan(d*x + sqrt(3)*I/3)/(9*d**4*e**2*x**3 + 9*
sqrt(3)*I*d**3*e**2*x**2 + 2*sqrt(3)*I*d*e**2) - 6*sqrt(3)*I*b*d*x*atan(d*x
 + sqrt(3)*I/3)/(9*d**4*e**2*x**3 + 9*sqrt(3)*I*d**3*e**2*x**2 + 2*sqrt(3)*
I*d*e**2) + 2*sqrt(3)*I*b*log(x + sqrt(3)*I/(3*d))/(9*d**4*e**2*x**3 + 9*sq
rt(3)*I*d**3*e**2*x**2 + 2*sqrt(3)*I*d*e**2) - 2*sqrt(3)*I*b*log(x - I/d +
sqrt(3)*I/(3*d))/(9*d**4*e**2*x**3 + 9*sqrt(3)*I*d**3*e**2*x**2 + 2*sqrt(3)
*I*d*e**2) - 6*b*atan(d*x + sqrt(3)*I/3)/(9*d**4*e**2*x**3 + 9*sqrt(3)*I*d*
*3*e**2*x**2 + 2*sqrt(3)*I*d*e**2) - 2*sqrt(3)*b*atan(d*x + sqrt(3)*I/3)/(9
*d**4*e**2*x**3 + 9*sqrt(3)*I*d**3*e**2*x**2 + 2*sqrt(3)*I*d*e**2), Eq(c, s
qrt(3)*I/3)), (zoo*a*x, Eq(c, -d*x)), (-2*a*c**2/(6*c**3*d*e**2 + 6*c**2*d*
*2*e**2*x + 2*c*d*e**2 + 2*d**2*e**2*x) + 4*a*c*d*x/(6*c**3*d*e**2 + 6*c**2
*d**2*e**2*x + 2*c*d*e**2 + 2*d**2*e**2*x) - 2*a/(6*c**3*d*e**2 + 6*c**2*d*
```

```
*2*e**2*x + 2*c*d*e**2 + 2*d**2*e**2*x) + 6*b*c**3*log(c/d + x)/(6*c**3*d*e
**2 + 6*c**2*d**2*e**2*x + 2*c*d*e**2 + 2*d**2*e**2*x) - 3*b*c**3*log(c**2/
d**2 + 2*c*x/d + x**2 + d**(-2))/(6*c**3*d*e**2 + 6*c**2*d**2*e**2*x + 2*c*
d*e**2 + 2*d**2*e**2*x) + 6*b*c**2*d*x*log(c/d + x)/(6*c**3*d*e**2 + 6*c**2
*d**2*e**2*x + 2*c*d*e**2 + 2*d**2*e**2*x) - 3*b*c**2*d*x*log(c**2/d**2 + 2
*c*x/d + x**2 + d**(-2))/(6*c**3*d*e**2 + 6*c**2*d**2*e**2*x + 2*c*d*e**2 +
 2*d**2*e**2*x) - 6*b*c**2*atan(c + d*x)/(6*c**3*d*e**2 + 6*c**2*d**2*e**2*
x + 2*c*d*e**2 + 2*d**2*e**2*x) + 2*b*c*log(c/d + x)/(6*c**3*d*e**2 + 6*c**
2*d**2*e**2*x + 2*c*d*e**2 + 2*d**2*e**2*x) - b*c*log(c**2/d**2 + 2*c*x/d +
 x**2 + d**(-2))/(6*c**3*d*e**2 + 6*c**2*d**2*e**2*x + 2*c*d*e**2 + 2*d**2*
e**2*x) + 2*b*d*x*log(c/d + x)/(6*c**3*d*e**2 + 6*c**2*d**2*e**2*x + 2*c*d*
e**2 + 2*d**2*e**2*x) - b*d*x*log(c**2/d**2 + 2*c*x/d + x**2 + d**(-2))/(6*
c**3*d*e**2 + 6*c**2*d**2*e**2*x + 2*c*d*e**2 + 2*d**2*e**2*x) - 2*b*atan(c
 + d*x)/(6*c**3*d*e**2 + 6*c**2*d**2*e**2*x + 2*c*d*e**2 + 2*d**2*e**2*x),
True))
```

Giac [A] time = 1.08692, size = 100, normalized size = 1.64

$$-\frac{1}{2}\,b\left(\frac{e^{(-2)}\log\left(\frac{e^2}{(dxe+ce)^2}+1\right)}{d}+\frac{2\,\arctan\left(dx+c\right)e^{(-1)}}{(dxe+ce)d}\right)-\frac{ae^{(-1)}}{(dxe+ce)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] -1/2*b*(e^(-2)*log(e^2/(d*x*e + c*e)^2 + 1)/d + 2*arctan(d*x + c)*e^(-1)/((
d*x*e + c*e)*d)) - a*e^(-1)/((d*x*e + c*e)*d)
```

3.6 $\int \frac{a+b\tan^{-1}(c+dx)}{(ce+dex)^3}\,dx$

Optimal. Leaf size=63

$$-\frac{a+b\tan^{-1}(c+dx)}{2de^3(c+dx)^2}-\frac{b}{2de^3(c+dx)}-\frac{b\tan^{-1}(c+dx)}{2de^3}$$

```
[Out] -b/(2*d*e^3*(c + d*x)) - (b*ArcTan[c + d*x])/(2*d*e^3) - (a + b*ArcTan[c +
d*x])/(2*d*e^3*(c + d*x)^2)
```

Rubi [A] time = 0.0446105, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5043, 12, 4852, 325, 203}

$$-\frac{a+b\tan^{-1}(c+dx)}{2de^3(c+dx)^2}-\frac{b}{2de^3(c+dx)}-\frac{b\tan^{-1}(c+dx)}{2de^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])/(c*e + d*e*x)^3,x]
```

```
[Out] -b/(2*d*e^3*(c + d*x)) - (b*ArcTan[c + d*x])/(2*d*e^3) - (a + b*ArcTan[c +
d*x])/(2*d*e^3*(c + d*x)^2)
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
```

```
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tan^{-1}(c+dx)}{(ce+dex)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{e^3x^3}\,dx, x, c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x^3}\,dx, x, c+dx\right)}{de^3}\\
&= -\frac{a+b\tan^{-1}(c+dx)}{2de^3(c+dx)^2}+\frac{b\,\text{Subst}\left(\int \frac{1}{x^2(1+x^2)}\,dx, x, c+dx\right)}{2de^3}\\
&= -\frac{b}{2de^3(c+dx)}-\frac{a+b\tan^{-1}(c+dx)}{2de^3(c+dx)^2}-\frac{b\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, c+dx\right)}{2de^3}\\
&= -\frac{b}{2de^3(c+dx)}-\frac{b\tan^{-1}(c+dx)}{2de^3}-\frac{a+b\tan^{-1}(c+dx)}{2de^3(c+dx)^2}
\end{aligned}
$$

Mathematica [C] time = 0.0131616, size = 51, normalized size = 0.81

$$
-\frac{b(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-(c+dx)^2\right)+a+b\tan^{-1}(c+dx)}{2de^3(c+dx)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])/(c*e + d*e*x)^3,x]
```

```
[Out] -(a + b*ArcTan[c + d*x] + b*(c + d*x)*Hypergeometric2F1[-1/2, 1, 1/2, -(c +
 d*x)^2])/(2*d*e^3*(c + d*x)^2)
```

Maple [A] time = 0.041, size = 71, normalized size = 1.1

$$-\frac{a}{2\,d e^{3} \left(d x+c\right)^{2}}-\frac{b \arctan \left(d x+c\right)}{2\,d e^{3} \left(d x+c\right)^{2}}-\frac{b \arctan \left(d x+c\right)}{2\,d e^{3}}-\frac{b}{2\,d e^{3} \left(d x+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))/(d*e*x+c*e)^3,x)
```

```
[Out] -1/2/d*a/e^3/(d*x+c)^2-1/2/d*b/e^3/(d*x+c)^2*arctan(d*x+c)-1/2*b*arctan(d*x
+c)/d/e^3-1/2*b/d/e^3/(d*x+c)
```

Maxima [B] time = 1.48335, size = 162, normalized size = 2.57

$$-\frac{1}{2}\left(d\left(\frac{1}{d^{3}e^{3}x+cd^{2}e^{3}}+\frac{\arctan\left(\frac{d^{2}x+cd}{d}\right)}{d^{2}e^{3}}\right)+\frac{\arctan\left(dx+c\right)}{d^{3}e^{3}x^{2}+2\,cd^{2}e^{3}x+c^{2}de^{3}}\right)b-\frac{a}{2\left(d^{3}e^{3}x^{2}+2\,cd^{2}e^{3}x+c^{2}de^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(d*(1/(d^3*e^3*x + c*d^2*e^3) + arctan((d^2*x + c*d)/d)/(d^2*e^3)) + a
rctan(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3))*b - 1/2*a/(d^3*e^
3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

Fricas [A] time = 1.68845, size = 162, normalized size = 2.57

$$-\frac{bdx+bc+\left(bd^{2}x^{2}+2\,bcdx+bc^{2}+b\right)\arctan\left(dx+c\right)+a}{2\left(d^{3}e^{3}x^{2}+2\,cd^{2}e^{3}x+c^{2}de^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b*d*x + b*c + (b*d^2*x^2 + 2*b*c*d*x + b*c^2 + b)*arctan(d*x + c) + a
)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

Sympy [A] time = 5.35019, size = 314, normalized size = 4.98

$$\begin{cases} -\frac{a}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} - \frac{bc^2\operatorname{atan}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} - \frac{2bcdx\operatorname{atan}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} - \frac{bc}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} - \frac{bd^2x^2\operatorname{atan}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} - \frac{}{2c^2de^3} \\ \frac{x(a+b\operatorname{atan}(c))}{c^3e^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))/(d*e*x+c*e)**3,x)
```

```
[Out] Piecewise((-a/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b*c**2
*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*b*c
*d*x*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b
*c/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b*d**2*x**2*atan(
c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b*d*x/(2*c*
*2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b*atan(c + d*x)/(2*c**2*d
*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2), Ne(d, 0)), (x*(a + b*atan(c))/
(c**3*e**3), True))
```

Giac [A] time = 1.12582, size = 113, normalized size = 1.79

$$-\frac{bd^2x^2\arctan(dx+c) + 2\,bcdx\arctan(dx+c) + bc^2\arctan(dx+c) + bdx + bc + b\arctan(dx+c) + a}{2\left(d^3x^2e^3 + 2\,cd^2xe^3 + c^2de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] -1/2*(b*d^2*x^2*arctan(d*x + c) + 2*b*c*d*x*arctan(d*x + c) + b*c^2*arctan(
d*x + c) + b*d*x + b*c + b*arctan(d*x + c) + a)/(d^3*x^2*e^3 + 2*c*d^2*x*e^
3 + c^2*d*e^3)
```

3.7 $\int (ce+dex)^3 \left(a+b\tan^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=157

$$\frac{e^3(c+dx)^4\left(a+b\tan^{-1}(c+dx)\right)^2}{4d}-\frac{be^3(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)}{6d}-\frac{e^3\left(a+b\tan^{-1}(c+dx)\right)^2}{4d}+\frac{1}{2}abe^3x+\frac{b^2e^3(c+}{12d}$$

```
[Out] (a*b*e^3*x)/2 + (b^2*e^3*(c + d*x)^2)/(12*d) + (b^2*e^3*(c + d*x)*ArcTan[c
+ d*x])/(2*d) - (b*e^3*(c + d*x)^3*(a + b*ArcTan[c + d*x]))/(6*d) - (e^3*(a
 + b*ArcTan[c + d*x])^2)/(4*d) + (e^3*(c + d*x)^4*(a + b*ArcTan[c + d*x])^2
)/(4*d) - (b^2*e^3*Log[1 + (c + d*x)^2])/(3*d)
```

Rubi [A] time = 0.22059, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5043, 12, 4852, 4916, 266, 43, 4846, 260, 4884}

$$\frac{e^3(c+dx)^4\left(a+b\tan^{-1}(c+dx)\right)^2}{4d}-\frac{be^3(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)}{6d}-\frac{e^3\left(a+b\tan^{-1}(c+dx)\right)^2}{4d}+\frac{1}{2}abe^3x+\frac{b^2e^3(c+}{12d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (a*b*e^3*x)/2 + (b^2*e^3*(c + d*x)^2)/(12*d) + (b^2*e^3*(c + d*x)*ArcTan[c
+ d*x])/(2*d) - (b*e^3*(c + d*x)^3*(a + b*ArcTan[c + d*x]))/(6*d) - (e^3*(a
 + b*ArcTan[c + d*x])^2)/(4*d) + (e^3*(c + d*x)^4*(a + b*ArcTan[c + d*x])^2
)/(4*d) - (b^2*e^3*Log[1 + (c + d*x)^2])/(3*d)
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\tan^{-1}(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int e^3x^3\left(a+b\tan^{-1}(x)\right)^2 dx,x,c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int x^3\left(a+b\tan^{-1}(x)\right)^2 dx,x,c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4\left(a+b\tan^{-1}(c+dx)\right)^2}{4d} - \frac{\left(be^3\right)\text{Subst}\left(\int \frac{x^4\left(a+b\tan^{-1}(x)\right)}{1+x^2} dx,x,c+d\right.}{2d}$$

$$= \frac{e^3(c+dx)^4\left(a+b\tan^{-1}(c+dx)\right)^2}{4d} - \frac{\left(be^3\right)\text{Subst}\left(\int x^2\left(a+b\tan^{-1}(x)\right) dx,x,\right.}{2d}$$

$$= -\frac{be^3(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)}{6d} + \frac{e^3(c+dx)^4\left(a+b\tan^{-1}(c+dx)\right)^2}{4d} + \frac{(b}{}$$

$$= \frac{1}{2}abe^3x - \frac{be^3(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)}{6d} - \frac{e^3\left(a+b\tan^{-1}(c+dx)\right)^2}{4d} + \frac{e^3(}{}$$

$$= \frac{1}{2}abe^3x + \frac{b^2e^3(c+dx)\tan^{-1}(c+dx)}{2d} - \frac{be^3(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)}{6d} - \frac{e^3}{}$$

$$= \frac{1}{2}abe^3x + \frac{b^2e^3(c+dx)^2}{12d} + \frac{b^2e^3(c+dx)\tan^{-1}(c+dx)}{2d} - \frac{be^3(c+dx)^3\left(a+b\tan\right.}{6d}$$

Mathematica [A] time = 0.0980365, size = 216, normalized size = 1.38

$$\frac{e^3\left((c+dx)\left(3a^2(c+dx)^3-2ab\left(c^2+2cdx+d^2x^2-3\right)+b^2(c+dx)\right)+2b\tan^{-1}(c+dx)\left(3a\left(6c^2d^2x^2+4c^3dx+c^4+4cd\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (e^3*((c + d*x)*(b^2*(c + d*x) + 3*a^2*(c + d*x)^3 - 2*a*b*(-3 + c^2 + 2*c*
d*x + d^2*x^2)) + 2*b*(-(b*(-3*c + c^3 - 3*d*x + 3*c^2*d*x + 3*c*d^2*x^2 +
d^3*x^3)) + 3*a*(-1 + c^4 + 4*c^3*d*x + 6*c^2*d^2*x^2 + 4*c*d^3*x^3 + d^4*x
^4))*ArcTan[c + d*x] + 3*b^2*(-1 + c^4 + 4*c^3*d*x + 6*c^2*d^2*x^2 + 4*c*d^
3*x^3 + d^4*x^4)*ArcTan[c + d*x]^2 - 4*b^2*Log[1 + (c + d*x)^2]))/(12*d)
```

Maple [B] time = 0.046, size = 543, normalized size = 3.5

$$\frac{b^2c^2e^3}{12d}+\frac{a^2c^4e^3}{4d}-\frac{dx^2abce^3}{2}-\frac{d\arctan(dx+c)\,x^2b^2ce^3}{2}+\frac{d^3\arctan(dx+c)\,x^4abe^3}{2}+\frac{\arctan(dx+c)\,abc^4e^3}{2d}+d^2\,(\mathrm{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arctan(d*x+c))^2,x)
```

```
[Out] 1/12/d*b^2*c^2*e^3+1/4/d*a^2*c^4*e^3-1/2*d*x^2*a*b*c*e^3-1/2*d*arctan(d*x+c
)*x^2*b^2*c*e^3+1/2*d^3*arctan(d*x+c)*x^4*a*b*e^3+1/2/d*arctan(d*x+c)*a*b*c
^4*e^3+d^2*arctan(d*x+c)^2*x^3*b^2*c*e^3+2*d^2*arctan(d*x+c)*x^3*a*b*c*e^3+
3*d*arctan(d*x+c)*x^2*a*b*c^2*e^3-1/2*x*a*b*c^2*e^3+1/4*d^3*x^4*a^2*e^3-1/4
/d*e^3*b^2*arctan(d*x+c)^2+1/2*arctan(d*x+c)*x*b^2*e^3+1/12*d*x^2*b^2*e^3+1
/6*x*b^2*c*e^3+x*a^2*c^3*e^3-1/2*arctan(d*x+c)*x*b^2*c^2*e^3-1/6*d^2*x^3*a*
b*e^3+d^2*x^3*a^2*c*e^3+3/2*d*x^2*a^2*c^2*e^3+1/4*d^3*arctan(d*x+c)^2*x^4*b
^2*e^3-1/6*d^2*arctan(d*x+c)*x^3*b^2*e^3-1/6/d*arctan(d*x+c)*b^2*c^3*e^3-1/
2/d*e^3*a*b*arctan(d*x+c)+1/4/d*arctan(d*x+c)^2*b^2*c^4*e^3+1/2/d*arctan(d*
x+c)*b^2*c*e^3+1/2/d*a*b*c*e^3-1/3*b^2*e^3*ln(1+(d*x+c)^2)/d+arctan(d*x+c)^
2*x*b^2*c^3*e^3+3/2*d*arctan(d*x+c)^2*x^2*b^2*c^2*e^3+2*arctan(d*x+c)*x*a*b
*c^3*e^3+1/2*a*b*e^3*x-1/6/d*a*b*c^3*e^3
```

Maxima [B] time = 5.51433, size = 806, normalized size = 5.13

$$\frac{1}{4}\,a^2d^3e^3x^4+a^2cd^2e^3x^3+\frac{3}{2}\,a^2c^2de^3x^2+3\left(x^2\arctan(dx+c)-d\left(\frac{x}{d^2}+\frac{\left(c^2-1\right)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3}-\frac{c\log\left(d^2x^2+2\,cdx\right.}{d^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*d^3*e^3*x^4 + a^2*c*d^2*e^3*x^3 + 3/2*a^2*c^2*d*e^3*x^2 + 3*(x^2*ar
ctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^
2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a*b*c^2*d*e^3 + (2*x^3*arctan(d*x + c) - d
*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2
- 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*a*b*c*d^2*e^3 + 1/6*(3*x^4*arct
an(d*x + c) - d*((d^2*x^3 - 3*c*d*x^2 + 3*(3*c^2 - 1)*x)/d^4 + 3*(c^4 - 6*c
^2 + 1)*arctan((d^2*x + c*d)/d)/d^5 - 6*(c^3 - c)*log(d^2*x^2 + 2*c*d*x + c
^2 + 1)/d^5))*a*b*d^3*e^3 + a^2*c^3*e^3*x + (2*(d*x + c)*arctan(d*x + c) -
log((d*x + c)^2 + 1))*a*b*c^3*e^3/d + 1/12*(b^2*d^2*e^3*x^2 + 2*b^2*c*d*e^3
```

```
*x - 4*b^2*e^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 3*(b^2*d^4*e^3*x^4 + 4*b^
2*c*d^3*e^3*x^3 + 6*b^2*c^2*d^2*e^3*x^2 + 4*b^2*c^3*d*e^3*x + (b^2*c^4 - b^
2)*e^3)*arctan(d*x + c)^2 - 2*(b^2*d^3*e^3*x^3 + 3*b^2*c*d^2*e^3*x^2 + 3*(b
^2*c^2 - b^2)*d*e^3*x + (b^2*c^3 - 3*b^2*c)*e^3)*arctan(d*x + c))/d
```

Fricas [B] time = 1.79675, size = 703, normalized size = 4.48

$$\frac{3\,a^2d^4e^3x^4 + 2\left(6\,a^2c - ab\right)d^3e^3x^3 + \left(18\,a^2c^2 - 6\,abc + b^2\right)d^2e^3x^2 + 2\left(6\,a^2c^3 - 3\,abc^2 + b^2c + 3\,ab\right)de^3x - 4\,b^2e^3\log\left(d^2x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arctan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^2*d^4*e^3*x^4 + 2*(6*a^2*c - a*b)*d^3*e^3*x^3 + (18*a^2*c^2 - 6*a
*b*c + b^2)*d^2*e^3*x^2 + 2*(6*a^2*c^3 - 3*a*b*c^2 + b^2*c + 3*a*b)*d*e^3*x
 - 4*b^2*e^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 3*(b^2*d^4*e^3*x^4 + 4*b^2*
c*d^3*e^3*x^3 + 6*b^2*c^2*d^2*e^3*x^2 + 4*b^2*c^3*d*e^3*x + (b^2*c^4 - b^2)
*e^3)*arctan(d*x + c)^2 + 2*(3*a*b*d^4*e^3*x^4 + (12*a*b*c - b^2)*d^3*e^3*x
^3 + 3*(6*a*b*c^2 - b^2*c)*d^2*e^3*x^2 + 3*(4*a*b*c^3 - b^2*c^2 + b^2)*d*e^
3*x + (3*a*b*c^4 - b^2*c^3 + 3*b^2*c - 3*a*b)*e^3)*arctan(d*x + c))/d
```

Sympy [A] time = 11.7642, size = 575, normalized size = 3.66

$$\begin{cases} a^2c^3e^3x + \frac{3a^2c^2de^3x^2}{2} + a^2cd^2e^3x^3 + \frac{a^2d^3e^3x^4}{4} + \frac{abc^4e^3\operatorname{atan}(c+dx)}{2d} + 2abc^3e^3x\operatorname{atan}(c+dx) + 3abc^2de^3x^2\operatorname{atan}(c+dx) - \frac{abc^2e}{2} \\ c^3e^3x\left(a + b\operatorname{atan}(c)\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*atan(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c**3*e**3*x + 3*a**2*c**2*d*e**3*x**2/2 + a**2*c*d**2*e**3*
x**3 + a**2*d**3*e**3*x**4/4 + a*b*c**4*e**3*atan(c + d*x)/(2*d) + 2*a*b*c*
*3*e**3*x*atan(c + d*x) + 3*a*b*c**2*d*e**3*x**2*atan(c + d*x) - a*b*c**2*e
**3*x/2 + 2*a*b*c*d**2*e**3*x**3*atan(c + d*x) - a*b*c*d*e**3*x**2/2 + a*b*
d**3*e**3*x**4*atan(c + d*x)/2 - a*b*d**2*e**3*x**3/6 + a*b*e**3*x/2 - a*b*
e**3*atan(c + d*x)/(2*d) + b**2*c**4*e**3*atan(c + d*x)**2/(4*d) + b**2*c**
3*e**3*x*atan(c + d*x)**2 - b**2*c**3*e**3*atan(c + d*x)/(6*d) + 3*b**2*c**
```

```
2*d*e**3*x**2*atan(c + d*x)**2/2 - b**2*c**2*e**3*x*atan(c + d*x)/2 + b**2*
c*d**2*e**3*x**3*atan(c + d*x)**2 - b**2*c*d*e**3*x**2*atan(c + d*x)/2 + b*
*2*c*e**3*x/6 + b**2*c*e**3*atan(c + d*x)/(2*d) + b**2*d**3*e**3*x**4*atan(
c + d*x)**2/4 - b**2*d**2*e**3*x**3*atan(c + d*x)/6 + b**2*d*e**3*x**2/12 +
 b**2*e**3*x*atan(c + d*x)/2 - b**2*e**3*log(c**2/d**2 + 2*c*x/d + x**2 + d
**(-2))/(3*d) - b**2*e**3*atan(c + d*x)**2/(4*d), Ne(d, 0)), (c**3*e**3*x*(
a + b*atan(c))**2, True))
```

Giac [B] time = 1.37464, size = 784, normalized size = 4.99

$$\frac{3\,b^2d^4x^4\arctan\left(dx+c\right)^2e^3+6\,abd^4x^4\arctan\left(dx+c\right)e^3+12\,b^2cd^3x^3\arctan\left(dx+c\right)^2e^3+3\,a^2d^4x^4e^3+24\,abcd^3x^3\,\mathrm{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arctan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(3*b^2*d^4*x^4*arctan(d*x + c)^2*e^3 + 6*a*b*d^4*x^4*arctan(d*x + c)*e
^3 + 12*b^2*c*d^3*x^3*arctan(d*x + c)^2*e^3 + 3*a^2*d^4*x^4*e^3 + 24*a*b*c*
d^3*x^3*arctan(d*x + c)*e^3 + 18*b^2*c^2*d^2*x^2*arctan(d*x + c)^2*e^3 + 12
*a^2*c*d^3*x^3*e^3 + 36*a*b*c^2*d^2*x^2*arctan(d*x + c)*e^3 - 2*b^2*d^3*x^3
*arctan(d*x + c)*e^3 + 12*b^2*c^3*d*x*arctan(d*x + c)^2*e^3 + 18*a^2*c^2*d^
2*x^2*e^3 - 2*a*b*d^3*x^3*e^3 + 24*a*b*c^3*d*x*arctan(d*x + c)*e^3 - 6*b^2*
c*d^2*x^2*arctan(d*x + c)*e^3 + 3*b^2*c^4*arctan(d*x + c)^2*e^3 + 3*pi*a*b*
c^4*e^3*sgn(d*x + c) - 3*pi*a*b*c^4*e^3 + 12*a^2*c^3*d*x*e^3 - 6*a*b*c*d^2*
x^2*e^3 - 6*b^2*c^2*d*x*arctan(d*x + c)*e^3 - 6*a*b*c^4*arctan(1/(d*x + c))
*e^3 - pi*b^2*c^3*e^3*sgn(d*x + c) + pi*b^2*c^3*e^3 - 6*a*b*c^2*d*x*e^3 + b
^2*d^2*x^2*e^3 + 2*b^2*c^3*arctan(1/(d*x + c))*e^3 + 2*b^2*c*d*x*e^3 + 6*b^
2*d*x*arctan(d*x + c)*e^3 + 3*pi*b^2*c*e^3*sgn(d*x + c) - 3*pi*b^2*c*e^3 +
6*a*b*d*x*e^3 - 3*b^2*arctan(d*x + c)^2*e^3 - 6*b^2*c*arctan(1/(d*x + c))*e
^3 - 3*pi*a*b*e^3*sgn(d*x + c) + 3*pi*a*b*e^3 + 6*a*b*arctan(1/(d*x + c))*e
^3 - 4*b^2*e^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

3.8 $\int (ce+dex)^2 \left(a+b\tan^{-1}(c+dx)\right)^2 dx$

Optimal. Leaf size=183

$$-\frac{ib^2e^2\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{3d}+\frac{e^2(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)^2}{3d}-\frac{be^2(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)}{3d}-\frac{ie^2\left(a+b\tan\right.}{}$$

```
[Out] (b^2*e^2*x)/3 - (b^2*e^2*ArcTan[c + d*x])/(3*d) - (b*e^2*(c + d*x)^2*(a + b
*ArcTan[c + d*x]))/(3*d) - ((I/3)*e^2*(a + b*ArcTan[c + d*x])^2)/d + (e^2*(
c + d*x)^3*(a + b*ArcTan[c + d*x])^2)/(3*d) - (2*b*e^2*(a + b*ArcTan[c + d*
x])*Log[2/(1 + I*(c + d*x))])/(3*d) - ((I/3)*b^2*e^2*PolyLog[2, 1 - 2/(1 +
I*(c + d*x))])/d
```

Rubi [A] time = 0.221134, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {5043, 12, 4852, 4916, 321, 203, 4920, 4854, 2402, 2315}

$$-\frac{ib^2e^2\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{3d}+\frac{e^2(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)^2}{3d}-\frac{be^2(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)}{3d}-\frac{ie^2\left(a+b\tan\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (b^2*e^2*x)/3 - (b^2*e^2*ArcTan[c + d*x])/(3*d) - (b*e^2*(c + d*x)^2*(a + b
*ArcTan[c + d*x]))/(3*d) - ((I/3)*e^2*(a + b*ArcTan[c + d*x])^2)/d + (e^2*(
c + d*x)^3*(a + b*ArcTan[c + d*x])^2)/(3*d) - (2*b*e^2*(a + b*ArcTan[c + d*
x])*Log[2/(1 + I*(c + d*x))])/(3*d) - ((I/3)*b^2*e^2*PolyLog[2, 1 - 2/(1 +
I*(c + d*x))])/d
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\tan^{-1}(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int e^2x^2\left(a+b\tan^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int x^2\left(a+b\tan^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)^2}{3d} - \frac{\left(2be^2\right)\text{Subst}\left(\int \frac{x^3\left(a+b\tan^{-1}(x)\right)}{1+x^2} dx, x, c+
{3d}$$

$$= \frac{e^2(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)^2}{3d} - \frac{\left(2be^2\right)\text{Subst}\left(\int x\left(a+b\tan^{-1}(x)\right) dx, x,
{3d}$$

$$= -\frac{be^2(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)}{3d} - \frac{ie^2\left(a+b\tan^{-1}(c+dx)\right)^2}{3d} + \frac{e^2(c+dx)^3
$$

$$= \frac{1}{3}b^2e^2x - \frac{be^2(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)}{3d} - \frac{ie^2\left(a+b\tan^{-1}(c+dx)\right)^2}{3d} + \frac{e^2(
$$

$$= \frac{1}{3}b^2e^2x - \frac{b^2e^2\tan^{-1}(c+dx)}{3d} - \frac{be^2(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)}{3d} - \frac{ie^2\left(a+b
$$

$$= \frac{1}{3}b^2e^2x - \frac{b^2e^2\tan^{-1}(c+dx)}{3d} - \frac{be^2(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)}{3d} - \frac{ie^2\left(a+b
$$

Mathematica [A] time = 0.36222, size = 163, normalized size = 0.89

$$e^2\left(b^2\left(i\text{PolyLog}\left(2,-e^{2i\tan^{-1}(c+dx)}\right)+(c+dx)^3\tan^{-1}(c+dx)^2-(c+dx)^2\tan^{-1}(c+dx)+i\tan^{-1}(c+dx)^2-\tan^{-1}(c+
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (e^2*(a^2*(c + d*x)^3 + a*b*(-(c + d*x)^2 + 2*(c + d*x)^3*ArcTan[c + d*x] +
 Log[1 + (c + d*x)^2]) + b^2*(c + d*x - ArcTan[c + d*x] - (c + d*x)^2*ArcTa
n[c + d*x] + I*ArcTan[c + d*x]^2 + (c + d*x)^3*ArcTan[c + d*x]^2 - 2*ArcTan
[c + d*x]*Log[1 + E^((2*I)*ArcTan[c + d*x])] + I*PolyLog[2, -E^((2*I)*ArcTa
n[c + d*x])])))/(3*d)
```

Maple [B] time = 0.123, size = 593, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^2,x)
```

```
[Out] 1/6*I/d*e^2*b^2*ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))-1/6*I/d*e^2*b^2*ln(1+(d*x+c
)^2)*ln(d*x+c+I)-1/6*I/d*e^2*b^2*ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I))+1/6*I/d*e
^2*b^2*ln(1+(d*x+c)^2)*ln(d*x+c-I)+2/3*d^2*arctan(d*x+c)*x^3*a*b*e^2-1/3*d*
arctan(d*x+c)*x^2*b^2*e^2+1/3/d*arctan(d*x+c)^2*b^2*c^3*e^2-1/3/d*arctan(d*
x+c)*b^2*c^2*e^2+1/3/d*e^2*b^2*arctan(d*x+c)*ln(1+(d*x+c)^2)-2/3*x*a*b*c*e^
2+1/3/d*e^2*a*b*ln(1+(d*x+c)^2)+1/3*d^2*arctan(d*x+c)^2*x^3*b^2*e^2-1/12*I/
d*e^2*b^2*ln(d*x+c-I)^2+1/12*I/d*e^2*b^2*ln(d*x+c+I)^2-1/6*I/d*e^2*b^2*dilo
g(-1/2*I*(d*x+c+I))+1/6*I/d*e^2*b^2*dilog(1/2*I*(d*x+c-I))+1/3*b^2*e^2*x-1/
3/d*a*b*c^2*e^2+x*a^2*c^2*e^2+1/3*d^2*x^3*a^2*e^2+2*d*arctan(d*x+c)*x^2*a*b
*c*e^2+1/3/d*b^2*c*e^2+1/3/d*a^2*c^3*e^2+d*x^2*a^2*c*e^2-2/3*arctan(d*x+c)*
x*b^2*c*e^2+arctan(d*x+c)^2*x*b^2*c^2*e^2-1/3*d*x^2*a*b*e^2+2*arctan(d*x+c)
*x*a*b*c^2*e^2+2/3/d*arctan(d*x+c)*a*b*c^3*e^2+d*arctan(d*x+c)^2*x^2*b^2*c*
e^2-1/3*b^2*e^2*arctan(d*x+c)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 3/4*b^2*c^4*e^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 1/4*(3*arctan
(d*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*b^2*c^
```

```
4*e^2 + 1/3*a^2*d^2*e^2*x^3 + 36*b^2*d^4*e^2*integrate(1/48*x^4*arctan(d*x
+ c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*d^4*e^2*integrate(1/48*x^4
*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 144
*b^2*c*d^3*e^2*integrate(1/48*x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^
2 + 1), x) + 4*b^2*d^4*e^2*integrate(1/48*x^4*log(d^2*x^2 + 2*c*d*x + c^2 +
 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^2*c*d^3*e^2*integrate(1/48*x^3
*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 216
*b^2*c^2*d^2*e^2*integrate(1/48*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x +
c^2 + 1), x) + 16*b^2*c*d^3*e^2*integrate(1/48*x^3*log(d^2*x^2 + 2*c*d*x +
c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 18*b^2*c^2*d^2*e^2*integrate(1
/48*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x
) + 144*b^2*c^3*d*e^2*integrate(1/48*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x
 + c^2 + 1), x) + 24*b^2*c^2*d^2*e^2*integrate(1/48*x^2*log(d^2*x^2 + 2*c*d
*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^2*c^3*d*e^2*integrat
e(1/48*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1),
x) + 12*b^2*c^3*d*e^2*integrate(1/48*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^
2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*c^4*e^2*integrate(1/48*log(d^2*x^2 +
 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + a^2*c*d*e^2*x^2 +
 3/4*b^2*c^2*e^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 8*b^2*d^3*e^
2*integrate(1/48*x^3*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 24
*b^2*c*d^2*e^2*integrate(1/48*x^2*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2
+ 1), x) - 24*b^2*c^2*d*e^2*integrate(1/48*x*arctan(d*x + c)/(d^2*x^2 + 2*c
*d*x + c^2 + 1), x) - 1/4*(3*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^2/d -
arctan((d^2*x + c*d)/d)^3/d)*b^2*c^2*e^2 + 2*(x^2*arctan(d*x + c) - d*(x/d^
2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 +
 1)/d^3))*a*b*c*d*e^2 + 1/3*(2*x^3*arctan(d*x + c) - d*((d*x^2 - 4*c*x)/d^3
 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*
c*d*x + c^2 + 1)/d^4))*a*b*d^2*e^2 + a^2*c^2*e^2*x + 36*b^2*d^2*e^2*integra
te(1/48*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*d^2
*e^2*integrate(1/48*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d
*x + c^2 + 1), x) + 72*b^2*c*d*e^2*integrate(1/48*x*arctan(d*x + c)^2/(d^2*
x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^2*c*d*e^2*integrate(1/48*x*log(d^2*x^2 +
 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*c^2*e^2*int
egrate(1/48*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1
), x) + (2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*a*b*c^2*e^2/d
+ 1/12*(b^2*d^2*e^2*x^3 + 3*b^2*c*d*e^2*x^2 + 3*b^2*c^2*e^2*x)*arctan(d*x +
 c)^2 - 1/48*(b^2*d^2*e^2*x^3 + 3*b^2*c*d*e^2*x^2 + 3*b^2*c^2*e^2*x)*log(d^
2*x^2 + 2*c*d*x + c^2 + 1)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(a^2d^2e^2x^2 + 2\,a^2cde^2x + a^2c^2e^2 + \left(b^2d^2e^2x^2 + 2\,b^2cde^2x + b^2c^2e^2\right)\arctan\left(dx+c\right)^2 + 2\left(abd^2e^2x^2 + 2\,abcde^2x + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*d^2*e^2*x^2 + 2*a^2*c*d*e^2*x + a^2*c^2*e^2 + (b^2*d^2*e^2*x^2
 + 2*b^2*c*d*e^2*x + b^2*c^2*e^2)*arctan(d*x + c)^2 + 2*(a*b*d^2*e^2*x^2 +
2*a*b*c*d*e^2*x + a*b*c^2*e^2)*arctan(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^2\left(\int a^2c^2\,dx + \int a^2d^2x^2\,dx + \int b^2c^2\,\mathrm{atan}^2\,(c+dx)\,dx + \int 2abc^2\,\mathrm{atan}\,(c+dx)\,dx + \int 2a^2cdx\,dx + \int b^2d^2x^2\,\mathrm{atan}^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*atan(d*x+c))**2,x)
```

```
[Out] e**2*(Integral(a**2*c**2, x) + Integral(a**2*d**2*x**2, x) + Integral(b**2*
c**2*atan(c + d*x)**2, x) + Integral(2*a*b*c**2*atan(c + d*x), x) + Integra
l(2*a**2*c*d*x, x) + Integral(b**2*d**2*x**2*atan(c + d*x)**2, x) + Integra
l(2*a*b*d**2*x**2*atan(c + d*x), x) + Integral(2*b**2*c*d*x*atan(c + d*x)**
2, x) + Integral(4*a*b*c*d*x*atan(c + d*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex+ce)^2(b\arctan(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arctan(d*x + c) + a)^2, x)
```

3.9 $\int (ce+dex)\left(a+b\tan^{-1}(c+dx)\right)^2\,dx$

Optimal. Leaf size=95

$$\frac{e(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}+\frac{e\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}-abex+\frac{b^2e\log\left((c+dx)^2+1\right)}{2d}-\frac{b^2e(c+dx)\tan^{-1}(c+dx)}{d}$$

```
[Out] -(a*b*e*x) - (b^2*e*(c + d*x)*ArcTan[c + d*x])/d + (e*(a + b*ArcTan[c + d*x
])^2)/(2*d) + (e*(c + d*x)^2*(a + b*ArcTan[c + d*x])^2)/(2*d) + (b^2*e*Log[
1 + (c + d*x)^2])/(2*d)
```

Rubi [A] time = 0.119431, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5043, 12, 4852, 4916, 4846, 260, 4884}

$$\frac{e(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}+\frac{e\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}-abex+\frac{b^2e\log\left((c+dx)^2+1\right)}{2d}-\frac{b^2e(c+dx)\tan^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] -(a*b*e*x) - (b^2*e*(c + d*x)*ArcTan[c + d*x])/d + (e*(a + b*ArcTan[c + d*x
])^2)/(2*d) + (e*(c + d*x)^2*(a + b*ArcTan[c + d*x])^2)/(2*d) + (b^2*e*Log[
1 + (c + d*x)^2])/(2*d)
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int (ce+dex)\left(a+b\tan^{-1}(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int ex\left(a+b\tan^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$
$$=\frac{e\,\text{Subst}\left(\int x\left(a+b\tan^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$
$$=\frac{e(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}-\frac{(be)\,\text{Subst}\left(\int\frac{x^2\left(a+b\tan^{-1}(x)\right)}{1+x^2}\,dx,x,c+dx\right)}{d}$$
$$=\frac{e(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}-\frac{(be)\,\text{Subst}\left(\int\left(a+b\tan^{-1}(x)\right)\,dx,x,c+dx\right.}{d}$$
$$=-abex+\frac{e\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}+\frac{e(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}-\frac{\left(b^2e\right)\text{S}}{}$$
$$=-abex-\frac{b^2e(c+dx)\tan^{-1}(c+dx)}{d}+\frac{e\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}+\frac{e(c+dx)^2\left(a+\right.}{}$$
$$=-abex-\frac{b^2e(c+dx)\tan^{-1}(c+dx)}{d}+\frac{e\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}+\frac{e(c+dx)^2\left(a+\right.}{}$$

Mathematica [A] time = 0.0576755, size = 107, normalized size = 1.13

$$\frac{e\left(2b\tan^{-1}(c+dx)\left(a\left(c^2+2cdx+d^2x^2+1\right)-b(c+dx)\right)+a(c+dx)(ac+adx-2b)+b^2\left(c^2+2cdx+d^2x^2+1\right)\tan^{-1}(c\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (e*(a*(c + d*x)*(-2*b + a*c + a*d*x) + 2*b*(-(b*(c + d*x)) + a*(1 + c^2 + 2
*c*d*x + d^2*x^2))*ArcTan[c + d*x] + b^2*(1 + c^2 + 2*c*d*x + d^2*x^2)*ArcT
an[c + d*x]^2 + b^2*Log[1 + (c + d*x)^2]))/(2*d)
```

Maple [B] time = 0.046, size = 220, normalized size = 2.3

$$\frac{a^2x^2de}{2}+xa^2ce+\frac{a^2c^2e}{2\,d}+\frac{d\,(\arctan(dx+c))^2\,x^2b^2e}{2}+(\arctan(dx+c))^2\,xb^2ce+\frac{(\arctan(dx+c))^2\,b^2c^2e}{2\,d}+\frac{eb^2\,(\text{arctar}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arctan(d*x+c))^2,x)
```

```
[Out] 1/2*a^2*x^2*d*e+x*a^2*c*e+1/2/d*a^2*c^2*e+1/2*d*arctan(d*x+c)^2*x^2*b^2*e+a
rctan(d*x+c)^2*x*b^2*c*e+1/2/d*arctan(d*x+c)^2*b^2*c^2*e+1/2/d*e*b^2*arctan
(d*x+c)^2-arctan(d*x+c)*x*b^2*e-1/d*arctan(d*x+c)*b^2*c*e+1/2*b^2*e*ln(1+(d
*x+c)^2)/d+d*arctan(d*x+c)*x^2*a*b*e+2*arctan(d*x+c)*x*a*b*c*e+1/d*arctan(d
*x+c)*a*b*c^2*e+1/d*e*a*b*arctan(d*x+c)-a*b*e*x-1/d*a*b*c*e
```

Maxima [B] time = 5.20193, size = 294, normalized size = 3.09

$$\frac{1}{2}a^2dex^2 + \left(x^2\arctan(dx+c) - d\left(\frac{x}{d^2} + \frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3} - \frac{c\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^3}\right)\right)abde + a^2cex + \frac{(2(\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*d*e*x^2 + (x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x
 + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a*b*d*e + a^2*c*e
*x + (2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*a*b*c*e/d + 1/2*(
b^2*e*log(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^2*d^2*e*x^2 + 2*b^2*c*d*e*x + (
b^2*c^2 + b^2)*e)*arctan(d*x + c)^2 - 2*(b^2*d*e*x + b^2*c*e)*arctan(d*x +
c))/d
```

Fricas [A] time = 1.72326, size = 335, normalized size = 3.53

$$\frac{a^2d^2ex^2 + 2\left(a^2c - ab\right)dex + b^2e\log\left(d^2x^2+2\,cdx+c^2+1\right) + \left(b^2d^2ex^2 + 2\,b^2cdex + \left(b^2c^2+b^2\right)e\right)\arctan(dx+c)^2 + 2\cdots}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*d^2*e*x^2 + 2*(a^2*c - a*b)*d*e*x + b^2*e*log(d^2*x^2 + 2*c*d*x +
c^2 + 1) + (b^2*d^2*e*x^2 + 2*b^2*c*d*e*x + (b^2*c^2 + b^2)*e)*arctan(d*x +
 c)^2 + 2*(a*b*d^2*e*x^2 + (2*a*b*c - b^2)*d*e*x + (a*b*c^2 - b^2*c + a*b)*
e)*arctan(d*x + c))/d
```

Sympy [A] time = 3.04498, size = 240, normalized size = 2.53

$$\begin{cases} a^2cex + \frac{a^2dex^2}{2} + \frac{abc^2e\,\mathrm{atan}\,(c+dx)}{d} + 2abcex\,\mathrm{atan}\,(c+dx) + abdex^2\,\mathrm{atan}\,(c+dx) - abex + \frac{abe\,\mathrm{atan}\,(c+dx)}{d} + \frac{b^2c^2e\,\mathrm{atan}^2\,(c+dx)}{2d} + \\ cex\,(a+b\,\mathrm{atan}\,(c))^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*atan(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c*e*x + a**2*d*e*x**2/2 + a*b*c**2*e*atan(c + d*x)/d + 2*a*
b*c*e*x*atan(c + d*x) + a*b*d*e*x**2*atan(c + d*x) - a*b*e*x + a*b*e*atan(c
 + d*x)/d + b**2*c**2*e*atan(c + d*x)**2/(2*d) + b**2*c*e*x*atan(c + d*x)**
2 - b**2*c*e*atan(c + d*x)/d + b**2*d*e*x**2*atan(c + d*x)**2/2 - b**2*e*x*
atan(c + d*x) + b**2*e*log(c**2/d**2 + 2*c*x/d + x**2 + d**(-2))/(2*d) + b*
*2*e*atan(c + d*x)**2/(2*d), Ne(d, 0)), (c*e*x*(a + b*atan(c))**2, True))
```

Giac [B] time = 1.23824, size = 294, normalized size = 3.09

$$\frac{b^2d^2x^2\arctan(dx+c)^2 e + 2\,abd^2x^2\arctan(dx+c)\,e + 2\,b^2cdx\arctan(dx+c)^2 e + a^2d^2x^2e + 4\,abcdx\arctan(dx+c)\,e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^2*x^2*arctan(d*x + c)^2*e + 2*a*b*d^2*x^2*arctan(d*x + c)*e + 2*
b^2*c*d*x*arctan(d*x + c)^2*e + a^2*d^2*x^2*e + 4*a*b*c*d*x*arctan(d*x + c)
*e + b^2*c^2*arctan(d*x + c)^2*e + 2*a^2*c*d*x*e + 2*a*b*c^2*arctan(d*x + c
)*e - 2*b^2*d*x*arctan(d*x + c)*e - 2*a*b*d*x*e - 2*b^2*c*arctan(d*x + c)*e
 + b^2*arctan(d*x + c)^2*e + 2*a*b*arctan(d*x + c)*e + b^2*e*log(d^2*x^2 +
2*c*d*x + c^2 + 1))/d
```

3.10 $\int \frac{(a+b\tan^{-1}(c+dx))^2}{ce+dex}\,dx$

Optimal. Leaf size=183

$$-\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{de}+\frac{ib\text{PolyLog}\left(2,-1+\frac{2}{1+i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{de}-\frac{b^2\text{PolyLo}}{}$$

```
[Out] (2*(a + b*ArcTan[c + d*x])^2*ArcTanh[1 - 2/(1 + I*(c + d*x))])/(d*e) - (I*b
*(a + b*ArcTan[c + d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/(d*e) + (I*b*
(a + b*ArcTan[c + d*x])*PolyLog[2, -1 + 2/(1 + I*(c + d*x))])/(d*e) - (b^2*
PolyLog[3, 1 - 2/(1 + I*(c + d*x))])/(2*d*e) + (b^2*PolyLog[3, -1 + 2/(1 +
I*(c + d*x))])/(2*d*e)
```

Rubi [A] time = 0.338879, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5043, 12, 4850, 4988, 4884, 4994, 6610}

$$-\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{de}+\frac{ib\text{PolyLog}\left(2,-1+\frac{2}{1+i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{de}-\frac{b^2\text{PolyLo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x),x]
```

```
[Out] (2*(a + b*ArcTan[c + d*x])^2*ArcTanh[1 - 2/(1 + I*(c + d*x))])/(d*e) - (I*b
*(a + b*ArcTan[c + d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/(d*e) + (I*b*
(a + b*ArcTan[c + d*x])*PolyLog[2, -1 + 2/(1 + I*(c + d*x))])/(d*e) - (b^2*
PolyLog[3, 1 - 2/(1 + I*(c + d*x))])/(2*d*e) + (b^2*PolyLog[3, -1 + 2/(1 +
I*(c + d*x))])/(2*d*e)
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{ce+dex}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{ex}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{x}\,dx, x, c+dx\right)}{de}$$

$$= \frac{2\left(a+b\tan^{-1}(c+dx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} - \frac{(4b)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)\tanh^{-1}\left(1-\frac{2}{1+ix}\right.}{1+x^2}\right.}{de}$$

$$= \frac{2\left(a+b\tan^{-1}(c+dx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} - \frac{(2b)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)\log\left(2-\frac{2}{1+ix}\right)}{1+x^2}\,d\right.}{de}$$

$$= \frac{2\left(a+b\tan^{-1}(c+dx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} - \frac{ib\left(a+b\tan^{-1}(c+dx)\right)\text{Li}_2\left(1-\frac{2}{1+i(c+\right.}{de}$$

$$= \frac{2\left(a+b\tan^{-1}(c+dx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} - \frac{ib\left(a+b\tan^{-1}(c+dx)\right)\text{Li}_2\left(1-\frac{2}{1+i(c+\right.}{de}$$

Mathematica [A] time = 0.0626482, size = 170, normalized size = 0.93

$$\frac{2ib\text{PolyLog}\left(2,-\frac{c+dx+i}{c+dx-i}\right)\left(a+b\tan^{-1}(c+dx)\right) - 2ib\text{PolyLog}\left(2,\frac{c+dx+i}{c+dx-i}\right)\left(a+b\tan^{-1}(c+dx)\right) + b^2\text{PolyLog}\left(3,-\frac{c+dx+}{c+dx-}\right.}{2de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x),x]
```

```
[Out] (4*(a + b*ArcTan[c + d*x])^2*ArcTanh[(I + c + d*x)/(-I + c + d*x)] + (2*I)*
b*(a + b*ArcTan[c + d*x])*PolyLog[2, -((I + c + d*x)/(-I + c + d*x))] - (2*
I)*b*(a + b*ArcTan[c + d*x])*PolyLog[2, (I + c + d*x)/(-I + c + d*x)] + b^2
*PolyLog[3, -((I + c + d*x)/(-I + c + d*x))] - b^2*PolyLog[3, (I + c + d*x)
/(-I + c + d*x)])/(2*d*e)
```

Maple [C] time = 0.588, size = 1433, normalized size = 7.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^2/(d*e*x+c*e),x)
```

```
[Out] -1/2*I/d*b^2/e*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1))*csgn(I*((1+I*(d
*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*arctan(d*x+c
)^2-1/2*I/d*b^2/e*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c)
)^2/(1+(d*x+c)^2)+1))*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))
^2/(1+(d*x+c)^2)+1))^2*arctan(d*x+c)^2+1/2*I/d*b^2/e*Pi*csgn(I*((1+I*(d*x+c
))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(((1+I*(d*x+c)
)^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*arctan(d*x+c)^2-1/2
*I/d*b^2/e*Pi*csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(I*((1+I*(d*x+c
))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*arctan(d*x+c)^2+
1/2*I/d*b^2/e*Pi*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1
+(d*x+c)^2)+1))^3*arctan(d*x+c)^2+2/d*b^2/e*polylog(3,-(1+I*(d*x+c))/(1+(d*
x+c)^2)^(1/2))+2/d*b^2/e*polylog(3,(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))-1/2/d
*b^2/e*polylog(3,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))+I/d*b^2/e*arctan(d*x+c)*po
lylog(2,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))+2/d*a*b/e*ln(d*x+c)*arctan(d*x+c)+1
/2*I/d*b^2/e*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1))*csgn(I/((1+I*(d*x
+c))^2/(1+(d*x+c)^2)+1))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*
x+c))^2/(1+(d*x+c)^2)+1))*arctan(d*x+c)^2-I/d*a*b/e*dilog(1-I*(d*x+c))+1/2*
I/d*b^2/e*Pi*arctan(d*x+c)^2-2*I/d*b^2/e*arctan(d*x+c)*polylog(2,(1+I*(d*x+
c))/(1+(d*x+c)^2)^(1/2))-2*I/d*b^2/e*arctan(d*x+c)*polylog(2,-(1+I*(d*x+c))
/(1+(d*x+c)^2)^(1/2))+1/d*a^2/e*ln(d*x+c)+I/d*a*b/e*dilog(1+I*(d*x+c))+1/d*
b^2/e*ln(d*x+c)*arctan(d*x+c)^2-1/d*b^2/e*arctan(d*x+c)^2*ln((1+I*(d*x+c))^
2/(1+(d*x+c)^2)-1)+1/d*b^2/e*arctan(d*x+c)^2*ln(1-(1+I*(d*x+c))/(1+(d*x+c)^
2)^(1/2))+1/d*b^2/e*arctan(d*x+c)^2*ln(1+(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))
+I/d*a*b/e*ln(d*x+c)*ln(1+I*(d*x+c))-1/2*I/d*b^2/e*Pi*csgn(((1+I*(d*x+c))^2
/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*arctan(d*x+c)^2-I/d*
a*b/e*ln(d*x+c)*ln(1-I*(d*x+c))+1/2*I/d*b^2/e*Pi*csgn(I*((1+I*(d*x+c))^2/(1
+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^3*arctan(d*x+c)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2\log(dex+ce)}{de}+\int\frac{12\,b^2\arctan(dx+c)^2+b^2\log\left(d^2x^2+2\,cdx+c^2+1\right)^2+32\,ab\arctan(dx+c)}{16\,(dex+ce)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] a^2*log(d*e*x + c*e)/(d*e) + integrate(1/16*(12*b^2*arctan(d*x + c)^2 + b^2
*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 32*a*b*arctan(d*x + c))/(d*e*x + c*e)
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \arctan\left(dx+c\right)^2 + 2\,ab\arctan\left(dx+c\right) + a^2}{dex+ce}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2)/(d*e*x + c*e
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2}{c+dx}\,dx + \int \frac{b^2 \operatorname{atan}^2\left(c+dx\right)}{c+dx}\,dx + \int \frac{2ab\operatorname{atan}\left(c+dx\right)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**2/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**2/(c + d*x), x) + Integral(b**2*atan(c + d*x)**2/(c + d*x), x)
 + Integral(2*a*b*atan(c + d*x)/(c + d*x), x))/e
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\arctan\left(dx+c\right)+a\right)^2}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(d*x + c) + a)^2/(d*e*x + c*e), x)
```

3.11 $\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{(ce+dex)^2}\,dx$

Optimal. Leaf size=119

$$-\frac{ib^2\text{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)}{de^2}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{de^2(c+dx)}-\frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{de^2}+\frac{2b\log\left(2-\frac{2}{1-i(c+dx)}\right)\left(a+b\tan^{-1}\right.}{de^2}$$

```
[Out] ((-I)*(a + b*ArcTan[c + d*x])^2)/(d*e^2) - (a + b*ArcTan[c + d*x])^2/(d*e^2
*(c + d*x)) + (2*b*(a + b*ArcTan[c + d*x])*Log[2 - 2/(1 - I*(c + d*x))])/(d
*e^2) - (I*b^2*PolyLog[2, -1 + 2/(1 - I*(c + d*x))])/(d*e^2)
```

Rubi [A] time = 0.186854, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5043, 12, 4852, 4924, 4868, 2447}

$$-\frac{ib^2\text{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)}{de^2}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{de^2(c+dx)}-\frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{de^2}+\frac{2b\log\left(2-\frac{2}{1-i(c+dx)}\right)\left(a+b\tan^{-1}\right.}{de^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^2,x]
```

```
[Out] ((-I)*(a + b*ArcTan[c + d*x])^2)/(d*e^2) - (a + b*ArcTan[c + d*x])^2/(d*e^2
*(c + d*x)) + (2*b*(a + b*ArcTan[c + d*x])*Log[2 - 2/(1 - I*(c + d*x))])/(d
*e^2) - (I*b^2*PolyLog[2, -1 + 2/(1 - I*(c + d*x))])/(d*e^2)
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{(ce+dex)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{e^2x^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{x^2}\,dx, x, c+dx\right)}{de^2}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{de^2(c+dx)} + \frac{(2b)\,\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x(1+x^2)}\,dx, x, c+dx\right)}{de^2}$$

$$= -\frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{de^2} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{de^2(c+dx)} + \frac{(2ib)\,\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x(i+x)}\,dx, x, c+\right.}{de^2}$$

$$= -\frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{de^2} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{de^2(c+dx)} + \frac{2b\left(a+b\tan^{-1}(c+dx)\right)\log\left(2-\frac{}{1-}\right.}{de^2}$$

$$= -\frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{de^2} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{de^2(c+dx)} + \frac{2b\left(a+b\tan^{-1}(c+dx)\right)\log\left(2-\frac{}{1-}\right.}{de^2}$$

Mathematica [A] time = 0.190932, size = 135, normalized size = 1.13

$$\frac{-ib^2(c+dx)\text{PolyLog}\left(2, e^{2i\tan^{-1}(c+dx)}\right) + a\left(2b(c+dx)\log\left(\frac{c+dx}{\sqrt{(c+dx)^2+1}}\right)-a\right) + 2b\tan^{-1}(c+dx)\left(-a+b(c+dx)\log\left(1-e\right.\right.}{de^2(c+dx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^2,x]
```

```
[Out] ((-I)*b^2*(-I + c + d*x)*ArcTan[c + d*x]^2 + 2*b*ArcTan[c + d*x]*(-a + b*(c
 + d*x)*Log[1 - E^((2*I)*ArcTan[c + d*x])]) + a*(-a + 2*b*(c + d*x)*Log[(c
+ d*x)/Sqrt[1 + (c + d*x)^2]]) - I*b^2*(c + d*x)*PolyLog[2, E^((2*I)*ArcTan
[c + d*x])])/(d*e^2*(c + d*x))
```

Maple [B] time = 0.125, size = 471, normalized size = 4.

$$-\frac{a^2}{de^2(dx+c)} - \frac{b^2(\arctan(dx+c))^2}{de^2(dx+c)} - \frac{b^2\arctan(dx+c)\ln\left(1+(dx+c)^2\right)}{de^2} + 2\frac{b^2\ln(dx+c)\arctan(dx+c)}{de^2} - \frac{ib^2\ln(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^2,x)
```

```
[Out] -1/d*a^2/e^2/(d*x+c)-1/d*b^2/e^2/(d*x+c)*arctan(d*x+c)^2-1/d*b^2/e^2*arctan
(d*x+c)*ln(1+(d*x+c)^2)+2/d*b^2/e^2*ln(d*x+c)*arctan(d*x+c)-I/d*b^2/e^2*ln(
d*x+c)*ln(1-I*(d*x+c))-I/d*b^2/e^2*dilog(1-I*(d*x+c))+1/2*I/d*b^2/e^2*ln(d*
x+c-I)*ln(-1/2*I*(d*x+c+I))+1/2*I/d*b^2/e^2*dilog(-1/2*I*(d*x+c+I))+1/2*I/d
*b^2/e^2*ln(1+(d*x+c)^2)*ln(d*x+c+I)-1/2*I/d*b^2/e^2*dilog(1/2*I*(d*x+c-I))
+1/4*I/d*b^2/e^2*ln(d*x+c-I)^2-1/2*I/d*b^2/e^2*ln(1+(d*x+c)^2)*ln(d*x+c-I)-
1/4*I/d*b^2/e^2*ln(d*x+c+I)^2+I/d*b^2/e^2*ln(d*x+c)*ln(1+I*(d*x+c))-1/2*I/d
*b^2/e^2*ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))+I/d*b^2/e^2*dilog(1+I*(d*x+c))-2/d
*a*b/e^2/(d*x+c)*arctan(d*x+c)-1/d*a*b/e^2*ln(1+(d*x+c)^2)+2/d*a*b/e^2*ln(d
*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \arctan(dx+c)^2 + 2\,ab\arctan(dx+c) + a^2}{d^2e^2x^2 + 2\,cde^2x + c^2e^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2)/(d^2*e^2*x^2
 + 2*c*d*e^2*x + c^2*e^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2}{c^2+2cdx+d^2x^2}\,dx+\int \frac{b^2\operatorname{atan}^2(c+dx)}{c^2+2cdx+d^2x^2}\,dx+\int \frac{2ab\operatorname{atan}(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**2/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**2*atan(c + d*
x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(2*a*b*atan(c + d*x)/(c**2
 + 2*c*d*x + d**2*x**2), x))/e**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(dx+c)+a)^2}{(dex+ce)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(d*x + c) + a)^2/(d*e*x + c*e)^2, x)
```

3.12 $\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{(ce+dex)^3}\,dx$

Optimal. Leaf size=117

$$-\frac{b\left(a+b\tan^{-1}(c+dx)\right)}{de^3(c+dx)}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3}+\frac{b^2\log(c+dx)}{de^3}-\frac{b^2\log\left((c+dx)^2+1\right)}{2de^3}$$

```
[Out] -((b*(a + b*ArcTan[c + d*x]))/(d*e^3*(c + d*x))) - (a + b*ArcTan[c + d*x])^
2/(2*d*e^3) - (a + b*ArcTan[c + d*x])^2/(2*d*e^3*(c + d*x)^2) + (b^2*Log[c
+ d*x])/(d*e^3) - (b^2*Log[1 + (c + d*x)^2])/(2*d*e^3)
```

Rubi [A] time = 0.152844, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5043, 12, 4852, 4918, 266, 36, 29, 31, 4884}

$$-\frac{b\left(a+b\tan^{-1}(c+dx)\right)}{de^3(c+dx)}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3}+\frac{b^2\log(c+dx)}{de^3}-\frac{b^2\log\left((c+dx)^2+1\right)}{2de^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^3,x]
```

```
[Out] -((b*(a + b*ArcTan[c + d*x]))/(d*e^3*(c + d*x))) - (a + b*ArcTan[c + d*x])^
2/(2*d*e^3) - (a + b*ArcTan[c + d*x])^2/(2*d*e^3*(c + d*x)^2) + (b^2*Log[c
+ d*x])/(d*e^3) - (b^2*Log[1 + (c + d*x)^2])/(2*d*e^3)
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{(ce+dex)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{e^3x^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{x^3}\,dx, x, c+dx\right)}{de^3}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2} + \frac{b\,\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x^2(1+x^2)}\,dx, x, c+dx\right)}{de^3}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2} + \frac{b\,\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x^2}\,dx, x, c+dx\right)}{de^3} - \frac{b\,\text{Subst}\left(\int \frac{a+b\tan^{-1}(x}{1+x^2}\right.}{de^3}$$

$$= -\frac{b\left(a+b\tan^{-1}(c+dx)\right)}{de^3(c+dx)} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2} + \frac{b^2\,\text{Subst}}{}$$

$$= -\frac{b\left(a+b\tan^{-1}(c+dx)\right)}{de^3(c+dx)} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2} + \frac{b^2\,\text{Subst}}{}$$

$$= -\frac{b\left(a+b\tan^{-1}(c+dx)\right)}{de^3(c+dx)} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2} + \frac{b^2\,\text{Subst}}{}$$

$$= -\frac{b\left(a+b\tan^{-1}(c+dx)\right)}{de^3(c+dx)} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2} + \frac{b^2\log(c}{de^3}$$

Mathematica [A] time = 0.108117, size = 194, normalized size = 1.66

$$-\frac{a^2+2b\tan^{-1}(c+dx)\left(a\left(c^2+2cdx+d^2x^2+1\right)+b(c+dx)\right)+2abc+2abdx+b^2c^2\log\left(c^2+2cdx+d^2x^2+1\right)+b^2d^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^3,x]
```

```
[Out] -(a^2 + 2*a*b*c + 2*a*b*d*x + 2*b*(b*(c + d*x) + a*(1 + c^2 + 2*c*d*x + d^2
*x^2))*ArcTan[c + d*x] + b^2*(1 + c^2 + 2*c*d*x + d^2*x^2)*ArcTan[c + d*x]^
2 - 2*b^2*(c + d*x)^2*Log[c + d*x] + b^2*c^2*Log[1 + c^2 + 2*c*d*x + d^2*x^
2] + 2*b^2*c*d*x*Log[1 + c^2 + 2*c*d*x + d^2*x^2] + b^2*d^2*x^2*Log[1 + c^2
 + 2*c*d*x + d^2*x^2])/(2*d*e^3*(c + d*x)^2)
```

Maple [A] time = 0.053, size = 182, normalized size = 1.6

$$-\frac{a^2}{2\,de^3\,(dx+c)^2}-\frac{b^2\,(\arctan\,(dx+c))^2}{2\,de^3\,(dx+c)^2}-\frac{b^2\,(\arctan\,(dx+c))^2}{2\,de^3}-\frac{b^2\arctan\,(dx+c)}{de^3\,(dx+c)}-\frac{b^2\ln\left(1+(dx+c)^2\right)}{2\,de^3}+\frac{b^2\ln\,(dx}{de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^3,x)
```

```
[Out] -1/2/d*a^2/e^3/(d*x+c)^2-1/2/d*b^2/e^3/(d*x+c)^2*arctan(d*x+c)^2-1/2/d*b^2/
e^3*arctan(d*x+c)^2-1/d*b^2/e^3*arctan(d*x+c)/(d*x+c)-1/2*b^2*ln(1+(d*x+c)^
2)/d/e^3+b^2*ln(d*x+c)/d/e^3-1/d*a*b/e^3/(d*x+c)^2*arctan(d*x+c)-1/d*a*b/e^
3*arctan(d*x+c)-1/d*a*b/e^3/(d*x+c)
```

Maxima [B] time = 1.62124, size = 362, normalized size = 3.09

$$-\left(d\left(\frac{1}{d^3e^3x+cd^2e^3}+\frac{\arctan\left(\frac{d^2x+cd}{d}\right)}{d^2e^3}\right)+\frac{\arctan\,(dx+c)}{d^3e^3x^2+2\,cd^2e^3x+c^2de^3}\right)ab-\frac{1}{2}\left(2\,d\left(\frac{1}{d^3e^3x+cd^2e^3}+\frac{\arctan\left(\frac{d^2x+cd}{d}\right)}{d^2e^3}\right)\text{arcta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] -(d*(1/(d^3*e^3*x + c*d^2*e^3) + arctan((d^2*x + c*d)/d)/(d^2*e^3)) + arcta
n(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3))*a*b - 1/2*(2*d*(1/(d^
3*e^3*x + c*d^2*e^3) + arctan((d^2*x + c*d)/d)/(d^2*e^3))*arctan(d*x + c) -
 (arctan(d*x + c)^2 - log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*log(d*x + c))/(d
*e^3))*b^2 - 1/2*b^2*arctan(d*x + c)^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d
*e^3) - 1/2*a^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

Fricas [A] time = 1.87855, size = 470, normalized size = 4.02

$$-\frac{2\,abdx+2\,abc+\left(b^2d^2x^2+2\,b^2cdx+b^2c^2+b^2\right)\arctan\,(dx+c)^2+a^2+2\left(abd^2x^2+abc^2+b^2c+\left(2\,abc+b^2\right)dx+ab\right)}{2\left(d^3e^3x^2+2\,cd^2e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*d*x + 2*a*b*c + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + b^2)*arc
tan(d*x + c)^2 + a^2 + 2*(a*b*d^2*x^2 + a*b*c^2 + b^2*c + (2*a*b*c + b^2)*d
*x + a*b)*arctan(d*x + c) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(d^2*x
^2 + 2*c*d*x + c^2 + 1) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(d*x +
 c))/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

Sympy [A] time = 6.51563, size = 986, normalized size = 8.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**2/(d*e*x+c*e)**3,x)
```

```
[Out] Piecewise((-a**2/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*a
*b*c**2*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2)
- 4*a*b*c*d*x*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*
x**2) - 2*a*b*c/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*a*
b*d**2*x**2*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x*
*2) - 2*a*b*d*x/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*a*
b*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + 2*b*
*2*c**2*log(c/d + x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) -
 b**2*c**2*log(c**2/d**2 + 2*c*x/d + x**2 + d**(-2))/(2*c**2*d*e**3 + 4*c*d
**2*e**3*x + 2*d**3*e**3*x**2) - b**2*c**2*atan(c + d*x)**2/(2*c**2*d*e**3
+ 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + 4*b**2*c*d*x*log(c/d + x)/(2*c**2*d
*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*b**2*c*d*x*log(c**2/d**2 +
2*c*x/d + x**2 + d**(-2))/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x*
*2) - 2*b**2*c*d*x*atan(c + d*x)**2/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d*
*3*e**3*x**2) - 2*b**2*c*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2
*d**3*e**3*x**2) + 2*b**2*d**2*x**2*log(c/d + x)/(2*c**2*d*e**3 + 4*c*d**2*
e**3*x + 2*d**3*e**3*x**2) - b**2*d**2*x**2*log(c**2/d**2 + 2*c*x/d + x**2
+ d**(-2))/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b**2*d**2
*x**2*atan(c + d*x)**2/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2)
 - 2*b**2*d*x*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*
x**2) - b**2*atan(c + d*x)**2/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**
3*x**2), Ne(d, 0)), (x*(a + b*atan(c))**2/(c**3*e**3), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(dx+c)+a)^2}{(dex+ce)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(d*x + c) + a)^2/(d*e*x + c*e)^3, x)
```

3.13 $\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{(ce+dex)^4}\,dx$

Optimal. Leaf size=194

$$\frac{ib^2\text{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)}{3de^4}-\frac{b\left(a+b\tan^{-1}(c+dx)\right)}{3de^4(c+dx)^2}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{3de^4(c+dx)^3}+\frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{3de^4}-2b\log($$

```
[Out] -b^2/(3*d*e^4*(c + d*x)) - (b^2*ArcTan[c + d*x])/(3*d*e^4) - (b*(a + b*ArcT
an[c + d*x]))/(3*d*e^4*(c + d*x)^2) + ((I/3)*(a + b*ArcTan[c + d*x])^2)/(d*
e^4) - (a + b*ArcTan[c + d*x])^2/(3*d*e^4*(c + d*x)^3) - (2*b*(a + b*ArcTan
[c + d*x])*Log[2 - 2/(1 - I*(c + d*x))])/(3*d*e^4) + ((I/3)*b^2*PolyLog[2,
-1 + 2/(1 - I*(c + d*x))])/(d*e^4)
```

Rubi [A] time = 0.257446, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5043, 12, 4852, 4918, 325, 203, 4924, 4868, 2447}

$$\frac{ib^2\text{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)}{3de^4}-\frac{b\left(a+b\tan^{-1}(c+dx)\right)}{3de^4(c+dx)^2}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{3de^4(c+dx)^3}+\frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{3de^4}-2b\log($$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^4,x]
```

```
[Out] -b^2/(3*d*e^4*(c + d*x)) - (b^2*ArcTan[c + d*x])/(3*d*e^4) - (b*(a + b*ArcT
an[c + d*x]))/(3*d*e^4*(c + d*x)^2) + ((I/3)*(a + b*ArcTan[c + d*x])^2)/(d*
e^4) - (a + b*ArcTan[c + d*x])^2/(3*d*e^4*(c + d*x)^3) - (2*b*(a + b*ArcTan
[c + d*x])*Log[2 - 2/(1 - I*(c + d*x))])/(3*d*e^4) + ((I/3)*b^2*PolyLog[2,
-1 + 2/(1 - I*(c + d*x))])/(d*e^4)
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
```

```
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{(ce+dex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{e^4x^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{x^4}\,dx, x, c+dx\right)}{de^4}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{3de^4(c+dx)^3} + \frac{(2b)\,\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x^3\left(1+x^2\right)}\,dx, x, c+dx\right)}{3de^4}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{3de^4(c+dx)^3} + \frac{(2b)\,\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x^3}\,dx, x, c+dx\right)}{3de^4} - \frac{(2b)\,\text{Subst}\left(\int \frac{a+b}{x(}\right.}{3}$$

$$= -\frac{b\left(a+b\tan^{-1}(c+dx)\right)}{3de^4(c+dx)^2} + \frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{3de^4} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{3de^4(c+dx)^3} - \frac{(2ib)\,\text{Su}}{}$$

$$= -\frac{b^2}{3de^4(c+dx)} - \frac{b\left(a+b\tan^{-1}(c+dx)\right)}{3de^4(c+dx)^2} + \frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{3de^4} - \frac{\left(a+b\tan^{-1}(c+\right.}{3de^4(c+dx)^3}$$

$$= -\frac{b^2}{3de^4(c+dx)} - \frac{b^2\tan^{-1}(c+dx)}{3de^4} - \frac{b\left(a+b\tan^{-1}(c+dx)\right)}{3de^4(c+dx)^2} + \frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{3de^4}$$

Mathematica [A] time = 0.606005, size = 163, normalized size = 0.84

$$-\frac{-ib^2\text{PolyLog}\left(2, e^{2i\tan^{-1}(c+dx)}\right) + \frac{a^2}{(c+dx)^3} + \frac{ab}{(c+dx)^2} + 2ab\log\left(\frac{c+dx}{\sqrt{(c+dx)^2+1}}\right) + b\tan^{-1}(c+dx)\left(\frac{2a}{(c+dx)^3} + \frac{b}{(c+dx)^2} + 2b\log\left(\right.\right.}{3de^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^4,x]
```

```
[Out] -(a*b + a^2/(c + d*x)^3 + (a*b)/(c + d*x)^2 + b^2/(c + d*x) + b^2*(-I + (c
+ d*x)^(-3))*ArcTan[c + d*x]^2 + b*ArcTan[c + d*x]*(b + (2*a)/(c + d*x)^3 +
 b/(c + d*x)^2 + 2*b*Log[1 - E^((2*I)*ArcTan[c + d*x])]) + 2*a*b*Log[(c + d
*x)/Sqrt[1 + (c + d*x)^2]] - I*b^2*PolyLog[2, E^((2*I)*ArcTan[c + d*x])])/(
3*d*e^4)
```

Maple [B] time = 0.125, size = 547, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^4,x)
```

```
[Out] -1/3/d*a^2/e^4/(d*x+c)^3-1/3/d*b^2/e^4/(d*x+c)^3*arctan(d*x+c)^2+1/3/d*b^2/
e^4*arctan(d*x+c)*ln(1+(d*x+c)^2)-1/3/d*b^2/e^4*arctan(d*x+c)/(d*x+c)^2-2/3
/d*b^2/e^4*ln(d*x+c)*arctan(d*x+c)-1/6*I/d*b^2/e^4*ln(d*x+c+I)*ln(1+(d*x+c)
^2)-1/6*I/d*b^2/e^4*ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I))-1/3*I/d*b^2/e^4*dilog(
1+I*(d*x+c))+1/3*I/d*b^2/e^4*dilog(1-I*(d*x+c))+1/3*I/d*b^2/e^4*ln(d*x+c)*l
n(1-I*(d*x+c))+1/6*I/d*b^2/e^4*dilog(1/2*I*(d*x+c-I))+1/12*I/d*b^2/e^4*ln(d
*x+c+I)^2-1/12*I/d*b^2/e^4*ln(d*x+c-I)^2-1/3*b^2*arctan(d*x+c)/d/e^4-1/3*b^
2/d/e^4/(d*x+c)+1/6*I/d*b^2/e^4*ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))-1/3*I/d*b^2
/e^4*ln(d*x+c)*ln(1+I*(d*x+c))-1/6*I/d*b^2/e^4*dilog(-1/2*I*(d*x+c+I))+1/6*
I/d*b^2/e^4*ln(d*x+c-I)*ln(1+(d*x+c)^2)-2/3/d*a*b/e^4/(d*x+c)^3*arctan(d*x+
c)+1/3/d*a*b/e^4*ln(1+(d*x+c)^2)-1/3/d*a*b/e^4/(d*x+c)^2-2/3/d*a*b/e^4*ln(d
*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\arctan(dx+c)^2+2\,ab\arctan(dx+c)+a^2}{d^4e^4x^4+4\,cd^3e^4x^3+6\,c^2d^2e^4x^2+4\,c^3de^4x+c^4e^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2)/(d^4*e^4*x^4
 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a^2}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int\frac{b^2\operatorname{atan}^2(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int\frac{2ab\operatorname{atan}(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**2/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(b**2*atan(c + d*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*d**
2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(2*a*b*atan(c + d*x)/(c**
4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\arctan(dx+c)+a)^2}{(dex+ce)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(d*x + c) + a)^2/(d*e*x + c*e)^4, x)
```

3.14 $\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{(ce+dex)^5} dx$

Optimal. Leaf size=170

$$\frac{b\left(a+b\tan^{-1}(c+dx)\right)}{2de^5(c+dx)}-\frac{b\left(a+b\tan^{-1}(c+dx)\right)}{6de^5(c+dx)^3}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{4de^5(c+dx)^4}+\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{4de^5}-\frac{b^2}{12de^5(c+dx)^2}-$$

```
[Out] -b^2/(12*d*e^5*(c + d*x)^2) - (b*(a + b*ArcTan[c + d*x]))/(6*d*e^5*(c + d*x
)^3) + (b*(a + b*ArcTan[c + d*x]))/(2*d*e^5*(c + d*x)) + (a + b*ArcTan[c +
d*x])^2/(4*d*e^5) - (a + b*ArcTan[c + d*x])^2/(4*d*e^5*(c + d*x)^4) - (2*b^
2*Log[c + d*x])/(3*d*e^5) + (b^2*Log[1 + (c + d*x)^2])/(3*d*e^5)
```

Rubi [A] time = 0.226746, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {5043, 12, 4852, 4918, 266, 44, 36, 29, 31, 4884}

$$\frac{b\left(a+b\tan^{-1}(c+dx)\right)}{2de^5(c+dx)}-\frac{b\left(a+b\tan^{-1}(c+dx)\right)}{6de^5(c+dx)^3}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{4de^5(c+dx)^4}+\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{4de^5}-\frac{b^2}{12de^5(c+dx)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^5,x]
```

```
[Out] -b^2/(12*d*e^5*(c + d*x)^2) - (b*(a + b*ArcTan[c + d*x]))/(6*d*e^5*(c + d*x
)^3) + (b*(a + b*ArcTan[c + d*x]))/(2*d*e^5*(c + d*x)) + (a + b*ArcTan[c +
d*x])^2/(4*d*e^5) - (a + b*ArcTan[c + d*x])^2/(4*d*e^5*(c + d*x)^4) - (2*b^
2*Log[c + d*x])/(3*d*e^5) + (b^2*Log[1 + (c + d*x)^2])/(3*d*e^5)
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{(ce+dex)^5}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{e^5x^5}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{x^5}\,dx, x, c+dx\right)}{de^5}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{4de^5(c+dx)^4} + \frac{b\,\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x^4(1+x^2)}\,dx, x, c+dx\right)}{2de^5}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{4de^5(c+dx)^4} + \frac{b\,\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x^4}\,dx, x, c+dx\right)}{2de^5} - \frac{b\,\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x^2(1+x^2)}\right.}{2de^5}$$

$$= -\frac{b\left(a+b\tan^{-1}(c+dx)\right)}{6de^5(c+dx)^3} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{4de^5(c+dx)^4} - \frac{b\,\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x^2}\,dx, x, c+dx\right)}{2de^5}$$

$$= -\frac{b\left(a+b\tan^{-1}(c+dx)\right)}{6de^5(c+dx)^3} + \frac{b\left(a+b\tan^{-1}(c+dx)\right)}{2de^5(c+dx)} + \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{4de^5} - \frac{\left(a+b\tan\right.}{4de^5(}$$

$$= -\frac{b\left(a+b\tan^{-1}(c+dx)\right)}{6de^5(c+dx)^3} + \frac{b\left(a+b\tan^{-1}(c+dx)\right)}{2de^5(c+dx)} + \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{4de^5} - \frac{\left(a+b\tan\right.}{4de^5(}$$

$$= -\frac{b^2}{12de^5(c+dx)^2} - \frac{b\left(a+b\tan^{-1}(c+dx)\right)}{6de^5(c+dx)^3} + \frac{b\left(a+b\tan^{-1}(c+dx)\right)}{2de^5(c+dx)} + \frac{\left(a+b\tan^{-1}(c+\right.}{4de^5}$$

$$= -\frac{b^2}{12de^5(c+dx)^2} - \frac{b\left(a+b\tan^{-1}(c+dx)\right)}{6de^5(c+dx)^3} + \frac{b\left(a+b\tan^{-1}(c+dx)\right)}{2de^5(c+dx)} + \frac{\left(a+b\tan^{-1}(c+\right.}{4de^5}$$

Mathematica [A] time = 0.270523, size = 245, normalized size = 1.44

$$-\frac{3a^2 - 2b\tan^{-1}(c+dx)\left(3a\left(6c^2d^2x^2+4c^3dx+c^4+4cd^3x^3+d^4x^4-1\right)+b\left(9c^2dx+3c^3+9cd^2x^2-c+3d^3x^3-dx\right)\right)-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^5,x]
```

```
[Out] -(3*a^2 + 2*a*b*(c + d*x) + b^2*(c + d*x)^2 - 6*a*b*(c + d*x)^3 - 2*b*(b*(-
c + 3*c^3 - d*x + 9*c^2*d*x + 9*c*d^2*x^2 + 3*d^3*x^3) + 3*a*(-1 + c^4 + 4*
c^3*d*x + 6*c^2*d^2*x^2 + 4*c*d^3*x^3 + d^4*x^4))*ArcTan[c + d*x] - 3*b^2*(
-1 + c^4 + 4*c^3*d*x + 6*c^2*d^2*x^2 + 4*c*d^3*x^3 + d^4*x^4)*ArcTan[c + d*
x]^2 + 8*b^2*(c + d*x)^4*Log[c + d*x] - 4*b^2*(c + d*x)^4*Log[1 + c^2 + 2*c
*d*x + d^2*x^2])/(12*d*e^5*(c + d*x)^4)
```

Maple [A] time = 0.054, size = 242, normalized size = 1.4

$$-\frac{a^2}{4\,de^5\,(dx+c)^4}-\frac{b^2\,(\arctan(dx+c))^2}{4\,de^5\,(dx+c)^4}+\frac{b^2\,(\arctan(dx+c))^2}{4\,de^5}-\frac{b^2\arctan(dx+c)}{6\,de^5\,(dx+c)^3}+\frac{b^2\arctan(dx+c)}{2\,de^5\,(dx+c)}+\frac{b^2\ln\left(1+\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^5,x)
```

```
[Out] -1/4/d*a^2/e^5/(d*x+c)^4-1/4/d*b^2/e^5/(d*x+c)^4*arctan(d*x+c)^2+1/4/d*b^2/
e^5*arctan(d*x+c)^2-1/6/d*b^2/e^5*arctan(d*x+c)/(d*x+c)^3+1/2/d*b^2/e^5*arc
tan(d*x+c)/(d*x+c)+1/3*b^2*ln(1+(d*x+c)^2)/d/e^5-1/12*b^2/d/e^5/(d*x+c)^2-2
/3*b^2*ln(d*x+c)/d/e^5-1/2/d*a*b/e^5/(d*x+c)^4*arctan(d*x+c)+1/2/d*a*b/e^5*
arctan(d*x+c)-1/6/d*a*b/e^5/(d*x+c)^3+1/2/d*a*b/e^5/(d*x+c)
```

Maxima [B] time = 1.8787, size = 721, normalized size = 4.24

$$\frac{1}{6}\left(d\left(\frac{3\,d^2x^2+6\,cdx+3\,c^2-1}{d^5e^5x^3+3\,cd^4e^5x^2+3\,c^2d^3e^5x+c^3d^2e^5}+\frac{3\arctan\left(\frac{d^2x+cd}{d}\right)}{d^2e^5}\right)-\frac{3\arctan(dx+c)}{d^5e^5x^4+4\,cd^4e^5x^3+6\,c^2d^3e^5x^2+4\,c^3d^2e^5x+c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^5,x, algorithm="maxima")
```

```
[Out] 1/6*(d*((3*d^2*x^2 + 6*c*d*x + 3*c^2 - 1)/(d^5*e^5*x^3 + 3*c*d^4*e^5*x^2 +
3*c^2*d^3*e^5*x + c^3*d^2*e^5) + 3*arctan((d^2*x + c*d)/d)/(d^2*e^5)) - 3*a
rctan(d*x + c)/(d^5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d
^2*e^5*x + c^4*d*e^5))*a*b + 1/12*(2*d*((3*d^2*x^2 + 6*c*d*x + 3*c^2 - 1)/(
```

```
d^5*e^5*x^3 + 3*c*d^4*e^5*x^2 + 3*c^2*d^3*e^5*x + c^3*d^2*e^5) + 3*arctan((
d^2*x + c*d)/d)/(d^2*e^5))*arctan(d*x + c) - (3*(d^2*x^2 + 2*c*d*x + c^2)*a
rctan(d*x + c)^2 - 4*(d^2*x^2 + 2*c*d*x + c^2)*log(d^2*x^2 + 2*c*d*x + c^2
+ 1) + 8*(d^2*x^2 + 2*c*d*x + c^2)*log(d*x + c) + 1)*d^2/(d^5*e^5*x^2 + 2*c
*d^4*e^5*x + c^2*d^3*e^5))*b^2 - 1/4*b^2*arctan(d*x + c)^2/(d^5*e^5*x^4 + 4
*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d^2*e^5*x + c^4*d*e^5) - 1/4*a^2
/(d^5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d^2*e^5*x + c^4
*d*e^5)
```

Fricas [B] time = 2.09413, size = 937, normalized size = 5.51

$$\frac{6\,abd^3x^3 + 6\,abc^3 + \left(18\,abc - b^2\right)d^2x^2 - b^2c^2 - 2\,abc + 2\left(9\,abc^2 - b^2c - ab\right)dx + 3\left(b^2d^4x^4 + 4\,b^2cd^3x^3 + 6\,b^2c^2d^2x^2 + 4\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(6*a*b*d^3*x^3 + 6*a*b*c^3 + (18*a*b*c - b^2)*d^2*x^2 - b^2*c^2 - 2*a*
b*c + 2*(9*a*b*c^2 - b^2*c - a*b)*d*x + 3*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 +
6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 - b^2)*arctan(d*x + c)^2 - 3*a^
2 + 2*(3*a*b*d^4*x^4 + 3*(4*a*b*c + b^2)*d^3*x^3 + 3*a*b*c^4 + 3*b^2*c^3 +
9*(2*a*b*c^2 + b^2*c)*d^2*x^2 - b^2*c + (12*a*b*c^3 + 9*b^2*c^2 - b^2)*d*x
- 3*a*b)*arctan(d*x + c) + 4*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2
*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*log(d^2*x^2 + 2*c*d*x + c^2 + 1) - 8*(b^2*d
^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*log
(d*x + c))/(d^5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d^2*e
^5*x + c^4*d*e^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**2/(d*e*x+c*e)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.49382, size = 1902, normalized size = 11.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^5,x, algorithm="giac")
```

```
[Out] 1/24*(6*pi^2*b^2*floor(-1/2*(pi*sgn(1/(d*x*e + c*e))*sgn(d)^2 - 2*arctan((d
*x*e + c*e)*e^(-1)))/pi)*sgn(1/(d*x*e + c*e))*sgn(d)^2 + 3*pi^2*b^2*sgn(1/(
d*x*e + c*e))*sgn(d)^2 - 6*pi*b^2*arctan((d*x*e + c*e)*e^(-1))*sgn(1/(d*x*e
 + c*e))*sgn(d)^2 + 6*pi^2*b^2*floor(-1/2*(pi*sgn(1/(d*x*e + c*e))*sgn(d)^2
 - 2*arctan((d*x*e + c*e)*e^(-1)))/pi)^2 + 12*pi*a*b*sgn(1/(d*x*e + c*e))*s
gn(d)^2 - 6*pi*b^2*e*sgn(1/(d*x*e + c*e))*sgn(d)^2/(d*x*e + c*e) + 6*pi^2*b
^2*floor(-1/2*(pi*sgn(1/(d*x*e + c*e))*sgn(d)^2 - 2*arctan((d*x*e + c*e)*e^
(-1)))/pi) - 12*pi*b^2*arctan((d*x*e + c*e)*e^(-1))*floor(-1/2*(pi*sgn(1/(d
*x*e + c*e))*sgn(d)^2 - 2*arctan((d*x*e + c*e)*e^(-1)))/pi) - 6*pi^2*b^2*e^
4*floor(-1/2*(pi*sgn(1/(d*x*e + c*e))*sgn(d)^2 - 2*arctan((d*x*e + c*e)*e^(
-1)))/pi)*sgn(1/(d*x*e + c*e))*sgn(d)^2/(d*x*e + c*e)^4 + 3*pi^2*b^2 - 6*pi
*b^2*arctan((d*x*e + c*e)*e^(-1)) + 6*b^2*arctan((d*x*e + c*e)*e^(-1))^2 -
12*pi*b^2*e*floor(-1/2*(pi*sgn(1/(d*x*e + c*e))*sgn(d)^2 - 2*arctan((d*x*e
+ c*e)*e^(-1)))/pi)/(d*x*e + c*e) - 3*pi^2*b^2*e^4*sgn(1/(d*x*e + c*e))*sgn
(d)^2/(d*x*e + c*e)^4 + 6*pi*b^2*arctan((d*x*e + c*e)*e^(-1))*e^4*sgn(1/(d*
x*e + c*e))*sgn(d)^2/(d*x*e + c*e)^4 + 2*pi*b^2*e^3*sgn(1/(d*x*e + c*e))*sg
n(d)^2/(d*x*e + c*e)^3 - 12*a*b*arctan(e/(d*x*e + c*e)) - 6*pi*b^2*e/(d*x*e
 + c*e) + 12*b^2*arctan((d*x*e + c*e)*e^(-1))*e/(d*x*e + c*e) - 6*pi^2*b^2*
e^4*floor(-1/2*(pi*sgn(1/(d*x*e + c*e))*sgn(d)^2 - 2*arctan((d*x*e + c*e)*e
^(-1)))/pi)^2/(d*x*e + c*e)^4 + 8*b^2*log(e^2/(d*x*e + c*e)^2 + 1) + 6*pi*a
*b*e^4*sgn(1/(d*x*e + c*e))*sgn(d)^2/(d*x*e + c*e)^4 + 12*a*b*e/(d*x*e + c*
e) - 6*pi^2*b^2*e^4*floor(-1/2*(pi*sgn(1/(d*x*e + c*e))*sgn(d)^2 - 2*arctan
((d*x*e + c*e)*e^(-1)))/pi)/(d*x*e + c*e)^4 + 12*pi*b^2*arctan((d*x*e + c*e
)*e^(-1))*e^4*floor(-1/2*(pi*sgn(1/(d*x*e + c*e))*sgn(d)^2 - 2*arctan((d*x*
e + c*e)*e^(-1)))/pi)/(d*x*e + c*e)^4 + 4*pi*b^2*e^3*floor(-1/2*(pi*sgn(1/(
d*x*e + c*e))*sgn(d)^2 - 2*arctan((d*x*e + c*e)*e^(-1)))/pi)/(d*x*e + c*e)^
3 - 3*pi^2*b^2*e^4/(d*x*e + c*e)^4 + 6*pi*b^2*arctan((d*x*e + c*e)*e^(-1))*
e^4/(d*x*e + c*e)^4 - 6*b^2*arctan((d*x*e + c*e)*e^(-1))^2*e^4/(d*x*e + c*e
)^4 + 2*pi*b^2*e^3/(d*x*e + c*e)^3 - 4*b^2*arctan((d*x*e + c*e)*e^(-1))*e^3
/(d*x*e + c*e)^3 - 2*b^2*e^2/(d*x*e + c*e)^2 + 12*pi*a*b*e^4*floor(-1/2*(pi
*sgn(1/(d*x*e + c*e))*sgn(d)^2 - 2*arctan((d*x*e + c*e)*e^(-1)))/pi)/(d*x*e
 + c*e)^4 + 6*pi*a*b*e^4/(d*x*e + c*e)^4 - 12*a*b*arctan((d*x*e + c*e)*e^(-
1))*e^4/(d*x*e + c*e)^4 - 4*a*b*e^3/(d*x*e + c*e)^3 - 6*a^2*e^4/(d*x*e + c*
e)^4)*e^(-5)/d
```

3.15 $\int (ce+dex)^2 \left(a+b\tan^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=271

$$-\frac{ib^2e^2\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{d}-\frac{b^3e^2\text{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d}+ab^2e^2x-\frac{be^2\left(a+b\tan^{-1}(c+\right.}{2d}$$

```
[Out] a*b^2*e^2*x + (b^3*e^2*(c + d*x)*ArcTan[c + d*x])/d - (b*e^2*(a + b*ArcTan[
c + d*x])^2)/(2*d) - (b*e^2*(c + d*x)^2*(a + b*ArcTan[c + d*x])^2)/(2*d) -
((I/3)*e^2*(a + b*ArcTan[c + d*x])^3)/d + (e^2*(c + d*x)^3*(a + b*ArcTan[c
+ d*x])^3)/(3*d) - (b*e^2*(a + b*ArcTan[c + d*x])^2*Log[2/(1 + I*(c + d*x))
])/d - (b^3*e^2*Log[1 + (c + d*x)^2])/(2*d) - (I*b^2*e^2*(a + b*ArcTan[c +
d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d - (b^3*e^2*PolyLog[3, 1 - 2/(1
 + I*(c + d*x))])/(2*d)
```

Rubi [A] time = 0.438706, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {5043, 12, 4852, 4916, 4846, 260, 4884, 4920, 4854, 4994, 6610}

$$-\frac{ib^2e^2\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{d}-\frac{b^3e^2\text{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d}+ab^2e^2x-\frac{be^2\left(a+b\tan^{-1}(c+\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] a*b^2*e^2*x + (b^3*e^2*(c + d*x)*ArcTan[c + d*x])/d - (b*e^2*(a + b*ArcTan[
c + d*x])^2)/(2*d) - (b*e^2*(c + d*x)^2*(a + b*ArcTan[c + d*x])^2)/(2*d) -
((I/3)*e^2*(a + b*ArcTan[c + d*x])^3)/d + (e^2*(c + d*x)^3*(a + b*ArcTan[c
+ d*x])^3)/(3*d) - (b*e^2*(a + b*ArcTan[c + d*x])^2*Log[2/(1 + I*(c + d*x))
])/d - (b^3*e^2*Log[1 + (c + d*x)^2])/(2*d) - (I*b^2*e^2*(a + b*ArcTan[c +
d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d - (b^3*e^2*PolyLog[3, 1 - 2/(1
 + I*(c + d*x))])/(2*d)
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\tan^{-1}(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int e^2x^2\left(a+b\tan^{-1}(x)\right)^3\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int x^2\left(a+b\tan^{-1}(x)\right)^3\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)^3}{3d} - \frac{\left(be^2\right)\text{Subst}\left(\int \frac{x^3\left(a+b\tan^{-1}(x)\right)^2}{1+x^2}\,dx, x, c+\right.}{d}$$

$$= \frac{e^2(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)^3}{3d} - \frac{\left(be^2\right)\text{Subst}\left(\int x\left(a+b\tan^{-1}(x)\right)^2\,dx, x\right.}{d}$$

$$= -\frac{be^2(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d} - \frac{ie^2\left(a+b\tan^{-1}(c+dx)\right)^3}{3d} + \frac{e^2(c+d}{}$$

$$= -\frac{be^2(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d} - \frac{ie^2\left(a+b\tan^{-1}(c+dx)\right)^3}{3d} + \frac{e^2(c+d}{}$$

$$= ab^2e^2x - \frac{be^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d} - \frac{be^2(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d} -$$

$$= ab^2e^2x + \frac{b^3e^2(c+dx)\tan^{-1}(c+dx)}{d} - \frac{be^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d} - \frac{be^2(c+}{}$$

$$= ab^2e^2x + \frac{b^3e^2(c+dx)\tan^{-1}(c+dx)}{d} - \frac{be^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d} - \frac{be^2(c+}{}$$

Mathematica [A] time = 0.529795, size = 349, normalized size = 1.29

$$\frac{e^2\left(6ab^2\left(i\text{PolyLog}\left(2,-e^{2i\tan^{-1}(c+dx)}\right)+(c+dx)^3\tan^{-1}(c+dx)^2-(c+dx)^2\tan^{-1}(c+dx)+i\tan^{-1}(c+dx)^2-\tan^{-1}(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (e^2*(-3*a^2*b*(c + d*x)^2 + 2*a^3*(c + d*x)^3 + 6*a^2*b*(c + d*x)^3*ArcTan
[c + d*x] + 3*a^2*b*Log[1 + (c + d*x)^2] + 6*a*b^2*(c + d*x - ArcTan[c + d*
x] - (c + d*x)^2*ArcTan[c + d*x] + I*ArcTan[c + d*x]^2 + (c + d*x)^3*ArcTan
```

```
[c + d*x]^2 - 2*ArcTan[c + d*x]*Log[1 + E^((2*I)*ArcTan[c + d*x])] + I*Poly
Log[2, -E^((2*I)*ArcTan[c + d*x])]) + b^3*(6*(c + d*x)*ArcTan[c + d*x] - 3*
(1 + (c + d*x)^2)*ArcTan[c + d*x]^2 + (2*I)*ArcTan[c + d*x]^3 - 2*(c + d*x)
*ArcTan[c + d*x]^3 + 2*(c + d*x)*(1 + (c + d*x)^2)*ArcTan[c + d*x]^3 - 6*Ar
cTan[c + d*x]^2*Log[1 + E^((2*I)*ArcTan[c + d*x])] + 6*Log[1/Sqrt[1 + (c +
d*x)^2]] + (6*I)*ArcTan[c + d*x]*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])] - 3
*PolyLog[3, -E^((2*I)*ArcTan[c + d*x])])))/(6*d)
```

Maple [C] time = 0.947, size = 3242, normalized size = 12.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^3,x)
```

```
[Out] -1/4*arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+I)*csgn(I*(1+I
*(d*x+c))^4/(1+(d*x+c)^2)^2+2*I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+I)^2*x*b^3*e^
2-1/8/d*arctan(d*x+c)^2*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^3*b^
3*c*e^2+1/8/d*arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))^4/(1+(d*x+c)^2)^2+2*I
*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+I)^3*b^3*c*e^2+3*d*arctan(d*x+c)*x^2*a^2*b*c
*e^2+3*d*arctan(d*x+c)^2*x^2*a*b^2*c*e^2-1/8*I/d*e^2*b^3*arctan(d*x+c)^2*Pi
*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^3+1/4*I/d*e^2*b^3*arctan(d*x+c
)^2*Pi*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))^3+1/4*I/d*e^2*b^3*arctan(d*x+c
)^2*Pi*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+
1)^2)^3-1/8*I/d*e^2*b^3*arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))^4/(1+(d*x+c
)^2)^2+2*I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+I)^3-1/8/d*arctan(d*x+c)^2*Pi*csgn
(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^
2)+1))^2*b^3*c*e^2-1/4/d*arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+
c)^2)+I)*csgn(I*(1+I*(d*x+c))^4/(1+(d*x+c)^2)^2+2*I*(1+I*(d*x+c))^2/(1+(d*x
+c)^2)+I)^2*b^3*c*e^2+1/4/d*arctan(d*x+c)^2*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(
d*x+c)^2)+1)^2)^2*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*b^3*c*e^2+1/4*I
/d*e^2*b^3*arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))*csgn(I*
(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))^2+1/8/d*arctan(d*x+c)^2*Pi*csgn(I*(1+I*(
d*x+c))^2/(1+(d*x+c)^2)+I)^2*csgn(I*(1+I*(d*x+c))^4/(1+(d*x+c)^2)^2+2*I*(1+
I*(d*x+c))^2/(1+(d*x+c)^2)+I)*b^3*c*e^2-1/4*I/d*e^2*b^3*arctan(d*x+c)^2*Pi*
csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/
((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^2-1/4*I/d*e^2*b^3*arctan(d*x+c)^2*Pi*c
sgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^2*
csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)-1/2*I/d*e^2*b^3*arctan(d*x+c)^2
*Pi*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2*csgn(I*(1+I*(d*x+c))/(1+(d*x+c)
^2)^(1/2))+1/4*I/d*e^2*b^3*arctan(d*x+c)^2*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d
```

```
*x+c)^2)+1))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^2-1/8*I/d*e^2*b^3*
arctan(d*x+c)^2*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*csgn(I*((1+I
*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)-1/8*I/d*e^2*b^3*arctan(d*x+c)^2*Pi*csgn(I*(
1+I*(d*x+c))^2/(1+(d*x+c)^2)+I)^2*csgn(I*(1+I*(d*x+c))^4/(1+(d*x+c)^2)^2+2*
I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+I)+1/4*I/d*e^2*b^3*arctan(d*x+c)^2*Pi*csgn(
I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+I)*csgn(I*(1+I*(d*x+c))^4/(1+(d*x+c)^2)^2+2
*I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+I)^2+d*arctan(d*x+c)^3*x^2*b^3*c*e^2+arcta
n(d*x+c)^3*x*b^3*c^2*e^2+1/3/d*arctan(d*x+c)^3*b^3*c^3*e^2-1/2*d*arctan(d*x
+c)^2*x^2*b^3*e^2+1/3*d^2*arctan(d*x+c)^3*x^3*b^3*e^2-1/d*e^2*a*b^2*arctan(
d*x+c)+1/2/d*e^2*a^2*b*ln(1+(d*x+c)^2)-1/d*e^2*b^3*arctan(d*x+c)^2*ln(2)+1/
2/d*e^2*b^3*arctan(d*x+c)^2*ln(1+(d*x+c)^2)-1/d*e^2*b^3*arctan(d*x+c)^2*ln(
(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+1/d*arctan(d*x+c)*b^3*c*e^2+1/4*I/d*e^2*
b^3*arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))*csgn(I*(1+I*(d
*x+c))^2/(1+(d*x+c)^2)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)*csgn(I/((1+I*(d
*x+c))^2/(1+(d*x+c)^2)+1)^2)-1/2/d*a^2*b*c^2*e^2+1/d*a*b^2*c*e^2-1/2/d*arct
an(d*x+c)^2*b^3*c^2*e^2-arctan(d*x+c)^2*x*b^3*c*e^2+1/3*I/d*e^2*b^3*arctan(
d*x+c)^3-I/d*e^2*b^3*arctan(d*x+c)-1/2*d*x^2*a^2*b*e^2+d*x^2*a^3*c*e^2+x*a^
3*c^2*e^2+1/3*d^2*x^3*a^3*e^2+arctan(d*x+c)*x*b^3*e^2-1/2/d*e^2*b^3*arctan(
d*x+c)^2-1/2/d*e^2*b^3*polylog(3,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))+1/d*e^2*b^
3*ln((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)+1/2*I/d*e^2*a*b^2*ln(1+(d*x+c)^2)*ln(
d*x+c-I)-1/2*I/d*e^2*a*b^2*ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I))-1/8*arctan(d*x+
c)^2*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)*csgn(I*((1+I*(d*x+c))^2
/(1+(d*x+c)^2)+1))^2*x*b^3*e^2-1/2*I/d*e^2*a*b^2*ln(1+(d*x+c)^2)*ln(d*x+c+I
)+1/2*I/d*e^2*a*b^2*ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))-x*a^2*b*c*e^2+3*arctan(
d*x+c)*x*a^2*b*c^2*e^2+3*arctan(d*x+c)^2*x*a*b^2*c^2*e^2-2*arctan(d*x+c)*x*
a*b^2*c*e^2-1/4*I/d*e^2*a*b^2*ln(d*x+c-I)^2-1/2*I/d*e^2*a*b^2*dilog(-1/2*I*
(d*x+c+I))+1/4*I/d*e^2*a*b^2*ln(d*x+c+I)^2+1/2*I/d*e^2*a*b^2*dilog(1/2*I*(d
*x+c-I))+1/4*arctan(d*x+c)^2*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)
^2*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*x*b^3*e^2+1/8*arctan(d*x+c)^2*
Pi*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+I)^2*csgn(I*(1+I*(d*x+c))^4/(1+(d*x
+c)^2)^2+2*I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+I)*x*b^3*e^2+1/3/d*a^3*c^3*e^2+I
/d*e^2*b^3*arctan(d*x+c)*polylog(2,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))+1/d*e^2*
a*b^2*arctan(d*x+c)*ln(1+(d*x+c)^2)+1/d*arctan(d*x+c)*a^2*b*c^3*e^2+1/d*arc
tan(d*x+c)^2*a*b^2*c^3*e^2-1/d*arctan(d*x+c)*a*b^2*c^2*e^2+d^2*arctan(d*x+c
)*x^3*a^2*b*e^2+d^2*arctan(d*x+c)^2*x^3*a*b^2*e^2-d*arctan(d*x+c)*x^2*a*b^2
*e^2-1/8*arctan(d*x+c)^2*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^3*x
*b^3*e^2+1/8*arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))^4/(1+(d*x+c)^2)^2+2*I*
(1+I*(d*x+c))^2/(1+(d*x+c)^2)+I)^3*x*b^3*e^2+a*b^2*e^2*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 7/8*b^3*c^4*e^2*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 3*a*b^2*c^4*e
^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - (3*arctan(d*x + c)*arctan(
(d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*c^4*e^2 - 7/32*(6
*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*arctan((
d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3*c^4*e^2 + 1/3*a^3*d^
2*e^2*x^3 + 7/8*b^3*c^2*e^2*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 2
8*b^3*d^4*e^2*integrate(1/32*x^4*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2
 + 1), x) + 3*b^3*d^4*e^2*integrate(1/32*x^4*arctan(d*x + c)*log(d^2*x^2 +
2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 96*a*b^2*d^4*e^2*i
ntegrate(1/32*x^4*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 112
*b^3*c*d^3*e^2*integrate(1/32*x^3*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^
2 + 1), x) + 4*b^3*d^4*e^2*integrate(1/32*x^4*arctan(d*x + c)*log(d^2*x^2 +
 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d^3*e^2*in
tegrate(1/32*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^
2 + 2*c*d*x + c^2 + 1), x) + 384*a*b^2*c*d^3*e^2*integrate(1/32*x^3*arctan(
d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 168*b^3*c^2*d^2*e^2*integrat
e(1/32*x^2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 16*b^3*c*d
^3*e^2*integrate(1/32*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/
(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 18*b^3*c^2*d^2*e^2*integrate(1/32*x^2*a
rctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2
+ 1), x) + 576*a*b^2*c^2*d^2*e^2*integrate(1/32*x^2*arctan(d*x + c)^2/(d^2*
x^2 + 2*c*d*x + c^2 + 1), x) + 112*b^3*c^3*d*e^2*integrate(1/32*x*arctan(d*
x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 24*b^3*c^2*d^2*e^2*integrate(1
/32*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x
 + c^2 + 1), x) + 12*b^3*c^3*d*e^2*integrate(1/32*x*arctan(d*x + c)*log(d^2
*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 384*a*b^2*c
^3*d*e^2*integrate(1/32*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1),
x) + 12*b^3*c^3*d*e^2*integrate(1/32*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*
x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*c^4*e^2*integrate(1/
32*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x +
c^2 + 1), x) + a^3*c*d*e^2*x^2 + 3*a*b^2*c^2*e^2*arctan(d*x + c)^2*arctan((
d^2*x + c*d)/d)/d - 4*b^3*d^3*e^2*integrate(1/32*x^3*arctan(d*x + c)^2/(d^2
*x^2 + 2*c*d*x + c^2 + 1), x) + b^3*d^3*e^2*integrate(1/32*x^3*log(d^2*x^2
+ 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 12*b^3*c*d^2*e^2
*integrate(1/32*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3
*b^3*c*d^2*e^2*integrate(1/32*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x
^2 + 2*c*d*x + c^2 + 1), x) - 12*b^3*c^2*d*e^2*integrate(1/32*x*arctan(d*x
+ c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*c^2*d*e^2*integrate(1/32*x
*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - (3*
arctan(d*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*
```

```
a*b^2*c^2*e^2 - 7/32*(6*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*a
rctan(d*x + c)*arctan((d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b
^3*c^2*e^2 + 3*(x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x +
c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a^2*b*c*d*e^2 + 1/2*
(2*x^3*arctan(d*x + c) - d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2
*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*a^2*b
*d^2*e^2 + a^3*c^2*e^2*x + 28*b^3*d^2*e^2*integrate(1/32*x^2*arctan(d*x + c
)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*d^2*e^2*integrate(1/32*x^2*ar
ctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 +
 1), x) + 96*a*b^2*d^2*e^2*integrate(1/32*x^2*arctan(d*x + c)^2/(d^2*x^2 +
2*c*d*x + c^2 + 1), x) + 56*b^3*c*d*e^2*integrate(1/32*x*arctan(d*x + c)^3/
(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*c*d*e^2*integrate(1/32*x*arctan(d
*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x
) + 192*a*b^2*c*d*e^2*integrate(1/32*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x
 + c^2 + 1), x) + 3*b^3*c^2*e^2*integrate(1/32*arctan(d*x + c)*log(d^2*x^2
+ 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3/2*(2*(d*x + c)
*arctan(d*x + c) - log((d*x + c)^2 + 1))*a^2*b*c^2*e^2/d + 1/24*(b^3*d^2*e^
2*x^3 + 3*b^3*c*d*e^2*x^2 + 3*b^3*c^2*e^2*x)*arctan(d*x + c)^3 - 1/32*(b^3*
d^2*e^2*x^3 + 3*b^3*c*d*e^2*x^2 + 3*b^3*c^2*e^2*x)*arctan(d*x + c)*log(d^2*
x^2 + 2*c*d*x + c^2 + 1)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(a^3d^2e^2x^2 + 2\,a^3cde^2x + a^3c^2e^2 + \left(b^3d^2e^2x^2 + 2\,b^3cde^2x + b^3c^2e^2\right)\arctan\left(dx+c\right)^3 + 3\left(ab^2d^2e^2x^2 + 2\,ab^2cde^2x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*d^2*e^2*x^2 + 2*a^3*c*d*e^2*x + a^3*c^2*e^2 + (b^3*d^2*e^2*x^2
 + 2*b^3*c*d*e^2*x + b^3*c^2*e^2)*arctan(d*x + c)^3 + 3*(a*b^2*d^2*e^2*x^2
+ 2*a*b^2*c*d*e^2*x + a*b^2*c^2*e^2)*arctan(d*x + c)^2 + 3*(a^2*b*d^2*e^2*x
^2 + 2*a^2*b*c*d*e^2*x + a^2*b*c^2*e^2)*arctan(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^2\left(\int a^3c^2\,dx + \int a^3d^2x^2\,dx + \int b^3c^2\,\mathrm{atan}^3\left(c+dx\right)dx + \int 3ab^2c^2\,\mathrm{atan}^2\left(c+dx\right)dx + \int 3a^2bc^2\,\mathrm{atan}\left(c+dx\right)dx + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*atan(d*x+c))**3,x)
```

```
[Out] e**2*(Integral(a**3*c**2, x) + Integral(a**3*d**2*x**2, x) + Integral(b**3*
c**2*atan(c + d*x)**3, x) + Integral(3*a*b**2*c**2*atan(c + d*x)**2, x) + I
ntegral(3*a**2*b*c**2*atan(c + d*x), x) + Integral(2*a**3*c*d*x, x) + Integ
ral(b**3*d**2*x**2*atan(c + d*x)**3, x) + Integral(3*a*b**2*d**2*x**2*atan(
c + d*x)**2, x) + Integral(3*a**2*b*d**2*x**2*atan(c + d*x), x) + Integral(
2*b**3*c*d*x*atan(c + d*x)**3, x) + Integral(6*a*b**2*c*d*x*atan(c + d*x)**
2, x) + Integral(6*a**2*b*c*d*x*atan(c + d*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^2(b \arctan (dx + c) + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arctan(d*x + c) + a)^3, x)
```

3.16 $\int (ce + dex) \left(a + b\tan^{-1}(c + dx)\right)^3 dx$

Optimal. Leaf size=164

$$-\frac{3ib^3e\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{2d}-\frac{3b^2e\log\left(\frac{2}{1+i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{d}-\frac{3ibe\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}-\frac{3be(c+d\ldots}{}$$

```
[Out] (((-3*I)/2)*b*e*(a + b*ArcTan[c + d*x])^2)/d - (3*b*e*(c + d*x)*(a + b*ArcT
an[c + d*x])^2)/(2*d) + (e*(a + b*ArcTan[c + d*x])^3)/(2*d) + (e*(c + d*x)^
2*(a + b*ArcTan[c + d*x])^3)/(2*d) - (3*b^2*e*(a + b*ArcTan[c + d*x])*Log[2
/(1 + I*(c + d*x))])/d - (((3*I)/2)*b^3*e*PolyLog[2, 1 - 2/(1 + I*(c + d*x)
)])/d
```

Rubi [A] time = 0.243152, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {5043, 12, 4852, 4916, 4846, 4920, 4854, 2402, 2315, 4884}

$$-\frac{3ib^3e\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{2d}-\frac{3b^2e\log\left(\frac{2}{1+i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{d}-\frac{3ibe\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}-\frac{3be(c+d\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcTan[c + d*x])^3,x]
```


```
[Out] (((-3*I)/2)*b*e*(a + b*ArcTan[c + d*x])^2)/d - (3*b*e*(c + d*x)*(a + b*ArcT
an[c + d*x])^2)/(2*d) + (e*(a + b*ArcTan[c + d*x])^3)/(2*d) + (e*(c + d*x)^
2*(a + b*ArcTan[c + d*x])^3)/(2*d) - (3*b^2*e*(a + b*ArcTan[c + d*x])*Log[2
/(1 + I*(c + d*x))])/d - (((3*I)/2)*b^3*e*PolyLog[2, 1 - 2/(1 + I*(c + d*x)
)])/d
```


Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int (ce+dex)\left(a+b\tan^{-1}(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int ex\left(a+b\tan^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$

$$=\frac{e\,\text{Subst}\left(\int x\left(a+b\tan^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$

$$=\frac{e(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)^3}{2d}-\frac{(3be)\,\text{Subst}\left(\int\frac{x^2\left(a+b\tan^{-1}(x)\right)^2}{1+x^2}\,dx,x,c+d\right.}{2d}$$

$$=\frac{e(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)^3}{2d}-\frac{(3be)\,\text{Subst}\left(\int\left(a+b\tan^{-1}(x)\right)^2\,dx,x,c\right.}{2d}$$

$$=-\frac{3be(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}+\frac{e\left(a+b\tan^{-1}(c+dx)\right)^3}{2d}+\frac{e(c+dx)^2\left(\right.}{}$$

$$=-\frac{3ibe\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}-\frac{3be(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}+\frac{e\left(a+b\,\mathrm{t}\right.}{}$$

$$=-\frac{3ibe\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}-\frac{3be(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}+\frac{e\left(a+b\,\mathrm{t}\right.}{}$$

$$=-\frac{3ibe\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}-\frac{3be(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}+\frac{e\left(a+b\,\mathrm{t}\right.}{}$$

$$=-\frac{3ibe\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}-\frac{3be(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}+\frac{e\left(a+b\,\mathrm{t}\right.}{}$$

Mathematica [A] time = 0.267458, size = 196, normalized size = 1.2

$$\frac{e\left(3ib^3\text{PolyLog}\left(2,-e^{2i\tan^{-1}(c+dx)}\right)+3b\tan^{-1}(c+dx)\left(a\left(a\left(c^2+2cdx+d^2x^2+1\right)-2b(c+dx)\right)-2b^2\log\left(1+e^{2i\tan^{-1}}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (e*(3*b^2*(-I + c + d*x)*(-b + a*(I + c + d*x))*ArcTan[c + d*x]^2 + b^3*(1
+ c^2 + 2*c*d*x + d^2*x^2)*ArcTan[c + d*x]^3 + 3*b*ArcTan[c + d*x]*(a*(-2*b
*(c + d*x) + a*(1 + c^2 + 2*c*d*x + d^2*x^2)) - 2*b^2*Log[1 + E^((2*I)*ArcT
an[c + d*x])]) + a*(a*(c + d*x)*(-3*b + a*c + a*d*x) - 6*b^2*Log[1/Sqrt[1 +
 (c + d*x)^2]]) + (3*I)*b^3*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])]))/(2*d)
```

Maple [B] time = 0.132, size = 567, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arctan(d*x+c))^3,x)
```

```
[Out] 3/4*I/d*e*b^3*ln(d*x+c-I)*ln(1+(d*x+c)^2)-3/4*I/d*e*b^3*ln(d*x+c+I)*ln(1+(d
*x+c)^2)+3*arctan(d*x+c)*x*a^2*b*c*e+3/4*I/d*e*b^3*ln(d*x+c+I)*ln(1/2*I*(d*
x+c-I))-3/4*I/d*e*b^3*ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I))-3/2/d*a^2*b*c*e+3/2/
d*e*b^3*arctan(d*x+c)*ln(1+(d*x+c)^2)+3/4*I/d*e*b^3*dilog(1/2*I*(d*x+c-I))-
3*arctan(d*x+c)*x*a*b^2*e+1/2*d*arctan(d*x+c)^3*x^2*b^3*e-3/2/d*arctan(d*x+
c)^2*b^3*c*e+arctan(d*x+c)^3*x*b^3*c*e-3/8*I/d*e*b^3*ln(d*x+c-I)^2-3/4*I/d*
e*b^3*dilog(-1/2*I*(d*x+c+I))+3/8*I/d*e*b^3*ln(d*x+c+I)^2+1/2/d*arctan(d*x+
c)^3*b^3*c^2*e+3/2/d*e*a*b^2*ln(1+(d*x+c)^2)+3/2/d*e*a^2*b*arctan(d*x+c)+3/
2/d*e*a*b^2*arctan(d*x+c)^2+3/2/d*arctan(d*x+c)^2*a*b^2*c^2*e+3/2*d*arctan(
d*x+c)^2*x^2*a*b^2*e+3*arctan(d*x+c)^2*x*a*b^2*c*e-3/d*arctan(d*x+c)*a*b^2*
c*e+3/2/d*arctan(d*x+c)*a^2*b*c^2*e+3/2*d*arctan(d*x+c)*x^2*a^2*b*e+1/2/d*e
*b^3*arctan(d*x+c)^3+1/2/d*a^3*c^2*e-3/2*e*x*a^2*b+x*a^3*c*e+1/2*d*x^2*a^3*
e-3/2*arctan(d*x+c)^2*x*b^3*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^3*d*e*x^2 + 3/2*(x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d
^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a^2*b*d*e + a
^3*c*e*x + 3/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*a^2*b*c
*e/d + 1/32*(8*(b^3*d^2*e*x^2 + 2*b^3*c*d*e*x + (b^3*c^2 + b^3)*e)*arctan(d
*x + c)^3 + 12*(a*b^2*d^2*e*x^2 + (2*a*b^2*c - b^3)*d*e*x)*arctan(d*x + c)^
2 - 3*(a*b^2*d^2*e*x^2 + (2*a*b^2*c - b^3)*d*e*x)*log(d^2*x^2 + 2*c*d*x + c
^2 + 1)^2 + 4*(4*b^3*c^3*e*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 18
*a*b^2*c^3*e*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 6*(3*arctan(d*x
+ c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*c^3*e
 - (6*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*arc
tan((d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3*c^3*e - 3*b^3*c
^2*e*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d + 4*b^3*c*e*arctan(d*x + c
)^3*arctan((d^2*x + c*d)/d)/d + 128*b^3*d^3*e*integrate(1/32*x^3*arctan(d*x
 + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 576*a*b^2*d^3*e*integrate(1/32*
x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 384*b^3*c*d^2*e*i
ntegrate(1/32*x^2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 48*
a*b^2*d^3*e*integrate(1/32*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2
+ 2*c*d*x + c^2 + 1), x) + 1728*a*b^2*c*d^2*e*integrate(1/32*x^2*arctan(d*x
 + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 384*b^3*c^2*d*e*integrate(1/32*
x*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 96*a*b^2*d^3*e*inte
grate(1/32*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 +
1), x) + 144*a*b^2*c*d^2*e*integrate(1/32*x^2*log(d^2*x^2 + 2*c*d*x + c^2 +
 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 1728*a*b^2*c^2*d*e*integrate(1/32
*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 288*a*b^2*c*d^2*e*
integrate(1/32*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^
2 + 1), x) + 144*a*b^2*c^2*d*e*integrate(1/32*x*log(d^2*x^2 + 2*c*d*x + c^2
 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*a*b^2*c^2*d*e*integrate(1/3
2*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 48
*a*b^2*c^3*e*integrate(1/32*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2
*c*d*x + c^2 + 1), x) + (3*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^2/d - ar
ctan((d^2*x + c*d)/d)^3/d)*b^3*c^2*e + 18*a*b^2*c*e*arctan(d*x + c)^2*arcta
n((d^2*x + c*d)/d)/d - 96*b^3*d^2*e*integrate(1/32*x^2*arctan(d*x + c)^2/(d
^2*x^2 + 2*c*d*x + c^2 + 1), x) - 24*b^3*d^2*e*integrate(1/32*x^2*log(d^2*x
^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 192*a*b^2*d^2
*e*integrate(1/32*x^2*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 1
92*b^3*c*d*e*integrate(1/32*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 +
1), x) - 96*b^3*d^2*e*integrate(1/32*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(
d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 48*b^3*c*d*e*integrate(1/32*x*log(d^2*x^
2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 384*a*b^2*c*d*
e*integrate(1/32*x*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 96*b
^3*c*d*e*integrate(1/32*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d
*x + c^2 + 1), x) - 24*b^3*c^2*e*integrate(1/32*log(d^2*x^2 + 2*c*d*x + c^2
 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 6*(3*arctan(d*x + c)*arctan((d^
2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*c*e - (6*arctan(d*x
+ c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*arctan((d^2*x + c*d)
```

```
/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3*c*e - 3*b^3*e*arctan(d*x + c)^2*
arctan((d^2*x + c*d)/d)/d + 128*b^3*d*e*integrate(1/32*x*arctan(d*x + c)^3/
(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 576*a*b^2*d*e*integrate(1/32*x*arctan(d
*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 48*a*b^2*d*e*integrate(1/32*x
*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192
*b^3*d*e*integrate(1/32*x*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
 + 48*a*b^2*c*e*integrate(1/32*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2
+ 2*c*d*x + c^2 + 1), x) + (3*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^2/d -
 arctan((d^2*x + c*d)/d)^3/d)*b^3*e - 24*b^3*e*integrate(1/32*log(d^2*x^2 +
 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x))*d)/d
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(a^3dex + a^3ce + \left(b^3dex + b^3ce\right)\arctan\left(dx+c\right)^3 + 3\left(ab^2dex + ab^2ce\right)\arctan\left(dx+c\right)^2 + 3\left(a^2bdex + a^2bce\right)\text{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*d*e*x + a^3*c*e + (b^3*d*e*x + b^3*c*e)*arctan(d*x + c)^3 + 3*
(a*b^2*d*e*x + a*b^2*c*e)*arctan(d*x + c)^2 + 3*(a^2*b*d*e*x + a^2*b*c*e)*a
rctan(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e\left(\int a^3c\,dx + \int a^3dx\,dx + \int b^3c\,\text{atan}^3\left(c+dx\right)dx + \int 3ab^2c\,\text{atan}^2\left(c+dx\right)dx + \int 3a^2bc\,\text{atan}\left(c+dx\right)dx + \int b^3dx\,\text{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*atan(d*x+c))**3,x)
```

```
[Out] e*(Integral(a**3*c, x) + Integral(a**3*d*x, x) + Integral(b**3*c*atan(c + d
*x)**3, x) + Integral(3*a*b**2*c*atan(c + d*x)**2, x) + Integral(3*a**2*b*c
*atan(c + d*x), x) + Integral(b**3*d*x*atan(c + d*x)**3, x) + Integral(3*a*
b**2*d*x*atan(c + d*x)**2, x) + Integral(3*a**2*b*d*x*atan(c + d*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)(b \arctan (dx + c) + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)*(b*arctan(d*x + c) + a)^3, x)
```

3.17 $\int \frac{(a+b\tan^{-1}(c+dx))^3}{ce+dex}\,dx$

Optimal. Leaf size=279

$$-\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{2de}+\frac{3b^2\text{PolyLog}\left(3,-1+\frac{2}{1+i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{2de}-\frac{3ib\text{PolyI}}{}$$

```
[Out] (2*(a + b*ArcTan[c + d*x])^3*ArcTanh[1 - 2/(1 + I*(c + d*x))])/(d*e) - (((3
*I)/2)*b*(a + b*ArcTan[c + d*x])^2*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/(d*
e) + (((3*I)/2)*b*(a + b*ArcTan[c + d*x])^2*PolyLog[2, -1 + 2/(1 + I*(c + d
*x))])/(d*e) - (3*b^2*(a + b*ArcTan[c + d*x])*PolyLog[3, 1 - 2/(1 + I*(c +
d*x))])/(2*d*e) + (3*b^2*(a + b*ArcTan[c + d*x])*PolyLog[3, -1 + 2/(1 + I*(
c + d*x))])/(2*d*e) + (((3*I)/4)*b^3*PolyLog[4, 1 - 2/(1 + I*(c + d*x))])/(
d*e) - (((3*I)/4)*b^3*PolyLog[4, -1 + 2/(1 + I*(c + d*x))])/(d*e)
```

Rubi [A] time = 0.458166, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5043, 12, 4850, 4988, 4884, 4994, 4998, 6610}

$$-\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{2de}+\frac{3b^2\text{PolyLog}\left(3,-1+\frac{2}{1+i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{2de}-\frac{3ib\text{PolyI}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x),x]
```

```
[Out] (2*(a + b*ArcTan[c + d*x])^3*ArcTanh[1 - 2/(1 + I*(c + d*x))])/(d*e) - (((3
*I)/2)*b*(a + b*ArcTan[c + d*x])^2*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/(d*
e) + (((3*I)/2)*b*(a + b*ArcTan[c + d*x])^2*PolyLog[2, -1 + 2/(1 + I*(c + d
*x))])/(d*e) - (3*b^2*(a + b*ArcTan[c + d*x])*PolyLog[3, 1 - 2/(1 + I*(c +
d*x))])/(2*d*e) + (3*b^2*(a + b*ArcTan[c + d*x])*PolyLog[3, -1 + 2/(1 + I*(
c + d*x))])/(2*d*e) + (((3*I)/4)*b^3*PolyLog[4, 1 - 2/(1 + I*(c + d*x))])/(
d*e) - (((3*I)/4)*b^3*PolyLog[4, -1 + 2/(1 + I*(c + d*x))])/(d*e)
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
```

```
IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4998

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{ce+dex}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^3}{ex}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^3}{x}\,dx, x, c+dx\right)}{de}$$

$$= \frac{2\left(a+b\tan^{-1}(c+dx)\right)^3\tanh^{-1}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} - \frac{(6b)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2\tanh^{-1}\left(1-\frac{2}{1+ix}\right.}{1+x^2}\right.}{de}$$

$$= \frac{2\left(a+b\tan^{-1}(c+dx)\right)^3\tanh^{-1}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} - \frac{(3b)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2\log\left(2-\frac{2}{1+ix}\right)}{1+x^2}\,dx\right.}{de}$$

$$= \frac{2\left(a+b\tan^{-1}(c+dx)\right)^3\tanh^{-1}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} - \frac{3ib\left(a+b\tan^{-1}(c+dx)\right)^2\text{Li}_2\left(1-\frac{2}{1+i(c\cdot\right.}}{2de}$$

$$= \frac{2\left(a+b\tan^{-1}(c+dx)\right)^3\tanh^{-1}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} - \frac{3ib\left(a+b\tan^{-1}(c+dx)\right)^2\text{Li}_2\left(1-\frac{2}{1+i(c\cdot\right.}}{2de}$$

$$= \frac{2\left(a+b\tan^{-1}(c+dx)\right)^3\tanh^{-1}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} - \frac{3ib\left(a+b\tan^{-1}(c+dx)\right)^2\text{Li}_2\left(1-\frac{2}{1+i(c\cdot\right.}}{2de}$$

Mathematica [A] time = 0.0917491, size = 252, normalized size = 0.9

$$\frac{6b^2\text{PolyLog}\left(3,-\frac{c+dx+i}{c+dx-i}\right)\left(a+b\tan^{-1}(c+dx)\right)-6b^2\text{PolyLog}\left(3,\frac{c+dx+i}{c+dx-i}\right)\left(a+b\tan^{-1}(c+dx)\right)+6ib\text{PolyLog}\left(2,-\frac{c+dx+}{c+dx-}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x),x]
```

```
[Out] (8*(a + b*ArcTan[c + d*x])^3*ArcTanh[(I + c + d*x)/(-I + c + d*x)] + (6*I)*
b*(a + b*ArcTan[c + d*x])^2*PolyLog[2, -((I + c + d*x)/(-I + c + d*x))] - (
6*I)*b*(a + b*ArcTan[c + d*x])^2*PolyLog[2, (I + c + d*x)/(-I + c + d*x)] +
 6*b^2*(a + b*ArcTan[c + d*x])*PolyLog[3, -((I + c + d*x)/(-I + c + d*x))]
- 6*b^2*(a + b*ArcTan[c + d*x])*PolyLog[3, (I + c + d*x)/(-I + c + d*x)] -
(3*I)*b^3*PolyLog[4, -((I + c + d*x)/(-I + c + d*x))] + (3*I)*b^3*PolyLog[4
, (I + c + d*x)/(-I + c + d*x)])/(4*d*e)
```

Maple [C] time = 0.377, size = 2894, normalized size = 10.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^3/(d*e*x+c*e),x)
```

```
[Out] 3/2*I/d*a*b^2/e*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^
2/(1+(d*x+c)^2)+1))*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2
/(1+(d*x+c)^2)+1))*arctan(d*x+c)^2-3/2*I/d*a*b^2/e*Pi*csgn(I/((1+I*(d*x+c))
^2/(1+(d*x+c)^2)+1))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c)
)^2/(1+(d*x+c)^2)+1))^2*arctan(d*x+c)^2+3/2*I/d*a*b^2/e*Pi*csgn(I*((1+I*(d*
x+c))^2/(1+(d*x+c)^2)-1))*csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(I*
((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*arctan
(d*x+c)^2+3/d*a*b^2/e*arctan(d*x+c)^2*ln(1+(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2
))+3/d*a^2*b/e*ln(d*x+c)*arctan(d*x+c)-3*I/d*b^3/e*arctan(d*x+c)^2*polylog(
2,-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+3/2*I/d*b^3/e*arctan(d*x+c)^2*polylog
(2,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))-3*I/d*b^3/e*arctan(d*x+c)^2*polylog(2,(1
+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+1/2*I/d*b^3/e*Pi*arctan(d*x+c)^3+3/2*I/d*a
^2*b/e*dilog(1+I*(d*x+c))-3/2*I/d*a^2*b/e*dilog(1-I*(d*x+c))+3/d*a*b^2/e*ln
(d*x+c)*arctan(d*x+c)^2-3/d*a*b^2/e*arctan(d*x+c)^2*ln((1+I*(d*x+c))^2/(1+(
d*x+c)^2)-1)+3/d*a*b^2/e*arctan(d*x+c)^2*ln(1-(1+I*(d*x+c))/(1+(d*x+c)^2)^(
1/2))+3/2*I/d*a*b^2/e*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*
x+c))^2/(1+(d*x+c)^2)+1))^3*arctan(d*x+c)^2-3/2*I/d*a*b^2/e*Pi*csgn(((1+I*(
d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*arctan(d*x+
c)^2+3/2*I/d*a*b^2/e*Pi*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c
))^2/(1+(d*x+c)^2)+1))^3*arctan(d*x+c)^2-1/2*I/d*b^3/e*Pi*csgn(I/((1+I*(d*x
+c))^2/(1+(d*x+c)^2)+1))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*
x+c))^2/(1+(d*x+c)^2)+1))^2*arctan(d*x+c)^3-1/2*I/d*b^3/e*Pi*csgn(I*((1+I*(
d*x+c))^2/(1+(d*x+c)^2)-1))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*
(d*x+c))^2/(1+(d*x+c)^2)+1))^2*arctan(d*x+c)^3+1/2*I/d*b^3/e*Pi*csgn(I*((1+
I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(((1+I
*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*arctan(d*x+
```

```
c)^3-1/2*I/d*b^3/e*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c
))^2/(1+(d*x+c)^2)+1))*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c)
)^2/(1+(d*x+c)^2)+1))^2*arctan(d*x+c)^3+1/d*a^3/e*ln(d*x+c)-3/4*I/d*b^3/e*p
olylog(4,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))+6*I/d*b^3/e*polylog(4,(1+I*(d*x+c)
)/(1+(d*x+c)^2)^(1/2))+6*I/d*b^3/e*polylog(4,-(1+I*(d*x+c))/(1+(d*x+c)^2)^(
1/2))+6/d*b^3/e*arctan(d*x+c)*polylog(3,(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+
1/d*b^3/e*arctan(d*x+c)^3*ln(1+(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+6/d*b^3/e
*arctan(d*x+c)*polylog(3,-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))-3/2/d*b^3/e*ar
ctan(d*x+c)*polylog(3,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))+1/d*b^3/e*ln(d*x+c)*a
rctan(d*x+c)^3-1/d*b^3/e*arctan(d*x+c)^3*ln((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1
)-3/2/d*a*b^2/e*polylog(3,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))+6/d*a*b^2/e*polyl
og(3,(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+6/d*a*b^2/e*polylog(3,-(1+I*(d*x+c)
)/(1+(d*x+c)^2)^(1/2))+1/d*b^3/e*arctan(d*x+c)^3*ln(1-(1+I*(d*x+c))/(1+(d*x
+c)^2)^(1/2))-3/2*I/d*a*b^2/e*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1))*
csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))
^2*arctan(d*x+c)^2-3/2*I/d*a*b^2/e*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)
-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-
1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*arctan(d*x+c)^2+3/2*I/d*a*b^2/e*Pi*
arctan(d*x+c)^2+3/2*I/d*a^2*b/e*ln(d*x+c)*ln(1+I*(d*x+c))-3/2*I/d*a^2*b/e*l
n(d*x+c)*ln(1-I*(d*x+c))-1/2*I/d*b^3/e*Pi*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^
2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*arctan(d*x+c)^3+1/2*I/d*b^3/e*Pi
*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^
3*arctan(d*x+c)^3+1/2*I/d*b^3/e*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)
/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^3*arctan(d*x+c)^3-6*I/d*a*b^2/e*arctan(
d*x+c)*polylog(2,(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+3*I/d*a*b^2/e*arctan(d*
x+c)*polylog(2,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))-6*I/d*a*b^2/e*arctan(d*x+c)*
polylog(2,-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+1/2*I/d*b^3/e*Pi*csgn(I*((1+I
*(d*x+c))^2/(1+(d*x+c)^2)-1))*csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csg
n(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*ar
ctan(d*x+c)^3
```

<u>Maxima</u> [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^3 \log(dex+ce)}{de} + \int \frac{28\,b^3 \arctan(dx+c)^3 + 3\,b^3 \arctan(dx+c) \log\left(d^2x^2+2\,cdx+c^2+1\right)^2 + 96\,ab^2 \arctan(dx+c)}{32\,(dex+ce)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] a^3*log(d*e*x + c*e)/(d*e) + integrate(1/32*(28*b^3*arctan(d*x + c)^3 + 3*b
^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 96*a*b^2*arctan(d*x
```

```
 + c)^2 + 96*a^2*b*arctan(d*x + c))/(d*e*x + c*e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3\arctan\left(dx+c\right)^3+3\,ab^2\arctan\left(dx+c\right)^2+3\,a^2b\arctan\left(dx+c\right)+a^3}{dex+ce},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arcta
n(d*x + c) + a^3)/(d*e*x + c*e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a^3}{c+dx}\,dx+\int\frac{b^3\operatorname{atan}^3(c+dx)}{c+dx}\,dx+\int\frac{3ab^2\operatorname{atan}^2(c+dx)}{c+dx}\,dx+\int\frac{3a^2b\operatorname{atan}(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**3/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**3/(c + d*x), x) + Integral(b**3*atan(c + d*x)**3/(c + d*x), x)
 + Integral(3*a*b**2*atan(c + d*x)**2/(c + d*x), x) + Integral(3*a**2*b*ata
n(c + d*x)/(c + d*x), x))/e
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\arctan\left(dx+c\right)+a\right)^3}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(d*x + c) + a)^3/(d*e*x + c*e), x)
```

3.18 $\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{(ce+dex)^2}\,dx$

Optimal. Leaf size=163

$$-\frac{3ib^2\text{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{de^2}+\frac{3b^3\text{PolyLog}\left(3,-1+\frac{2}{1-i(c+dx)}\right)}{2de^2}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{de^2(c+dx)}-\frac{i\left(\right.}{}$$

```
[Out] ((-I)*(a + b*ArcTan[c + d*x])^3)/(d*e^2) - (a + b*ArcTan[c + d*x])^3/(d*e^2
*(c + d*x)) + (3*b*(a + b*ArcTan[c + d*x])^2*Log[2 - 2/(1 - I*(c + d*x))])/
(d*e^2) - ((3*I)*b^2*(a + b*ArcTan[c + d*x])*PolyLog[2, -1 + 2/(1 - I*(c +
d*x))])/(d*e^2) + (3*b^3*PolyLog[3, -1 + 2/(1 - I*(c + d*x))])/(2*d*e^2)
```

Rubi [A] time = 0.299623, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5043, 12, 4852, 4924, 4868, 4884, 4992, 6610}

$$-\frac{3ib^2\text{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{de^2}+\frac{3b^3\text{PolyLog}\left(3,-1+\frac{2}{1-i(c+dx)}\right)}{2de^2}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{de^2(c+dx)}-\frac{i\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x)^2,x]
```


```
[Out] ((-I)*(a + b*ArcTan[c + d*x])^3)/(d*e^2) - (a + b*ArcTan[c + d*x])^3/(d*e^2
*(c + d*x)) + (3*b*(a + b*ArcTan[c + d*x])^2*Log[2 - 2/(1 - I*(c + d*x))])/
(d*e^2) - ((3*I)*b^2*(a + b*ArcTan[c + d*x])*PolyLog[2, -1 + 2/(1 - I*(c +
d*x))])/(d*e^2) + (3*b^3*PolyLog[3, -1 + 2/(1 - I*(c + d*x))])/(2*d*e^2)
```


Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{(ce+dex)^2}\,dx = \frac{\mathrm{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^3}{e^2x^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\mathrm{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^3}{x^2}\,dx, x, c+dx\right)}{de^2}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{(3b)\,\mathrm{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{x(1+x^2)}\,dx, x, c+dx\right)}{de^2}$$

$$= -\frac{i\left(a+b\tan^{-1}(c+dx)\right)^3}{de^2} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{(3ib)\,\mathrm{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{x(i+x)}\,dx, x, c\right.}{de^2}$$

$$= -\frac{i\left(a+b\tan^{-1}(c+dx)\right)^3}{de^2} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{3b\left(a+b\tan^{-1}(c+dx)\right)^2\log\left(2-\right.}{de^2}$$

$$= -\frac{i\left(a+b\tan^{-1}(c+dx)\right)^3}{de^2} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{3b\left(a+b\tan^{-1}(c+dx)\right)^2\log\left(2-\right.}{de^2}$$

$$= -\frac{i\left(a+b\tan^{-1}(c+dx)\right)^3}{de^2} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{3b\left(a+b\tan^{-1}(c+dx)\right)^2\log\left(2-\right.}{de^2}$$

Mathematica [A] time = 0.449064, size = 263, normalized size = 1.61

$$\frac{6ab^2\left(\tan^{-1}(c+dx)\left(\left(-\frac{1}{c+dx}-i\right)\tan^{-1}(c+dx)+2\log\left(1-e^{2i\tan^{-1}(c+dx)}\right)\right)-i\mathrm{PolyLog}\left(2,e^{2i\tan^{-1}(c+dx)}\right)\right)+2b^3\left(3i\tan^{-}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x)^2,x]
```

```
[Out] ((-2*a^3)/(c + d*x) - (6*a^2*b*ArcTan[c + d*x])/(c + d*x) + 6*a^2*b*Log[c +
 d*x] - 3*a^2*b*Log[1 + c^2 + 2*c*d*x + d^2*x^2] + 6*a*b^2*(ArcTan[c + d*x]
*((-I - (c + d*x)^(-1))*ArcTan[c + d*x] + 2*Log[1 - E^((2*I)*ArcTan[c + d*x
])]) - I*PolyLog[2, E^((2*I)*ArcTan[c + d*x])]) + 2*b^3*((-I/8)*Pi^3 + I*Ar
cTan[c + d*x]^3 - ArcTan[c + d*x]^3/(c + d*x) + 3*ArcTan[c + d*x]^2*Log[1 -
 E^((-2*I)*ArcTan[c + d*x])] + (3*I)*ArcTan[c + d*x]*PolyLog[2, E^((-2*I)*A
```

```
rcTan[c + d*x])] + (3*PolyLog[3, E^((-2*I)*ArcTan[c + d*x])])/2))/(2*d*e^2)
```

Maple [C] time = 0.4, size = 2696, normalized size = 16.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^2,x)
```

```
[Out] 3/2*I/d*b^3/e^2*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1))*csgn(I/((1+I*(
d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*
(d*x+c))^2/(1+(d*x+c)^2)+1))*arctan(d*x+c)^2-3/4*I/d*b^3/e^2*arctan(d*x+c)^
2*Pi*csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)*csgn(I*(1+I*(d*x+c))^2/(1+
(d*x+c)^2))*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/((1+I*(d*x+c))^2/(1+(d*x+c
)^2)+1)^2)-1/d*a^3/e^2/(d*x+c)+3/4*I/d*b^3/e^2*arctan(d*x+c)^2*Pi*csgn(I/((
1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/((1
+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^2+3/4*I/d*b^3/e^2*arctan(d*x+c)^2*Pi*csgn
(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^
2)+1)^2)-3/4*I/d*b^3/e^2*arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))/(1+(d*x+c)
^2)^(1/2))^2*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))+3/4*I/d*b^3/e^2*arctan(d
*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))*csgn(I*(1+I*(d*x+c))^2/(1+
(d*x+c)^2)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^2-3/2*I/d*b^3/e^2*Pi*csgn(I
/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1
)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*arctan(d*x+c)^2+3/2*I/d*b^3/e^2*Pi*c
sgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*
csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*a
rctan(d*x+c)^2+3/2*I/d*b^3/e^2*arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))/(1+(
d*x+c)^2)^(1/2))*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2-3/2*I/d*b^3/e^2*ar
ctan(d*x+c)^2*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(I*((1+I*(d*
x+c))^2/(1+(d*x+c)^2)+1)^2)^2-3/2*I/d*b^3/e^2*Pi*csgn(I*((1+I*(d*x+c))^2/(1
+(d*x+c)^2)-1))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(
1+(d*x+c)^2)+1))^2*arctan(d*x+c)^2-3/2*I/d*b^3/e^2*Pi*csgn(I*((1+I*(d*x+c))
^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(((1+I*(d*x+c))^
2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*arctan(d*x+c)^2+6/d
*b^3/e^2*polylog(3,-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+6/d*b^3/e^2*polylog(
3,(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))-3/d*a*b^2/e^2*arctan(d*x+c)*ln(1+(d*x+
c)^2)+6/d*a*b^2/e^2*arctan(d*x+c)*ln(d*x+c)-3/d*a^2*b/e^2/(d*x+c)*arctan(d*
x+c)-3/d*a*b^2/e^2/(d*x+c)*arctan(d*x+c)^2+3/2*I/d*b^3/e^2*Pi*arctan(d*x+c)
^2-6*I/d*b^3/e^2*arctan(d*x+c)*polylog(2,-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2)
)-6*I/d*b^3/e^2*arctan(d*x+c)*polylog(2,(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+
3*I/d*a*b^2/e^2*dilog(1+I*(d*x+c))-3*I/d*a*b^2/e^2*dilog(1-I*(d*x+c))+3/4*I
```

```
/d*a*b^2/e^2*ln(d*x+c-I)^2+3/2*I/d*a*b^2/e^2*dilog(-1/2*I*(d*x+c+I))-3/4*I/
d*a*b^2/e^2*ln(d*x+c+I)^2-3/2*I/d*a*b^2/e^2*dilog(1/2*I*(d*x+c-I))-3/2/d*a^
2*b/e^2*ln(1+(d*x+c)^2)+3/d*a^2*b/e^2*ln(d*x+c)+3/d*b^3/e^2*arctan(d*x+c)^2
*ln(2)+3/d*b^3/e^2*arctan(d*x+c)^2*ln(1+(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+
3/d*b^3/e^2*arctan(d*x+c)^2*ln(1-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+3/d*b^3
/e^2*arctan(d*x+c)^2*ln((1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))-1/d*b^3/e^2/(d*x
+c)*arctan(d*x+c)^3-3/d*b^3/e^2*arctan(d*x+c)^2*ln((1+I*(d*x+c))^2/(1+(d*x+
c)^2)-1)-3/2/d*b^3/e^2*arctan(d*x+c)^2*ln(1+(d*x+c)^2)+3/d*b^3/e^2*ln(d*x+c
)*arctan(d*x+c)^2-I/d*b^3/e^2*arctan(d*x+c)^3+3/2*I/d*a*b^2/e^2*ln(1+(d*x+c
)^2)*ln(d*x+c+I)-3/2*I/d*b^3/e^2*Pi*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/
((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*arctan(d*x+c)^2+3/2*I/d*b^3/e^2*Pi*csg
n(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^3*ar
ctan(d*x+c)^2-3/4*I/d*b^3/e^2*arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))^2/(1+
(d*x+c)^2))^3-3/4*I/d*b^3/e^2*arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))^2/(1+
(d*x+c)^2)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^3+3/4*I/d*b^3/e^2*arctan(d*
x+c)^2*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^3-3*I/d*a*b^2/e^2*ln(
d*x+c)*ln(1-I*(d*x+c))-3/2*I/d*a*b^2/e^2*ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))+3*
I/d*a*b^2/e^2*ln(d*x+c)*ln(1+I*(d*x+c))+3/2*I/d*b^3/e^2*Pi*csgn(I*((1+I*(d*
x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^3*arctan(d*x+c)
^2-3/2*I/d*a*b^2/e^2*ln(1+(d*x+c)^2)*ln(d*x+c-I)+3/2*I/d*a*b^2/e^2*ln(d*x+c
-I)*ln(-1/2*I*(d*x+c+I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{3}{2}\left(d\left(\frac{\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^2e^2}-\frac{2\,\log\left(dx+c\right)}{d^2e^2}\right)+\frac{2\,\arctan\left(dx+c\right)}{d^2e^2x+cde^2}\right)a^2b-\frac{a^3}{d^2e^2x+cde^2}-\frac{\frac{15}{2}\,b^3\arctan\left(dx+c\right)^3-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] -3/2*(d*(log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*e^2) - 2*log(d*x + c)/(d^2*e
^2)) + 2*arctan(d*x + c)/(d^2*e^2*x + c*d*e^2))*a^2*b - a^3/(d^2*e^2*x + c*
d*e^2) - 1/32*(4*b^3*arctan(d*x + c)^3 - 3*b^3*arctan(d*x + c)*log(d^2*x^2
+ 2*c*d*x + c^2 + 1)^2 - 32*(d^2*e^2*x + c*d*e^2)*integrate(1/32*(28*(b^3*d
^2*x^2 + 2*b^3*c*d*x + b^3*c^2 + b^3)*arctan(d*x + c)^3 + 12*(8*a*b^2*d^2*x
^2 + 8*a*b^2*c^2 + b^3*c + 8*a*b^2 + (16*a*b^2*c + b^3)*d*x)*arctan(d*x + c
)^2 - 12*(b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2)*arctan(d*x + c)*log(d^2*x^2
+ 2*c*d*x + c^2 + 1) - 3*(b^3*d*x + b^3*c - (b^3*d^2*x^2 + 2*b^3*c*d*x + b^
3*c^2 + b^3)*arctan(d*x + c))*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2)/(d^4*e^2*
x^4 + 4*c*d^3*e^2*x^3 + (6*c^2 + 1)*d^2*e^2*x^2 + 2*(2*c^3 + c)*d*e^2*x + (
```

```
c^4 + c^2)*e^2), x))/(d^2*e^2*x + c*d*e^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3\arctan\left(dx+c\right)^3+3\,ab^2\arctan\left(dx+c\right)^2+3\,a^2b\arctan\left(dx+c\right)+a^3}{d^2e^2x^2+2\,cde^2x+c^2e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arcta
n(d*x + c) + a^3)/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a^3}{c^2+2cdx+d^2x^2}\,dx+\int\frac{b^3\operatorname{atan}^3\left(c+dx\right)}{c^2+2cdx+d^2x^2}\,dx+\int\frac{3ab^2\operatorname{atan}^2\left(c+dx\right)}{c^2+2cdx+d^2x^2}\,dx+\int\frac{3a^2b\operatorname{atan}\left(c+dx\right)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**3/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**3*atan(c + d*
x)**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a*b**2*atan(c + d*x)**2
/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a**2*b*atan(c + d*x)/(c**2 +
 2*c*d*x + d**2*x**2), x))/e**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\arctan\left(dx+c\right)+a\right)^3}{\left(dex+ce\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(d*x + c) + a)^3/(d*e*x + c*e)^2, x)
```

3.19 $\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{(ce+dex)^3}\,dx$

Optimal. Leaf size=180

$$-\frac{3ib^3\text{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)}{2de^3}+\frac{3b^2\log\left(2-\frac{2}{1-i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{de^3}-\frac{3b\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3(c+dx)}-\frac{3ib\left(a+b\right.}{}$$

```
[Out] (((-3*I)/2)*b*(a + b*ArcTan[c + d*x])^2)/(d*e^3) - (3*b*(a + b*ArcTan[c + d
*x])^2)/(2*d*e^3*(c + d*x)) - (a + b*ArcTan[c + d*x])^3/(2*d*e^3) - (a + b*
ArcTan[c + d*x])^3/(2*d*e^3*(c + d*x)^2) + (3*b^2*(a + b*ArcTan[c + d*x])*L
og[2 - 2/(1 - I*(c + d*x))])/(d*e^3) - (((3*I)/2)*b^3*PolyLog[2, -1 + 2/(1
- I*(c + d*x))])/(d*e^3)
```

Rubi [A] time = 0.31859, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5043, 12, 4852, 4918, 4924, 4868, 2447, 4884}

$$-\frac{3ib^3\text{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)}{2de^3}+\frac{3b^2\log\left(2-\frac{2}{1-i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{de^3}-\frac{3b\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3(c+dx)}-\frac{3ib\left(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x)^3,x]
```

```
[Out] (((-3*I)/2)*b*(a + b*ArcTan[c + d*x])^2)/(d*e^3) - (3*b*(a + b*ArcTan[c + d
*x])^2)/(2*d*e^3*(c + d*x)) - (a + b*ArcTan[c + d*x])^3/(2*d*e^3) - (a + b*
ArcTan[c + d*x])^3/(2*d*e^3*(c + d*x)^2) + (3*b^2*(a + b*ArcTan[c + d*x])*L
og[2 - 2/(1 - I*(c + d*x))])/(d*e^3) - (((3*I)/2)*b^3*PolyLog[2, -1 + 2/(1
- I*(c + d*x))])/(d*e^3)
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
```

```
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{(ce+dex)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^3}{e^3x^3}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^3}{x^3}\,dx, x, c+dx\right)}{de^3}$$
$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{2de^3(c+dx)^2} + \frac{(3b)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{x^2(1+x^2)}\,dx, x, c+dx\right)}{2de^3}$$
$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{2de^3(c+dx)^2} + \frac{(3b)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{x^2}\,dx, x, c+dx\right)}{2de^3} - \frac{(3b)\,\text{Subst}\left(\int \frac{(a+\cdots}{\cdots}\right.}{\cdots}$$
$$= -\frac{3b\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3(c+dx)} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{2de^3} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{2de^3(c+dx)^2} + \frac{\left(3b^2\right)\text{Su}\cdots}{\cdots}$$
$$= -\frac{3ib\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3} - \frac{3b\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3(c+dx)} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{2de^3} - \frac{(a+\cdots}{\cdots}$$
$$= -\frac{3ib\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3} - \frac{3b\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3(c+dx)} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{2de^3} - \frac{(a+\cdots}{\cdots}$$
$$= -\frac{3ib\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3} - \frac{3b\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3(c+dx)} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{2de^3} - \frac{(a+\cdots}{\cdots}$$

Mathematica [A] time = 0.225925, size = 225, normalized size = 1.25

$$-\frac{3b^3(c+dx)\left(i(c+dx)\left(\tan^{-1}(c+dx)^2+\text{PolyLog}\left(2,e^{2i\tan^{-1}(c+dx)}\right)\right)+\tan^{-1}(c+dx)^2-2(c+dx)\tan^{-1}(c+dx)\log\left(1\cdots\right.\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x)^3,x]
```

```
[Out] -(a^3 + b^3*(1 + c^2 + 2*c*d*x + d^2*x^2)*ArcTan[c + d*x]^3 + 3*a^2*b*(c +
d*x + (1 + (c + d*x)^2)*ArcTan[c + d*x]) + 3*a*b^2*(2*(c + d*x)*ArcTan[c +
d*x] + (1 + (c + d*x)^2)*ArcTan[c + d*x]^2 - 2*(c + d*x)^2*Log[(c + d*x)/Sq
rt[1 + (c + d*x)^2]]) + 3*b^3*(c + d*x)*(ArcTan[c + d*x]^2 - 2*(c + d*x)*Ar
cTan[c + d*x]*Log[1 - E^((2*I)*ArcTan[c + d*x])] + I*(c + d*x)*(ArcTan[c +
d*x]^2 + PolyLog[2, E^((2*I)*ArcTan[c + d*x])])))/(2*d*e^3*(c + d*x)^2)
```

Maple [B] time = 0.138, size = 631, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^3,x)
```

```
[Out] -3/2/d*a*b^2/e^3*arctan(d*x+c)^2-3/2/d*a*b^2/e^3*ln(1+(d*x+c)^2)+3/d*a*b^2/
e^3*ln(d*x+c)-3/2/d*a^2*b/e^3*arctan(d*x+c)-3/4*I/d*b^3/e^3*dilog(1/2*I*(d*
x+c-I))-3/8*I/d*b^3/e^3*ln(d*x+c+I)^2+3/2*I/d*b^3/e^3*dilog(1+I*(d*x+c))+3/
8*I/d*b^3/e^3*ln(d*x+c-I)^2+3/4*I/d*b^3/e^3*dilog(-1/2*I*(d*x+c+I))-3/2*I/d
*b^3/e^3*dilog(1-I*(d*x+c))-3/2/d*a^2*b/e^3/(d*x+c)-1/2/d*b^3/e^3/(d*x+c)^2
*arctan(d*x+c)^3-3/2/d*b^3/e^3*arctan(d*x+c)^2/(d*x+c)-3/2/d*b^3/e^3*arctan
(d*x+c)*ln(1+(d*x+c)^2)+3/d*b^3/e^3*ln(d*x+c)*arctan(d*x+c)-3/4*I/d*b^3/e^3
*ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))-3/2/d*a^2*b/e^3/(d*x+c)^2*arctan(d*x+c)+3/
4*I/d*b^3/e^3*ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I))+3/4*I/d*b^3/e^3*ln(1+(d*x+c)
^2)*ln(d*x+c+I)+3/2*I/d*b^3/e^3*ln(d*x+c)*ln(1+I*(d*x+c))-3/4*I/d*b^3/e^3*l
n(1+(d*x+c)^2)*ln(d*x+c-I)-3/2*I/d*b^3/e^3*ln(d*x+c)*ln(1-I*(d*x+c))-3/2/d*
a*b^2/e^3/(d*x+c)^2*arctan(d*x+c)^2-3/d*a*b^2/e^3*arctan(d*x+c)/(d*x+c)-1/2
/d*b^3/e^3*arctan(d*x+c)^3-1/2/d*a^3/e^3/(d*x+c)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] -3/2*(d*(1/(d^3*e^3*x + c*d^2*e^3) + arctan((d^2*x + c*d)/d)/(d^2*e^3)) + a
rctan(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3))*a^2*b - 3/2*(2*d*
```

```
(1/(d^3*e^3*x + c*d^2*e^3) + arctan((d^2*x + c*d)/d)/(d^2*e^3))*arctan(d*x
+ c) - (arctan(d*x + c)^2 - log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*log(d*x +
c))/(d*e^3))*a*b^2 - 3/2*a*b^2*arctan(d*x + c)^2/(d^3*e^3*x^2 + 2*c*d^2*e^3
*x + c^2*d*e^3) - 1/32*(8*(d^2*x^2 + 2*c*d*x + c^2 + 1)*arctan(d*x + c)^3 +
 12*(d*x + c)*arctan(d*x + c)^2 - 3*(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 +
 1)^2 - 32*(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)*integrate(1/32*(16*(d^
2*x^2 + 2*c*d*x + c^2 + 1)*arctan(d*x + c)^3 + 12*(d^3*x^3 + 3*c*d^2*x^2 +
c^3 + (3*c^2 + 1)*d*x + c)*arctan(d*x + c)^2 + 3*(d^3*x^3 + 3*c*d^2*x^2 + c
^3 + (3*c^2 + 1)*d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 24*(d^2*x^2
+ 2*c*d*x + c^2)*arctan(d*x + c) - 12*(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x +
c^3)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^5*e^3*x^5 + 5*c*d^4*e^3*x^4 + (10
*c^2 + 1)*d^3*e^3*x^3 + (10*c^3 + 3*c)*d^2*e^3*x^2 + (5*c^4 + 3*c^2)*d*e^3*
x + (c^5 + c^3)*e^3), x))*b^3/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3) - 1
/2*a^3/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \arctan(dx+c)^3 + 3\,ab^2 \arctan(dx+c)^2 + 3\,a^2 b \arctan(dx+c) + a^3}{d^3e^3x^3 + 3\,cd^2e^3x^2 + 3\,c^2de^3x + c^3e^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arcta
n(d*x + c) + a^3)/(d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^3}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{b^3\,\mathrm{atan}^3(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{3ab^2\,\mathrm{atan}^2(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{3a^2b\,\mathrm{atan}(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**3/(d*e*x+c*e)**3,x)
```

```
[Out] (Integral(a**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(b**3*atan(c + d*x)**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x
```

```
) + Integral(3*a*b**2*atan(c + d*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 +
 d**3*x**3), x) + Integral(3*a**2*b*atan(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*
d**2*x**2 + d**3*x**3), x))/e**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan (dx + c) + a)^3}{(dex + ce)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(d*x + c) + a)^3/(d*e*x + c*e)^3, x)
```

3.20 $\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{(ce+dex)^4}\,dx$

Optimal. Leaf size=287

$$\frac{ib^2\text{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{de^4}-\frac{b^3\text{PolyLog}\left(3,-1+\frac{2}{1-i(c+dx)}\right)}{2de^4}-\frac{b^2\left(a+b\tan^{-1}(c+dx)\right)}{de^4(c+dx)}-\frac{b\left(a+\right.}{}$$

```
[Out] -((b^2*(a + b*ArcTan[c + d*x]))/(d*e^4*(c + d*x))) - (b*(a + b*ArcTan[c + d
*x])^2)/(2*d*e^4) - (b*(a + b*ArcTan[c + d*x])^2)/(2*d*e^4*(c + d*x)^2) + (
(I/3)*(a + b*ArcTan[c + d*x])^3)/(d*e^4) - (a + b*ArcTan[c + d*x])^3/(3*d*e
^4*(c + d*x)^3) + (b^3*Log[c + d*x])/(d*e^4) - (b^3*Log[1 + (c + d*x)^2])/(
2*d*e^4) - (b*(a + b*ArcTan[c + d*x])^2*Log[2 - 2/(1 - I*(c + d*x))])/(d*e^
4) + (I*b^2*(a + b*ArcTan[c + d*x])*PolyLog[2, -1 + 2/(1 - I*(c + d*x))])/(
d*e^4) - (b^3*PolyLog[3, -1 + 2/(1 - I*(c + d*x))])/(2*d*e^4)
```

Rubi [A] time = 0.499924, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {5043, 12, 4852, 4918, 266, 36, 29, 31, 4884, 4924, 4868, 4992, 6610}

$$\frac{ib^2\text{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{de^4}-\frac{b^3\text{PolyLog}\left(3,-1+\frac{2}{1-i(c+dx)}\right)}{2de^4}-\frac{b^2\left(a+b\tan^{-1}(c+dx)\right)}{de^4(c+dx)}-\frac{b\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x)^4,x]
```


```
[Out] -((b^2*(a + b*ArcTan[c + d*x]))/(d*e^4*(c + d*x))) - (b*(a + b*ArcTan[c + d
*x])^2)/(2*d*e^4) - (b*(a + b*ArcTan[c + d*x])^2)/(2*d*e^4*(c + d*x)^2) + (
(I/3)*(a + b*ArcTan[c + d*x])^3)/(d*e^4) - (a + b*ArcTan[c + d*x])^3/(3*d*e
^4*(c + d*x)^3) + (b^3*Log[c + d*x])/(d*e^4) - (b^3*Log[1 + (c + d*x)^2])/(
2*d*e^4) - (b*(a + b*ArcTan[c + d*x])^2*Log[2 - 2/(1 - I*(c + d*x))])/(d*e^
4) + (I*b^2*(a + b*ArcTan[c + d*x])*PolyLog[2, -1 + 2/(1 - I*(c + d*x))])/(
d*e^4) - (b^3*PolyLog[3, -1 + 2/(1 - I*(c + d*x))])/(2*d*e^4)
```


Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
```

```
IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{(ce+dex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^3}{e^4x^4}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^3}{x^4}\,dx,x,c+dx\right)}{de^4}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{3de^4(c+dx)^3} + \frac{b\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{x^3(1+x^2)}\,dx,x,c+dx\right)}{de^4}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{3de^4(c+dx)^3} + \frac{b\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{x^3}\,dx,x,c+dx\right)}{de^4} - \frac{b\,\text{Subst}\left(\int \frac{\left(a+b\tan\right.}{x(1+}\right.}{d}$$

$$= -\frac{b\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} + \frac{i\left(a+b\tan^{-1}(c+dx)\right)^3}{3de^4} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{3de^4(c+dx)^3} - \frac{(ib)\,\text{Su}}{}$$

$$= -\frac{b\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} + \frac{i\left(a+b\tan^{-1}(c+dx)\right)^3}{3de^4} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{3de^4(c+dx)^3} - \frac{b\left(a+\right.}{}$$

$$= -\frac{b^2\left(a+b\tan^{-1}(c+dx)\right)}{de^4(c+dx)} - \frac{b\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^4} - \frac{b\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} + \frac{i\left(a\right.}{}$$

$$= -\frac{b^2\left(a+b\tan^{-1}(c+dx)\right)}{de^4(c+dx)} - \frac{b\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^4} - \frac{b\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} + \frac{i\left(a\right.}{}$$

$$= -\frac{b^2\left(a+b\tan^{-1}(c+dx)\right)}{de^4(c+dx)} - \frac{b\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^4} - \frac{b\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} + \frac{i\left(a\right.}{}$$

$$= -\frac{b^2\left(a+b\tan^{-1}(c+dx)\right)}{de^4(c+dx)} - \frac{b\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^4} - \frac{b\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} + \frac{i\left(a\right.}{}$$

Mathematica [A] time = 0.823884, size = 360, normalized size = 1.25

$$\frac{24ab^2\left(i\text{PolyLog}\left(2,e^{2i\tan^{-1}(c+dx)}\right) - \frac{(c+dx)^2+\tan^{-1}(c+dx)^2}{(c+dx)^3} + \tan^{-1}(c+dx)\left(-\frac{1}{(c+dx)^2} + i\tan^{-1}(c+dx) - 2\log\left(1-e^{2i\tan^{-1}}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x)^4,x]
```

```
[Out] ((-8*a^3)/(c + d*x)^3 - (12*a^2*b)/(c + d*x)^2 - (24*a^2*b*ArcTan[c + d*x])
/(c + d*x)^3 - 24*a^2*b*Log[c + d*x] + 12*a^2*b*Log[1 + c^2 + 2*c*d*x + d^2
*x^2] + 24*a*b^2*(-(((c + d*x)^2 + ArcTan[c + d*x]^2)/(c + d*x)^3) + ArcTan
[c + d*x]*(-1 - (c + d*x)^(-2) + I*ArcTan[c + d*x] - 2*Log[1 - E^((2*I)*Arc
Tan[c + d*x])]) + I*PolyLog[2, E^((2*I)*ArcTan[c + d*x])]) + b^3*(I*Pi^3 -
(24*ArcTan[c + d*x])/(c + d*x) - 12*ArcTan[c + d*x]^2 - (12*ArcTan[c + d*x]
^2)/(c + d*x)^2 - (8*I)*ArcTan[c + d*x]^3 - (8*ArcTan[c + d*x]^3)/(c + d*x)
^3 - 24*ArcTan[c + d*x]^2*Log[1 - E^((-2*I)*ArcTan[c + d*x])] + 24*Log[(c +
 d*x)/Sqrt[1 + (c + d*x)^2]] - (24*I)*ArcTan[c + d*x]*PolyLog[2, E^((-2*I)*
ArcTan[c + d*x])] - 12*PolyLog[3, E^((-2*I)*ArcTan[c + d*x])]))/(24*d*e^4)
```

Maple [C] time = 0.864, size = 7083, normalized size = 24.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^4,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \arctan\left(dx+c\right)^3 + 3\,ab^2 \arctan\left(dx+c\right)^2 + 3\,a^2 b \arctan\left(dx+c\right) + a^3}{d^4e^4x^4 + 4\,cd^3e^4x^3 + 6\,c^2d^2e^4x^2 + 4\,c^3de^4x + c^4e^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arcta
n(d*x + c) + a^3)/(d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^
3*d*e^4*x + c^4*e^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^3}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{b^3\operatorname{atan}^3(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{3ab^2\operatorname{atan}^2(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{3a^2b\operatorname{atan}(c\cdot}{c^4+4c^3dx+6c^2d^2x^2+\cdot}}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**3/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a**3/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(b**3*atan(c + d*x)**3/(c**4 + 4*c**3*d*x + 6*c**2*d**
2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(3*a*b**2*atan(c + d*x)**
2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) +
Integral(3*a**2*b*atan(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c
*d**3*x**3 + d**4*x**4), x))/e**4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(dx+c)+a)^3}{(dex+ce)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(d*x + c) + a)^3/(d*e*x + c*e)^4, x)
```

3.21 $\int \frac{\tan^{-1}(1+x)}{2+2x}\,dx$

Optimal. Leaf size=31

$$\frac{1}{4}i\text{PolyLog}(2,-i(x+1))-\frac{1}{4}i\text{PolyLog}(2,i(x+1))$$

```
[Out] (I/4)*PolyLog[2, (-I)*(1 + x)] - (I/4)*PolyLog[2, I*(1 + x)]
```

Rubi [A] time = 0.0375427, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5043, 12, 4848, 2391}

$$\frac{1}{4}i\text{PolyLog}(2,-i(x+1))-\frac{1}{4}i\text{PolyLog}(2,i(x+1))$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[1 + x]/(2 + 2*x),x]
```

```
[Out] (I/4)*PolyLog[2, (-I)*(1 + x)] - (I/4)*PolyLog[2, I*(1 + x)]
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(1+x)}{2+2x}\,dx &= \mathrm{Subst}\left(\int \frac{\tan^{-1}(x)}{2x}\,dx, x, 1+x\right)\\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{\tan^{-1}(x)}{x}\,dx, x, 1+x\right)\\ &= \frac{1}{4}i\,\mathrm{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, 1+x\right) - \frac{1}{4}i\,\mathrm{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, 1+x\right)\\ &= \frac{1}{4}i\mathrm{Li}_2(-i(1+x)) - \frac{1}{4}i\mathrm{Li}_2(i(1+x))\end{aligned}$$

Mathematica [A] time = 0.0034877, size = 31, normalized size = 1.

$$\frac{1}{4}i\mathrm{PolyLog}(2,-i(x+1)) - \frac{1}{4}i\mathrm{PolyLog}(2,i(x+1))$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[1 + x]/(2 + 2*x),x]
```

```
[Out] (I/4)*PolyLog[2, (-I)*(1 + x)] - (I/4)*PolyLog[2, I*(1 + x)]
```

Maple [B] time = 0.046, size = 68, normalized size = 2.2

$$\frac{\ln(x+1)\arctan(x+1)}{2} + \frac{i}{4}\ln(x+1)\ln(1+i(x+1)) - \frac{i}{4}\ln(x+1)\ln(1-i(x+1)) + \frac{i}{4}\mathrm{dilog}(1+i(x+1)) - \frac{i}{4}\mathrm{dilo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x+1)/(2+2*x),x)
```

```
[Out] 1/2*ln(x+1)*arctan(x+1)+1/4*I*ln(x+1)*ln(1+I*(x+1))-1/4*I*ln(x+1)*ln(1-I*(x
+1))+1/4*I*dilog(1+I*(x+1))-1/4*I*dilog(1-I*(x+1))
```

Maxima [B] time = 1.62231, size = 59, normalized size = 1.9

$$-\frac{1}{4}\arctan\left(x+1,0\right)\log\left(x^2+2x+2\right)+\frac{1}{2}\arctan\left(x+1\right)\log\left(|x+1|\right)-\frac{1}{4}i\operatorname{Li}_2\left(ix+i+1\right)+\frac{1}{4}i\operatorname{Li}_2\left(-ix-i+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(1+x)/(2+2*x),x, algorithm="maxima")
```

```
[Out] -1/4*arctan2(x + 1, 0)*log(x^2 + 2*x + 2) + 1/2*arctan(x + 1)*log(abs(x + 1
)) - 1/4*I*dilog(I*x + I + 1) + 1/4*I*dilog(-I*x - I + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan\left(x+1\right)}{2\left(x+1\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(1+x)/(2+2*x),x, algorithm="fricas")
```

```
[Out] integral(1/2*arctan(x + 1)/(x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atan}\left(x+1\right)}{x+1}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(1+x)/(2+2*x),x)
```

```
[Out] Integral(atan(x + 1)/(x + 1), x)/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left(x+1\right)}{2\left(x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(1+x)/(2+2*x),x, algorithm="giac")
```

```
[Out] integrate(1/2*arctan(x + 1)/(x + 1), x)
```

3.22 $\int \frac{\tan^{-1}(a+bx)}{\frac{ad}{b}+dx} dx$

Optimal. Leaf size=41

$$\frac{i\text{PolyLog}(2,-i(a+bx))}{2d}-\frac{i\text{PolyLog}(2,i(a+bx))}{2d}$$

```
[Out] ((I/2)*PolyLog[2, (-I)*(a + b*x)])/d - ((I/2)*PolyLog[2, I*(a + b*x)])/d
```

Rubi [A] time = 0.0452878, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {5043, 12, 4848, 2391}

$$\frac{i\text{PolyLog}(2,-i(a+bx))}{2d}-\frac{i\text{PolyLog}(2,i(a+bx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] ((I/2)*PolyLog[2, (-I)*(a + b*x)])/d - ((I/2)*PolyLog[2, I*(a + b*x)])/d
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{b\tan^{-1}(x)}{dx}\,dx, x, a+bx\right)}{b}\\
&= \frac{\operatorname{Subst}\left(\int \frac{\tan^{-1}(x)}{x}\,dx, x, a+bx\right)}{d}\\
&= \frac{i\operatorname{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, a+bx\right)}{2d} - \frac{i\operatorname{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, a+bx\right)}{2d}\\
&= \frac{i\mathrm{Li}_2(-i(a+bx))}{2d} - \frac{i\mathrm{Li}_2(i(a+bx))}{2d}
\end{aligned}
$$

Mathematica [A] time = 0.0074928, size = 34, normalized size = 0.83

$$\frac{i(\operatorname{PolyLog}(2,-i(a+bx))-\operatorname{PolyLog}(2,i(a+bx)))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] ((I/2)*(PolyLog[2, (-I)*(a + b*x)] - PolyLog[2, I*(a + b*x)]))/d
```

Maple [B] time = 0.046, size = 98, normalized size = 2.4

$$\frac{\ln(bx+a)\arctan(bx+a)}{d} + \frac{\frac{i}{2}\ln(bx+a)\ln(1+i(bx+a))}{d} - \frac{\frac{i}{2}\ln(bx+a)\ln(1-i(bx+a))}{d} + \frac{\frac{i}{2}\operatorname{dilog}(1+i(bx+a))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(a*d/b+d*x),x)
```

```
[Out] 1/d*ln(b*x+a)*arctan(b*x+a)+1/2*I/d*ln(b*x+a)*ln(1+I*(b*x+a))-1/2*I/d*ln(b*
x+a)*ln(1-I*(b*x+a))+1/2*I/d*dilog(1+I*(b*x+a))-1/2*I/d*dilog(1-I*(b*x+a))
```

Maxima [B] time = 1.68232, size = 166, normalized size = 4.05

$$\frac{\arctan(bx+a)\log\left(dx+\frac{ad}{b}\right)}{d}-\frac{\arctan\left(\frac{b^2x+ab}{b}\right)\log\left(dx+\frac{ad}{b}\right)}{d}-\frac{\arctan(bx+a,0)\log\left(b^2x^2+2\,abx+a^2+1\right)-2\,\mathrm{arct}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
```

```
[Out] arctan(b*x + a)*log(d*x + a*d/b)/d - arctan((b^2*x + a*b)/b)*log(d*x + a*d/
b)/d - 1/2*(arctan2(b*x + a, 0)*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*arctan
(b*x + a)*log(abs(b*x + a)) + I*dilog(I*b*x + I*a + 1) - I*dilog(-I*b*x - I
*a + 1))/d
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\arctan(bx+a)}{bdx+ad},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
```

```
[Out] integral(b*arctan(b*x + a)/(b*d*x + a*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b\int\frac{\operatorname{atan}(a+bx)}{a+bx}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(a*d/b+d*x),x)
```

```
[Out] b*Integral(atan(a + b*x)/(a + b*x), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(bx+a)}{dx+\frac{ad}{b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
```

```
[Out] integrate(arctan(b*x + a)/(d*x + a*d/b), x)
```

3.23 $\int (a+bx)^2 \sqrt{\tan^{-1}(a+bx)}\, dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left((a+bx)^2\sqrt{\tan^{-1}(a+bx)},x\right)$$

```
[Out] Unintegrable[(a + b*x)^2*Sqrt[ArcTan[a + b*x]], x]
```

Rubi [A] time = 0.0165432, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (a+bx)^2\sqrt{\tan^{-1}(a+bx)}\, dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*x)^2*Sqrt[ArcTan[a + b*x]],x]
```

```
[Out] Defer[Int][(a + b*x)^2*Sqrt[ArcTan[a + b*x]], x]
```

Rubi steps

$$\int (a+bx)^2\sqrt{\tan^{-1}(a+bx)}\, dx = \int (a+bx)^2\sqrt{\tan^{-1}(a+bx)}\, dx$$

Mathematica [A] time = 5.40776, size = 0, normalized size = 0.

$$\int (a+bx)^2\sqrt{\tan^{-1}(a+bx)}\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*x)^2*Sqrt[ArcTan[a + b*x]],x]
```

```
[Out] Integrate[(a + b*x)^2*Sqrt[ArcTan[a + b*x]], x]
```

Maple [A] time = 0.472, size = 0, normalized size = 0.

$$\int (bx+a)^2 \sqrt{\arctan(bx+a)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*arctan(b*x+a)^(1/2),x)
```

```
[Out] int((b*x+a)^2*arctan(b*x+a)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (a+bx)^2 \sqrt{\operatorname{atan}(a+bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*atan(b*x+a)**(1/2),x)
```

```
[Out] Integral((a + b*x)**2*sqrt(atan(a + b*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^2 \sqrt{\arctan(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^2*sqrt(arctan(b*x + a)), x)
```

3.24 $\int (e+fx)^3 \left(a+b\tan^{-1}(c+dx)\right) dx$

Optimal. Leaf size=233

$$\frac{(e+fx)^4\left(a+b\tan^{-1}(c+dx)\right)}{4f}-\frac{bfx\left(-\left(1-6c^2\right)f^2-12cdef+6d^2e^2\right)}{4d^3}-\frac{b\left(-6\left(1-c^2\right)d^2e^2f^2+4c\left(3-c^2\right)def^3+\left(c\right.\right.}{4}$$

```
[Out] -(b*f*(6*d^2*e^2 - 12*c*d*e*f - (1 - 6*c^2)*f^2)*x)/(4*d^3) - (b*f^2*(d*e -
 c*f)*(c + d*x)^2)/(2*d^4) - (b*f^3*(c + d*x)^3)/(12*d^4) - (b*(d^4*e^4 - 4
*c*d^3*e^3*f - 6*(1 - c^2)*d^2*e^2*f^2 + 4*c*(3 - c^2)*d*e*f^3 + (1 - 6*c^2
 + c^4)*f^4)*ArcTan[c + d*x])/(4*d^4*f) + ((e + f*x)^4*(a + b*ArcTan[c + d*
x]))/(4*f) - (b*(d*e - c*f)*(d*e + f - c*f)*(d*e - (1 + c)*f)*Log[1 + (c +
d*x)^2])/(2*d^4)
```

Rubi [A] time = 0.381339, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5047, 4862, 702, 635, 203, 260}

$$\frac{(e+fx)^4\left(a+b\tan^{-1}(c+dx)\right)}{4f}-\frac{bfx\left(-\left(1-6c^2\right)f^2-12cdef+6d^2e^2\right)}{4d^3}-\frac{b\left(-6\left(1-c^2\right)d^2e^2f^2+4c\left(3-c^2\right)def^3+\left(c\right.\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] -(b*f*(6*d^2*e^2 - 12*c*d*e*f - (1 - 6*c^2)*f^2)*x)/(4*d^3) - (b*f^2*(d*e -
 c*f)*(c + d*x)^2)/(2*d^4) - (b*f^3*(c + d*x)^3)/(12*d^4) - (b*(d^4*e^4 - 4
*c*d^3*e^3*f - 6*(1 - c^2)*d^2*e^2*f^2 + 4*c*(3 - c^2)*d*e*f^3 + (1 - 6*c^2
 + c^4)*f^4)*ArcTan[c + d*x])/(4*d^4*f) + ((e + f*x)^4*(a + b*ArcTan[c + d*
x]))/(4*f) - (b*(d*e - c*f)*(d*e + f - c*f)*(d*e - (1 + c)*f)*Log[1 + (c +
d*x)^2])/(2*d^4)
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int (e+fx)^3 \left(a+b\tan^{-1}(c+dx)\right) dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^3 \left(a+b\tan^{-1}(x)\right) dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^4 \left(a+b\tan^{-1}(c+dx)\right)}{4f} - \frac{b\,\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^4}{1+x^2} dx, x, c+dx\right)}{4f}$$

$$= \frac{(e+fx)^4 \left(a+b\tan^{-1}(c+dx)\right)}{4f} - \frac{b\,\text{Subst}\left(\int \left(\frac{f^2\left(6d^2e^2-12cdef-\left(1-6c^2\right)f^2\right)}{d^4} + \frac{4f^3(de-\right.\right.}{}$$

$$= -\frac{bf\left(6d^2e^2-12cdef-\left(1-6c^2\right)f^2\right)x}{4d^3} - \frac{bf^2(de-cf)(c+dx)^2}{2d^4} - \frac{bf^3(c+dx)^3}{12d^4}$$

$$= -\frac{bf\left(6d^2e^2-12cdef-\left(1-6c^2\right)f^2\right)x}{4d^3} - \frac{bf^2(de-cf)(c+dx)^2}{2d^4} - \frac{bf^3(c+dx)^3}{12d^4}$$

$$= -\frac{bf\left(6d^2e^2-12cdef-\left(1-6c^2\right)f^2\right)x}{4d^3} - \frac{bf^2(de-cf)(c+dx)^2}{2d^4} - \frac{bf^3(c+dx)^3}{12d^4}$$

Mathematica [C] time = 0.252758, size = 157, normalized size = 0.67

$$\frac{(e+fx)^4 \left(a+b\tan^{-1}(c+dx)\right) - \frac{b\left(6dfx^2x\left(\left(6c^2-1\right)f^2-12cdef+6d^2e^2\right)+12f^3(c+dx)^2(de-cf)-3i(de-(c-i)f)^4\log(-c-dx+i)+3i(de-(c+i)f)^4\log(c+dx\right.}{6d^4}}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^3*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] ((e + f*x)^4*(a + b*ArcTan[c + d*x]) - (b*(6*d*f^2*(6*d^2*e^2 - 12*c*d*e*f
+ (-1 + 6*c^2)*f^2)*x + 12*f^3*(d*e - c*f)*(c + d*x)^2 + 2*f^4*(c + d*x)^3
- (3*I)*(d*e - (-I + c)*f)^4*Log[I - c - d*x] + (3*I)*(d*e - (I + c)*f)^4*L
og[I + c + d*x]))/(6*d^4))/(4*f)
```

Maple [B] time = 0.042, size = 494, normalized size = 2.1

$$-\frac{bf^3\arctan(dx+c)}{4d^4} - \frac{b\ln\left(1+(dx+c)^2\right)e^3}{2d} + \frac{bf^3\arctan(dx+c)x^4}{4} + \arctan(dx+c)xbe^3 + \frac{bf^3x}{4d^3} + \frac{3afx^2e^2}{2} + a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*(a+b*arctan(d*x+c)),x)
```

```
[Out] -1/4/d^4*b*f^3*arctan(d*x+c)-1/2/d*b*ln(1+(d*x+c)^2)*e^3+1/4*b*f^3*arctan(d
*x+c)*x^4+arctan(d*x+c)*x*b*e^3+1/4*b/d^3*f^3*x+3/2*a*f*x^2*e^2+a*f^2*x^3*e
-13/12/d^4*b*f^3*c^3+1/4/d^4*b*f^3*c-1/12/d*b*f^3*x^3+1/4*a/f*e^4+b*f^2*arc
tan(d*x+c)*e*x^3-1/2/d*b*f^2*e*x^2-3/4*b/d^3*f^3*c^2*x-3/2*b/d*f*e^2*x+3/2/
d^2*b*f*arctan(d*x+c)*e^2+1/2/d^4*b*f^3*ln(1+(d*x+c)^2)*c^3+1/4/d^2*b*f^3*x
^2*c+1/2/d^3*b*f^2*ln(1+(d*x+c)^2)*e+3/2*b*f*arctan(d*x+c)*e^2*x^2-1/4/d^4*
b*f^3*arctan(d*x+c)*c^4+1/d*arctan(d*x+c)*b*c*e^3-1/2/d^4*b*f^3*ln(1+(d*x+c
)^2)*c+5/2/d^3*b*f^2*c^2*e-3/2/d^2*b*f*c*e^2+a*x*e^3+1/4*a*f^3*x^4+3/2/d^4*
b*f^3*arctan(d*x+c)*c^2-3/2/d^2*b*f*arctan(d*x+c)*e^2*c^2-3/d^3*b*f^2*arcta
n(d*x+c)*c*e-3/2/d^3*b*f^2*ln(1+(d*x+c)^2)*c^2*e+3/2/d^2*b*f*ln(1+(d*x+c)^2
)*c*e^2+2*b/d^2*f^2*c*e*x+1/d^3*b*f^2*arctan(d*x+c)*e*c^3
```

Maxima [A] time = 1.54294, size = 467, normalized size = 2.

$$\frac{1}{4}af^3x^4 + aef^2x^3 + \frac{3}{2}ae^2fx^2 + \frac{3}{2}\left(x^2\arctan(dx+c) - d\left(\frac{x}{d^2} + \frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3} - \frac{c\log\left(d^2x^2+2cdx+c^2+1\right)}{d^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*a*f^3*x^4 + a*e*f^2*x^3 + 3/2*a*e^2*f*x^2 + 3/2*(x^2*arctan(d*x + c) -
d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x
+ c^2 + 1)/d^3))*b*e^2*f + 1/2*(2*x^3*arctan(d*x + c) - d*((d*x^2 - 4*c*x)/
d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 +
 2*c*d*x + c^2 + 1)/d^4))*b*e*f^2 + 1/12*(3*x^4*arctan(d*x + c) - d*((d^2*x
^3 - 3*c*d*x^2 + 3*(3*c^2 - 1)*x)/d^4 + 3*(c^4 - 6*c^2 + 1)*arctan((d^2*x +
 c*d)/d)/d^5 - 6*(c^3 - c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^5))*b*f^3 + a
*e^3*x + 1/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*b*e^3/d
```

Fricas [A] time = 1.62432, size = 667, normalized size = 2.86

$$3ad^4f^3x^4 + \left(12ad^4ef^2 - bd^3f^3\right)x^3 + 3\left(6ad^4e^2f - 2bd^3ef^2 + bcd^2f^3\right)x^2 + 3\left(4ad^4e^3 - 6bd^3e^2f + 8bcd^2ef^2 - \left(3bc^2 - b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*d^4*f^3*x^4 + (12*a*d^4*e*f^2 - b*d^3*f^3)*x^3 + 3*(6*a*d^4*e^2*f
 - 2*b*d^3*e*f^2 + b*c*d^2*f^3)*x^2 + 3*(4*a*d^4*e^3 - 6*b*d^3*e^2*f + 8*b*
c*d^2*e*f^2 - (3*b*c^2 - b)*d*f^3)*x + 3*(b*d^4*f^3*x^4 + 4*b*d^4*e*f^2*x^3
 + 6*b*d^4*e^2*f*x^2 + 4*b*d^4*e^3*x + 4*b*c*d^3*e^3 - 6*(b*c^2 - b)*d^2*e^
2*f + 4*(b*c^3 - 3*b*c)*d*e*f^2 - (b*c^4 - 6*b*c^2 + b)*f^3)*arctan(d*x + c
) - 6*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + (3*b*c^2 - b)*d*e*f^2 - (b*c^3 - b*c)*
f^3)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/d^4
```

Sympy [A] time = 10.2317, size = 627, normalized size = 2.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*(a+b*atan(d*x+c)),x)
```

```
[Out] Piecewise((a*e**3*x + 3*a*e**2*f*x**2/2 + a*e*f**2*x**3 + a*f**3*x**4/4 - b
*c**4*f**3*atan(c + d*x)/(4*d**4) + b*c**3*e*f**2*atan(c + d*x)/d**3 + b*c*
*3*f**3*log(c**2/d**2 + 2*c*x/d + x**2 + d**(-2))/(2*d**4) - 3*b*c**2*e**2*
f*atan(c + d*x)/(2*d**2) - 3*b*c**2*e*f**2*log(c**2/d**2 + 2*c*x/d + x**2 +
 d**(-2))/(2*d**3) - 3*b*c**2*f**3*x/(4*d**3) + 3*b*c**2*f**3*atan(c + d*x)
/(2*d**4) + b*c*e**3*atan(c + d*x)/d + 3*b*c*e**2*f*log(c**2/d**2 + 2*c*x/d
 + x**2 + d**(-2))/(2*d**2) + 2*b*c*e*f**2*x/d**2 + b*c*f**3*x**2/(4*d**2)
- 3*b*c*e*f**2*atan(c + d*x)/d**3 - b*c*f**3*log(c**2/d**2 + 2*c*x/d + x**2
 + d**(-2))/(2*d**4) + b*e**3*x*atan(c + d*x) + 3*b*e**2*f*x**2*atan(c + d*
x)/2 + b*e*f**2*x**3*atan(c + d*x) + b*f**3*x**4*atan(c + d*x)/4 - b*e**3*l
og(c**2/d**2 + 2*c*x/d + x**2 + d**(-2))/(2*d) - 3*b*e**2*f*x/(2*d) - b*e*f
**2*x**2/(2*d) - b*f**3*x**3/(12*d) + 3*b*e**2*f*atan(c + d*x)/(2*d**2) + b
*e*f**2*log(c**2/d**2 + 2*c*x/d + x**2 + d**(-2))/(2*d**3) + b*f**3*x/(4*d*
*3) - b*f**3*atan(c + d*x)/(4*d**4), Ne(d, 0)), ((a + b*atan(c))*(e**3*x +
3*e**2*f*x**2/2 + e*f**2*x**3 + f**3*x**4/4), True))
```

Giac [B] time = 3.44363, size = 961, normalized size = 4.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(6*b*d^4*f^3*x^4*arctan(d*x + c) + 6*a*d^4*f^3*x^4 + 24*b*d^4*f^2*x^3*
arctan(d*x + c)*e + 24*a*d^4*f^2*x^3*e - 2*b*d^3*f^3*x^3 + 36*b*d^4*f*x^2*a
rctan(d*x + c)*e^2 - 3*pi*b*c^4*f^3*sgn(d*x + c) + 12*pi*b*c^3*d*f^2*e*sgn(
d*x + c) + 3*pi*b*c^4*f^3 + 6*b*c*d^2*f^3*x^2 + 6*b*c^4*f^3*arctan(1/(d*x +
 c)) + 36*a*d^4*f*x^2*e^2 - 12*pi*b*c^3*d*f^2*e - 12*b*d^3*f^2*x^2*e - 24*b
*c^3*d*f^2*arctan(1/(d*x + c))*e - 18*pi*b*c^2*d^2*f*e^2*sgn(d*x + c) - 18*
b*c^2*d*f^3*x + 24*b*d^4*x*arctan(d*x + c)*e^3 + 18*pi*b*c^2*d^2*f*e^2 + 36
*b*c^2*d^2*f*arctan(1/(d*x + c))*e^2 + 48*b*c*d^2*f^2*x*e + 12*b*c^3*f^3*lo
g(d^2*x^2 + 2*c*d*x + c^2 + 1) - 36*b*c^2*d*f^2*e*log(d^2*x^2 + 2*c*d*x + c
^2 + 1) + 18*pi*b*c^2*f^3*sgn(d*x + c) - 24*pi*b*c*d^3*e^3*sgn(d*x + c) - 3
6*pi*b*c*d*f^2*e*sgn(d*x + c) - 18*pi*b*c^2*f^3 - 36*b*c^2*f^3*arctan(1/(d*
x + c)) + 24*a*d^4*x*e^3 + 24*b*c*d^3*arctan(d*x + c)*e^3 - 36*b*d^3*f*x*e^
2 + 36*pi*b*c*d*f^2*e + 72*b*c*d*f^2*arctan(1/(d*x + c))*e + 36*b*c*d^2*f*e
^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 18*pi*b*d^2*f*e^2*sgn(d*x + c) + 6*b*
d*f^3*x - 18*pi*b*d^2*f*e^2 - 36*b*d^2*f*arctan(1/(d*x + c))*e^2 - 12*b*c*f
^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1) - 12*b*d^3*e^3*log(d^2*x^2 + 2*c*d*x +
c^2 + 1) + 12*b*d*f^2*e*log(d^2*x^2 + 2*c*d*x + c^2 + 1) - 3*pi*b*f^3*sgn(d
*x + c) + 3*pi*b*f^3 + 6*b*f^3*arctan(1/(d*x + c)))/d^4
```

3.25 $\int (e+fx)^2 \left(a+b\tan^{-1}(c+dx)\right) dx$

Optimal. Leaf size=155

$$\frac{(e+fx)^3\left(a+b\tan^{-1}(c+dx)\right)}{3f}-\frac{b\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\log\left((c+dx)^2+1\right)}{6d^3}-\frac{b(de-cf)\left(-\left(3-c^2\right)f^2-2c\right.}{3d^3}$$

```
[Out] -((b*f*(d*e - c*f)*x)/d^2) - (b*f^2*(c + d*x)^2)/(6*d^3) - (b*(d*e - c*f)*(
d^2*e^2 - 2*c*d*e*f - (3 - c^2)*f^2)*ArcTan[c + d*x])/(3*d^3*f) + ((e + f*x
)^3*(a + b*ArcTan[c + d*x]))/(3*f) - (b*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*c^2
)*f^2)*Log[1 + (c + d*x)^2])/(6*d^3)
```

Rubi [A] time = 0.189819, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5047, 4862, 702, 635, 203, 260}

$$\frac{(e+fx)^3\left(a+b\tan^{-1}(c+dx)\right)}{3f}-\frac{b\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\log\left((c+dx)^2+1\right)}{6d^3}-\frac{b(de-cf)\left(-\left(3-c^2\right)f^2-2c\right.}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] -((b*f*(d*e - c*f)*x)/d^2) - (b*f^2*(c + d*x)^2)/(6*d^3) - (b*(d*e - c*f)*(
d^2*e^2 - 2*c*d*e*f - (3 - c^2)*f^2)*ArcTan[c + d*x])/(3*d^3*f) + ((e + f*x
)^3*(a + b*ArcTan[c + d*x]))/(3*f) - (b*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*c^2
)*f^2)*Log[1 + (c + d*x)^2])/(6*d^3)
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int (e+fx)^2 \left(a+b\tan^{-1}(c+dx)\right) dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2 \left(a+b\tan^{-1}(x)\right) dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^3 \left(a+b\tan^{-1}(c+dx)\right)}{3f} - \frac{b\,\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^3}{1+x^2} dx, x, c+dx\right)}{3f}$$

$$= \frac{(e+fx)^3 \left(a+b\tan^{-1}(c+dx)\right)}{3f} - \frac{b\,\text{Subst}\left(\int \left(\frac{3f^2(de-cf)}{d^3}+\frac{f^3x}{d^3}+\frac{(de-cf)\left(d^2e^2-2cde\right.}{}\right.\right.}{}$$

$$= -\frac{bf(de-cf)x}{d^2} - \frac{bf^2(c+dx)^2}{6d^3} + \frac{(e+fx)^3 \left(a+b\tan^{-1}(c+dx)\right)}{3f} - \frac{b\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{bf(de-cf)x}{d^2} - \frac{bf^2(c+dx)^2}{6d^3} + \frac{(e+fx)^3 \left(a+b\tan^{-1}(c+dx)\right)}{3f} - \frac{\left(b\left(3d^2e^2 - \right.\right.}{}$$

$$= -\frac{bf(de-cf)x}{d^2} - \frac{bf^2(c+dx)^2}{6d^3} - \frac{b(de-cf)\left(d^2e^2-2cdef-\left(3-c^2\right)f^2\right)\tan^{-1}(}{3d^3f}$$

Mathematica [C] time = 0.139037, size = 118, normalized size = 0.76

$$\frac{(e+fx)^3 \left(a+b\tan^{-1}(c+dx)\right) - \frac{b\left(6df^2x(de-cf)-i(de-(c-i)f)^3\log(-c-dx+i)+i(de-(c+i)f)^3\log(c+dx+i)+f^3(c+dx)^2\right)}{2d^3}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] ((e + f*x)^3*(a + b*ArcTan[c + d*x]) - (b*(6*d*f^2*(d*e - c*f)*x + f^3*(c +
 d*x)^2 - I*(d*e - (-I + c)*f)^3*Log[I - c - d*x] + I*(d*e - (I + c)*f)^3*L
og[I + c + d*x]))/(2*d^3))/(3*f)
```

Maple [A] time = 0.041, size = 283, normalized size = 1.8

$$-\frac{\arctan(dx+c)\,bfec^2}{d^2} + \frac{bf^2\ln\left(1+(dx+c)^2\right)}{6\,d^3} + \frac{b\arctan(dx+c)\,ce^2}{d} + \frac{5\,bf^2c^2}{6\,d^3} + bf\arctan(dx+c)\,ex^2 + \frac{af^2x^3}{3} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(a+b*arctan(d*x+c)),x)
```

```
[Out] -1/d^2*b*f*arctan(d*x+c)*e*c^2+1/6/d^3*b*f^2*ln(1+(d*x+c)^2)+1/d*arctan(d*x
+c)*b*c*e^2+5/6/d^3*b*f^2*c^2+b*f*arctan(d*x+c)*e*x^2+1/3*a*f^2*x^3+1/3*a/f
*e^3-1/6/d*b*f^2*x^2+1/d^2*b*f*ln(1+(d*x+c)^2)*c*e-1/2*b*e^2*ln(1+(d*x+c)^2
)/d+arctan(d*x+c)*x*b*e^2+1/3/d^3*b*f^2*arctan(d*x+c)*c^3+1/3*b*f^2*arctan(
d*x+c)*x^3+1/d^2*b*f*arctan(d*x+c)*e-1/d^3*b*f^2*arctan(d*x+c)*c-1/2/d^3*b*
f^2*ln(1+(d*x+c)^2)*c^2-b/d*f*e*x+2/3*b/d^2*f^2*c*x+a*f*x^2*e+a*x*e^2-1/d^2
*b*f*c*e
```

Maxima [A] time = 1.53185, size = 297, normalized size = 1.92

$$\frac{1}{3}af^2x^3 + aefx^2 + \left(x^2\arctan\left(dx+c\right) - d\left(\frac{x}{d^2} + \frac{\left(c^2-1\right)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3} - \frac{c\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^3}\right)\right)bef + \frac{1}{6}\Bigg(2\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*a*f^2*x^3 + a*e*f*x^2 + (x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arc
tan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*b*e*f +
 1/6*(2*x^3*arctan(d*x + c) - d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan
((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*
b*f^2 + a*e^2*x + 1/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*
b*e^2/d
```

Fricas [A] time = 1.46635, size = 432, normalized size = 2.79

$$\frac{2\,ad^3f^2x^3 + \left(6\,ad^3ef - bd^2f^2\right)x^2 + 2\left(3\,ad^3e^2 - 3\,bd^2ef + 2\,bcdf^2\right)x + 2\left(bd^3f^2x^3 + 3\,bd^3efx^2 + 3\,bd^3e^2x + 3\,bcd^2e^2 - 3\,\left(\right.\right.}{6\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*d^3*f^2*x^3 + (6*a*d^3*e*f - b*d^2*f^2)*x^2 + 2*(3*a*d^3*e^2 - 3*b
*d^2*e*f + 2*b*c*d*f^2)*x + 2*(b*d^3*f^2*x^3 + 3*b*d^3*e*f*x^2 + 3*b*d^3*e^
2*x + 3*b*c*d^2*e^2 - 3*(b*c^2 - b)*d*e*f + (b*c^3 - 3*b*c)*f^2)*arctan(d*x
```

```
 + c) - (3*b*d^2*e^2 - 6*b*c*d*e*f + (3*b*c^2 - b)*f^2)*log(d^2*x^2 + 2*c*d
*x + c^2 + 1))/d^3
```

Sympy [A] time = 4.9976, size = 357, normalized size = 2.3

$$\begin{cases} ae^2x + aefx^2 + \frac{af^2x^3}{3} + \frac{bc^3f^2\operatorname{atan}(c+dx)}{3d^3} - \frac{bc^2ef\operatorname{atan}(c+dx)}{d^2} - \frac{bc^2f^2\log\left(\frac{c^2}{d^2}+\frac{2cx}{d}+x^2+\frac{1}{d^2}\right)}{2d^3} + \frac{bce^2\operatorname{atan}(c+dx)}{d} + \frac{bcef\log\left(\frac{c^2}{d^2}+\frac{2cx}{d}+x^2+\frac{1}{d^2}\right)}{d^2} \\ (a+b\operatorname{atan}(c))\left(e^2x+efx^2+\frac{f^2x^3}{3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(a+b*atan(d*x+c)),x)
```

```
[Out] Piecewise((a*e**2*x + a*e*f*x**2 + a*f**2*x**3/3 + b*c**3*f**2*atan(c + d*x
)/(3*d**3) - b*c**2*e*f*atan(c + d*x)/d**2 - b*c**2*f**2*log(c**2/d**2 + 2*
c*x/d + x**2 + d**(-2))/(2*d**3) + b*c*e**2*atan(c + d*x)/d + b*c*e*f*log(c
**2/d**2 + 2*c*x/d + x**2 + d**(-2))/d**2 + 2*b*c*f**2*x/(3*d**2) - b*c*f**
2*atan(c + d*x)/d**3 + b*e**2*x*atan(c + d*x) + b*e*f*x**2*atan(c + d*x) +
b*f**2*x**3*atan(c + d*x)/3 - b*e**2*log(c**2/d**2 + 2*c*x/d + x**2 + d**(-
2))/(2*d) - b*e*f*x/d - b*f**2*x**2/(6*d) + b*e*f*atan(c + d*x)/d**2 + b*f*
*2*log(c**2/d**2 + 2*c*x/d + x**2 + d**(-2))/(6*d**3), Ne(d, 0)), ((a + b*a
tan(c))*(e**2*x + e*f*x**2 + f**2*x**3/3), True))
```

Giac [B] time = 1.51601, size = 562, normalized size = 3.63

$$\frac{2\,bd^3f^2x^3\arctan(dx+c) + 2\,ad^3f^2x^3 + 6\,bd^3fx^2\arctan(dx+c)\,e + 6\,ad^3fx^2e + \pi bc^3f^2\operatorname{sgn}(dx+c) - 3\,\pi bc^2dfe\operatorname{sgn}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*d^3*f^2*x^3*arctan(d*x + c) + 2*a*d^3*f^2*x^3 + 6*b*d^3*f*x^2*arct
an(d*x + c)*e + 6*a*d^3*f*x^2*e + pi*b*c^3*f^2*sgn(d*x + c) - 3*pi*b*c^2*d*
f*e*sgn(d*x + c) - pi*b*c^3*f^2 - b*d^2*f^2*x^2 - 2*b*c^3*f^2*arctan(1/(d*x
 + c)) + 6*b*d^3*x*arctan(d*x + c)*e^2 + 3*pi*b*c^2*d*f*e + 6*b*c^2*d*f*arc
tan(1/(d*x + c))*e - 6*pi*b*c*d^2*e^2*sgn(d*x + c) + 4*b*c*d*f^2*x + 6*a*d^
3*x*e^2 + 6*b*c*d^2*arctan(d*x + c)*e^2 - 6*b*d^2*f*x*e - 3*b*c^2*f^2*log(d
```

```
^2*x^2 + 2*c*d*x + c^2 + 1) + 6*b*c*d*f*e*log(d^2*x^2 + 2*c*d*x + c^2 + 1)
- 3*pi*b*c*f^2*sgn(d*x + c) + 3*pi*b*d*f*e*sgn(d*x + c) + 3*pi*b*c*f^2 + 6*
b*c*f^2*arctan(1/(d*x + c)) - 3*pi*b*d*f*e - 6*b*d*f*arctan(1/(d*x + c))*e
- 3*b*d^2*e^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1) + b*f^2*log(d^2*x^2 + 2*c*d*
x + c^2 + 1))/d^3
```

3.26 $\int (e+fx)\left(a+b\tan^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=97

$$\frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)}{2f}-\frac{b(de-cf)\log\left((c+dx)^2+1\right)}{2d^2}-\frac{b(-cf+de+f)(de-(c+1)f)\tan^{-1}(c+dx)}{2d^2f}-\frac{bfx}{2d}$$

```
[Out] -(b*f*x)/(2*d) - (b*(d*e + f - c*f)*(d*e - (1 + c)*f)*ArcTan[c + d*x])/(2*d
^2*f) + ((e + f*x)^2*(a + b*ArcTan[c + d*x]))/(2*f) - (b*(d*e - c*f)*Log[1
+ (c + d*x)^2])/(2*d^2)
```

Rubi [A] time = 0.112068, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5047, 4862, 702, 635, 203, 260}

$$\frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)}{2f}-\frac{b(de-cf)\log\left((c+dx)^2+1\right)}{2d^2}-\frac{b(-cf+de+f)(de-(c+1)f)\tan^{-1}(c+dx)}{2d^2f}-\frac{bfx}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] -(b*f*x)/(2*d) - (b*(d*e + f - c*f)*(d*e - (1 + c)*f)*ArcTan[c + d*x])/(2*d
^2*f) + ((e + f*x)^2*(a + b*ArcTan[c + d*x]))/(2*f) - (b*(d*e - c*f)*Log[1
+ (c + d*x)^2])/(2*d^2)
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int (e+fx)\left(a+b\tan^{-1}(c+dx)\right)dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)\left(a+b\tan^{-1}(x)\right)dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)}{2f} - \frac{b\,\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2}{1+x^2}dx, x, c+dx\right)}{2f}$$

$$= \frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)}{2f} - \frac{b\,\text{Subst}\left(\int \left(\frac{f^2}{d^2}+\frac{(de-f-cf)(de+f-cf)+2f(de-cf)x}{d^2(1+x^2)}\right)dx \ldots\right.}{2f}$$

$$= -\frac{bfx}{2d} + \frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)}{2f} - \frac{b\,\text{Subst}\left(\int \frac{(de-f-cf)(de+f-cf)+2f(de-cf)x}{1+x^2}d \ldots\right.}{2d^2f}$$

$$= -\frac{bfx}{2d} + \frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)}{2f} - \frac{(b(de-cf))\,\text{Subst}\left(\int \frac{x}{1+x^2}dx, x, c+dx \ldots\right.}{d^2}$$

$$= -\frac{bfx}{2d} - \frac{b(de+f-cf)(de-(1+c)f)\tan^{-1}(c+dx)}{2d^2f} + \frac{(e+fx)^2\left(a+b\tan^{-1}(c+ \ldots\right.}{2f}$$

Mathematica [C] time = 0.0548483, size = 163, normalized size = 1.68

$$aex+\frac{1}{2}afx^2-\frac{be\left(\log\left(c^2+2cdx+d^2x^2+1\right)-2c\tan^{-1}(c+dx)\right)}{2d}+\frac{bf\left(\frac{1}{2}d\left(\frac{c+dx}{d}-\frac{c}{d}\right)^2\tan^{-1}(c+dx)-\frac{1}{2}d\left(-\frac{i(-c+i)^2\log}{2d}\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] a*e*x + (a*f*x^2)/2 + b*e*x*ArcTan[c + d*x] + (b*f*((d*(-(c/d) + (c + d*x)/
d)^2*ArcTan[c + d*x])/2 - (d*(x/d - ((I/2)*(I - c)^2*Log[I - c - d*x])/d^2
+ ((I/2)*(I + c)^2*Log[I + c + d*x])/d^2))/2))/d - (b*e*(-2*c*ArcTan[c + d*
x] + Log[1 + c^2 + 2*c*d*x + d^2*x^2]))/(2*d)
```

Maple [A] time = 0.039, size = 146, normalized size = 1.5

$$\frac{ax^2f}{2}-\frac{ac^2f}{2d^2}+aex+\frac{ace}{d}+\frac{bf\arctan(dx+c)x^2}{2}-\frac{\arctan(dx+c)bc^2f}{2d^2}+\arctan(dx+c)xbe+\frac{b\arctan(dx+c)ce}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(a+b*arctan(d*x+c)),x)
```

```
[Out] 1/2*a*x^2*f-1/2/d^2*a*f*c^2+a*e*x+1/d*a*c*e+1/2*b*f*arctan(d*x+c)*x^2-1/2/d
^2*b*f*arctan(d*x+c)*c^2+arctan(d*x+c)*x*b*e+1/d*arctan(d*x+c)*b*c*e-1/2*b*
f*x/d-1/2/d^2*b*c*f+1/2/d^2*b*ln(1+(d*x+c)^2)*c*f-1/2/d*b*ln(1+(d*x+c)^2)*e
+1/2/d^2*b*f*arctan(d*x+c)
```

Maxima [A] time = 1.48311, size = 157, normalized size = 1.62

$$\frac{1}{2}afx^2+\frac{1}{2}\left(x^2\arctan(dx+c)-d\left(\frac{x}{d^2}+\frac{\left(c^2-1\right)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3}-\frac{c\log\left(d^2x^2+2cdx+c^2+1\right)}{d^3}\right)\right)bf+aex+\frac{\left(2(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*a*f*x^2 + 1/2*(x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x
 + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*b*f + a*e*x + 1/2
*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*b*e/d
```

Fricas [A] time = 1.42201, size = 232, normalized size = 2.39

$$\frac{ad^2fx^2 + \left(2\,ad^2e - bdf\right)x + \left(bd^2fx^2 + 2\,bd^2ex + 2\,bcde - \left(bc^2 - b\right)f\right)\arctan\left(dx + c\right) - \left(bde - bcf\right)\log\left(d^2x^2 + 2\,cdx + c\right.}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d^2*f*x^2 + (2*a*d^2*e - b*d*f)*x + (b*d^2*f*x^2 + 2*b*d^2*e*x + 2*b
*c*d*e - (b*c^2 - b)*f)*arctan(d*x + c) - (b*d*e - b*c*f)*log(d^2*x^2 + 2*c
*d*x + c^2 + 1))/d^2
```

Sympy [A] time = 2.4684, size = 177, normalized size = 1.82

$$\begin{cases} aex + \frac{afx^2}{2} - \frac{bc^2f\,\mathrm{atan}\,(c+dx)}{2d^2} + \frac{bce\,\mathrm{atan}\,(c+dx)}{d} + \frac{bcf\log\left(\frac{c^2}{d^2} + \frac{2cx}{d} + x^2 + \frac{1}{d^2}\right)}{2d^2} + bex\,\mathrm{atan}\,(c+dx) + \frac{bfx^2\,\mathrm{atan}\,(c+dx)}{2} - \frac{be\log\left(\frac{c^2}{d^2} + \frac{2cx}{d} + x^2 + \right.}{2d} \\ (a + b\,\mathrm{atan}\,(c))\left(ex + \frac{fx^2}{2}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*atan(d*x+c)),x)
```

```
[Out] Piecewise((a*e*x + a*f*x**2/2 - b*c**2*f*atan(c + d*x)/(2*d**2) + b*c*e*ata
n(c + d*x)/d + b*c*f*log(c**2/d**2 + 2*c*x/d + x**2 + d**(-2))/(2*d**2) + b
*e*x*atan(c + d*x) + b*f*x**2*atan(c + d*x)/2 - b*e*log(c**2/d**2 + 2*c*x/d
 + x**2 + d**(-2))/(2*d) - b*f*x/(2*d) + b*f*atan(c + d*x)/(2*d**2), Ne(d,
0)), ((a + b*atan(c))*(e*x + f*x**2/2), True))
```

Giac [B] time = 1.20523, size = 269, normalized size = 2.77

$$\frac{2\,bd^2fx^2\arctan\left(dx + c\right) + 2\,ad^2fx^2 + 4\,bd^2x\arctan\left(dx + c\right)e - \pi bc^2f\mathrm{sgn}\left(dx + c\right) - 4\,\pi bcde\mathrm{sgn}\left(dx + c\right) + \pi bc^2f + 2\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(2*b*d^2*f*x^2*arctan(d*x + c) + 2*a*d^2*f*x^2 + 4*b*d^2*x*arctan(d*x +
 c)*e - pi*b*c^2*f*sgn(d*x + c) - 4*pi*b*c*d*e*sgn(d*x + c) + pi*b*c^2*f +
2*b*c^2*f*arctan(1/(d*x + c)) + 4*a*d^2*x*e + 4*b*c*d*arctan(d*x + c)*e - 2
*b*d*f*x + 2*b*c*f*log(d^2*x^2 + 2*c*d*x + c^2 + 1) - 2*b*d*e*log(d^2*x^2 +
 2*c*d*x + c^2 + 1) + pi*b*f*sgn(d*x + c) - pi*b*f - 2*b*f*arctan(1/(d*x +
c)))/d^2
```

3.27 $\int \left(a+b\tan^{-1}(c+dx)\right) dx$

Optimal. Leaf size=38

$$ax-\frac{b\log\left((c+dx)^2+1\right)}{2d}+\frac{b(c+dx)\tan^{-1}(c+dx)}{d}$$

```
[Out] a*x + (b*(c + d*x)*ArcTan[c + d*x])/d - (b*Log[1 + (c + d*x)^2])/(2*d)
```

Rubi [A] time = 0.0184687, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5039, 4846, 260}

$$ax-\frac{b\log\left((c+dx)^2+1\right)}{2d}+\frac{b(c+dx)\tan^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTan[c + d*x],x]
```

```
[Out] a*x + (b*(c + d*x)*ArcTan[c + d*x])/d - (b*Log[1 + (c + d*x)^2])/(2*d)
```

Rule 5039

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+b\tan^{-1}(c+dx)\right)dx &= ax+b\int \tan^{-1}(c+dx)\,dx\\
&= ax+\frac{b\,\mathrm{Subst}\left(\int \tan^{-1}(x)\,dx,x,c+dx\right)}{d}\\
&= ax+\frac{b(c+dx)\tan^{-1}(c+dx)}{d}-\frac{b\,\mathrm{Subst}\left(\int \frac{x}{1+x^2}\,dx,x,c+dx\right)}{d}\\
&= ax+\frac{b(c+dx)\tan^{-1}(c+dx)}{d}-\frac{b\log\left(1+(c+dx)^2\right)}{2d}
\end{aligned}
$$

Mathematica [A] time = 0.0132084, size = 49, normalized size = 1.29

$$ax-\frac{b\left(\log\left(c^2+2cdx+d^2x^2+1\right)-2c\tan^{-1}(c+dx)\right)}{2d}+bx\tan^{-1}(c+dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTan[c + d*x],x]
```

```
[Out] a*x + b*x*ArcTan[c + d*x] - (b*(-2*c*ArcTan[c + d*x] + Log[1 + c^2 + 2*c*d*
x + d^2*x^2]))/(2*d)
```

Maple [A] time = 0.033, size = 42, normalized size = 1.1

$$ax+b\arctan(dx+c)\,x+\frac{b\arctan(dx+c)\,c}{d}-\frac{b\ln\left(1+(dx+c)^2\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctan(d*x+c),x)
```

```
[Out] a*x+b*arctan(d*x+c)*x+b/d*arctan(d*x+c)*c-1/2*b*ln(1+(d*x+c)^2)/d
```

Maxima [A] time = 0.973068, size = 49, normalized size = 1.29

$$ax+\frac{\left(2\,(dx+c)\arctan(dx+c)-\log\left((dx+c)^2+1\right)\right)b}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + 1/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*b/d
```

Fricas [A] time = 1.51522, size = 119, normalized size = 3.13

$$\frac{2\,adx + 2\,(bdx + bc)\arctan\left(dx + c\right) - b\log\left(d^{2}x^{2} + 2\,cdx + c^{2} + 1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*d*x + 2*(b*d*x + b*c)*arctan(d*x + c) - b*log(d^2*x^2 + 2*c*d*x +
c^2 + 1))/d
```

Sympy [A] time = 0.565511, size = 51, normalized size = 1.34

$$ax + b\left(\begin{cases} \frac{c\operatorname{atan}\left(c+dx\right)}{d} + x\operatorname{atan}\left(c + dx\right) - \frac{\log\left(c^{2}+2cdx+d^{2}x^{2}+1\right)}{2d} & \text{for}\: d \neq 0 \\ x\operatorname{atan}\left(c\right) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atan(d*x+c),x)
```

```
[Out] a*x + b*Piecewise((c*atan(c + d*x)/d + x*atan(c + d*x) - log(c**2 + 2*c*d*x
 + d**2*x**2 + 1)/(2*d), Ne(d, 0)), (x*atan(c), True))
```

Giac [A] time = 1.08183, size = 49, normalized size = 1.29

$$ax + \frac{\left(2\,(dx + c)\arctan\left(dx + c\right) - \log\left((dx + c)^{2} + 1\right)\right)b}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(d*x+c),x, algorithm="giac")

[Out] a*x + 1/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*b/d
```

3.28 $\int \frac{a+b\tan^{-1}(c+dx)}{e+fx}\,dx$

Optimal. Leaf size=162

$$-\frac{ib\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{2f}+\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{2f}+\frac{\left(a+b\tan^{-1}(c+dx)\right)\log\left(\frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{f}$$

```
[Out] -(((a + b*ArcTan[c + d*x])*Log[2/(1 - I*(c + d*x))])/f) + ((a + b*ArcTan[c
+ d*x])*Log[(2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f + ((I
/2)*b*PolyLog[2, 1 - 2/(1 - I*(c + d*x))])/f - ((I/2)*b*PolyLog[2, 1 - (2*d
*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f
```

Rubi [A] time = 0.150132, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5047, 4856, 2402, 2315, 2447}

$$-\frac{ib\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{2f}+\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{2f}+\frac{\left(a+b\tan^{-1}(c+dx)\right)\log\left(\frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])/(e + f*x),x]
```

```
[Out] -(((a + b*ArcTan[c + d*x])*Log[2/(1 - I*(c + d*x))])/f) + ((a + b*ArcTan[c
+ d*x])*Log[(2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f + ((I
/2)*b*PolyLog[2, 1 - 2/(1 - I*(c + d*x))])/f - ((I/2)*b*PolyLog[2, 1 - (2*d
*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
```

```
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(c+dx)}{e+fx}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{\frac{de-cf}{d}+\frac{fx}{d}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)\log\left(\frac{2}{1-i(c+dx)}\right)}{f} + \frac{\left(a+b\tan^{-1}(c+dx)\right)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f} + \frac{b}{-}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)\log\left(\frac{2}{1-i(c+dx)}\right)}{f} + \frac{\left(a+b\tan^{-1}(c+dx)\right)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f} - \frac{ib}{-}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)\log\left(\frac{2}{1-i(c+dx)}\right)}{f} + \frac{\left(a+b\tan^{-1}(c+dx)\right)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f} + \frac{ib}{-}$$

Mathematica [A] time = 0.104057, size = 160, normalized size = 0.99

$$\frac{-ib\text{PolyLog}\left(2,\frac{f(c+dx-i)}{-de+(c-i)f}\right)+ib\text{PolyLog}\left(2,\frac{f(c+dx+i)}{-de+(c+i)f}\right)+2a\log(d(e+fx))+ib\log(1-i(c+dx))\log\left(\frac{d(e+fx)}{de-(c+i)f}\right)-ib\log(1}{2f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c + d*x])/(e + f*x),x]
```

```
[Out] (2*a*Log[d*(e + f*x)] + I*b*Log[(d*(e + f*x))/(d*e - (I + c)*f)]*Log[1 - I*
(c + d*x)] - I*b*Log[(d*(e + f*x))/(d*e + I*f - c*f)]*Log[1 + I*(c + d*x)]
- I*b*PolyLog[2, (f*(-I + c + d*x))/(-(d*e) + (-I + c)*f)] + I*b*PolyLog[2,
 (f*(I + c + d*x))/(-(d*e) + (I + c)*f)])/(2*f)
```

Maple [A] time = 0.072, size = 224, normalized size = 1.4

$$\frac{a\ln\left(f(dx+c)-cf+de\right)}{f}+\frac{b\ln\left(f(dx+c)-cf+de\right)\arctan(dx+c)}{f}+\frac{\frac{i}{2}b\ln\left(f(dx+c)-cf+de\right)}{f}\ln\left(\frac{if-f(dx+c}{de+if-cf}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))/(f*x+e),x)
```

```
[Out] a*ln(f*(d*x+c)-c*f+d*e)/f+b*ln(f*(d*x+c)-c*f+d*e)/f*arctan(d*x+c)+1/2*I*b*l
n(f*(d*x+c)-c*f+d*e)/f*ln((I*f-f*(d*x+c))/(d*e+I*f-c*f))-1/2*I*b*ln(f*(d*x+
c)-c*f+d*e)/f*ln((I*f+f*(d*x+c))/(I*f+c*f-d*e))+1/2*I*b/f*dilog((I*f-f*(d*x
+c))/(d*e+I*f-c*f))-1/2*I*b/f*dilog((I*f+f*(d*x+c))/(I*f+c*f-d*e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2\,b\int\frac{\arctan(dx+c)}{2\left(fx+e\right)}\,dx+\frac{a\log\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e),x, algorithm="maxima")
```

```
[Out] 2*b*integrate(1/2*arctan(d*x + c)/(f*x + e), x) + a*log(f*x + e)/f
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\arctan\left(dx+c\right)+a}{fx+e},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(d*x + c) + a)/(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\operatorname{atan}\left(c+dx\right)}{e+fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))/(f*x+e),x)
```

```
[Out] Integral((a + b*atan(c + d*x))/(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\arctan\left(dx+c\right)+a}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(d*x + c) + a)/(f*x + e), x)
```

3.29 $\int \frac{a+b\tan^{-1}(c+dx)}{(e+fx)^2}\,dx$

Optimal. Leaf size=151

$$-\frac{a+b\tan^{-1}(c+dx)}{f(e+fx)}-\frac{bd\log\left(c^2+2cdx+d^2x^2+1\right)}{2\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)}+\frac{bd\log(e+fx)}{\left(c^2+1\right)f^2-2cdef+d^2e^2}+\frac{bd(de-cf)\tan^{-1}(c+dx)}{f\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)}$$

```
[Out] (b*d*(d*e - c*f)*ArcTan[c + d*x])/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2))
 - (a + b*ArcTan[c + d*x])/(f*(e + f*x)) + (b*d*Log[e + f*x])/(d^2*e^2 - 2*
c*d*e*f + (1 + c^2)*f^2) - (b*d*Log[1 + c^2 + 2*c*d*x + d^2*x^2])/(2*(d^2*e
^2 - 2*c*d*e*f + (1 + c^2)*f^2))
```

Rubi [A] time = 0.121052, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5045, 1982, 705, 31, 634, 618, 204, 628}

$$-\frac{a+b\tan^{-1}(c+dx)}{f(e+fx)}-\frac{bd\log\left(c^2+2cdx+d^2x^2+1\right)}{2\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)}+\frac{bd\log(e+fx)}{\left(c^2+1\right)f^2-2cdef+d^2e^2}+\frac{bd(de-cf)\tan^{-1}(c+dx)}{f\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])/(e + f*x)^2,x]
```

```
[Out] (b*d*(d*e - c*f)*ArcTan[c + d*x])/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2))
 - (a + b*ArcTan[c + d*x])/(f*(e + f*x)) + (b*d*Log[e + f*x])/(d^2*e^2 - 2*
c*d*e*f + (1 + c^2)*f^2) - (b*d*Log[1 + c^2 + 2*c*d*x + d^2*x^2])/(2*(d^2*e
^2 - 2*c*d*e*f + (1 + c^2)*f^2))
```

Rule 5045

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[((e + f*x)^(m + 1)*(a + b*ArcTan[c + d*x])^p)/(f*(m +
 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcTan[c
 + d*x])^(p - 1))/(1 + (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 1982

```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
```

```
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(c+dx)}{(e+fx)^2}\,dx = -\frac{a+b\tan^{-1}(c+dx)}{f(e+fx)} + \frac{(bd)\int \frac{1}{(e+fx)\left(1+(c+dx)^2\right)}\,dx}{f}$$

$$= -\frac{a+b\tan^{-1}(c+dx)}{f(e+fx)} + \frac{(bd)\int \frac{1}{(e+fx)\left(1+c^2+2cdx+d^2x^2\right)}\,dx}{f}$$

$$= -\frac{a+b\tan^{-1}(c+dx)}{f(e+fx)} + \frac{(bd)\int \frac{d^2e-2cdf-d^2fx}{1+c^2+2cdx+d^2x^2}\,dx}{f\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)} + \frac{(bdf)\int \frac{1}{e+fx}\,dx}{d^2e^2-2cdef+\left(1+c^2\right)f^2}$$

$$= -\frac{a+b\tan^{-1}(c+dx)}{f(e+fx)} + \frac{bd\log(e+fx)}{d^2e^2-2cdef+\left(1+c^2\right)f^2} - \frac{(bd)\int \frac{2cd+2d^2x}{1+c^2+2cdx+d^2x^2}\,dx}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)} + \frac{\left(bd^2\left(\cdots\right.\right.}{f\left(\cdots\right.}$$

$$= -\frac{a+b\tan^{-1}(c+dx)}{f(e+fx)} + \frac{bd\log(e+fx)}{d^2e^2-2cdef+\left(1+c^2\right)f^2} - \frac{bd\log\left(1+c^2+2cdx+d^2x^2\right)}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)} - \frac{\left(2bd^2\cdots\right.}{\cdots}$$

$$= \frac{bd(de-cf)\tan^{-1}(c+dx)}{f\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)} - \frac{a+b\tan^{-1}(c+dx)}{f(e+fx)} + \frac{bd\log(e+fx)}{d^2e^2-2cdef+\left(1+c^2\right)f^2} - \frac{bd\log\cdots}{2\left(d^2e^2\cdots\right.}$$

Mathematica [C] time = 0.156554, size = 121, normalized size = 0.8

$$\frac{-\frac{a+b\tan^{-1}(c+dx)}{e+fx} + \frac{bd(i(-de+(c+i)f)\log(-c-dx+i)+i(-cf+de+if)\log(c+dx+i)+2f\log(d(e+fx)))}{2\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])/(e + f*x)^2,x]
```

```
[Out] (-((a + b*ArcTan[c + d*x])/(e + f*x)) + (b*d*(I*(-(d*e) + (I + c)*f)*Log[I
- c - d*x] + I*(d*e + I*f - c*f)*Log[I + c + d*x] + 2*f*Log[d*(e + f*x)]))/
(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)))/f
```

Maple [A] time = 0.045, size = 205, normalized size = 1.4

$$-\frac{ad}{\left(dfx+de\right)f} - \frac{bd\arctan\left(dx+c\right)}{\left(dfx+de\right)f} - \frac{bd\ln\left(1+\left(dx+c\right)^2\right)}{2c^2f^2-4cdef+2d^2e^2+2f^2} - \frac{bd\arctan\left(dx+c\right)c}{c^2f^2-2cdef+d^2e^2+f^2} + \frac{bd^2\arctan\left(dx\cdots\right.}{f\left(c^2f^2-2cdef+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))/(f*x+e)^2,x)
```

```
[Out] -d*a/(d*f*x+d*e)/f-d*b/(d*f*x+d*e)/f*arctan(d*x+c)-1/2*d*b/(c^2*f^2-2*c*d*e
*f+d^2*e^2+f^2)*ln(1+(d*x+c)^2)-d*b/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(
d*x+c)*c+d^2*b/f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)*e+d*b/(c^2*f
^2-2*c*d*e*f+d^2*e^2+f^2)*ln(f*(d*x+c)-c*f+d*e)
```

Maxima [A] time = 1.53327, size = 239, normalized size = 1.58

$$\frac{1}{2}\left(d\left(\frac{2\left(d^2e-cdf\right)\arctan\left(\frac{d^2x+cd}{d}\right)}{\left(d^2e^2f-2\,cdef^2+\left(c^2+1\right)f^3\right)d}-\frac{\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^2e^2-2\,cdef+\left(c^2+1\right)f^2}+\frac{2\,\log\left(fx+e\right)}{d^2e^2-2\,cdef+\left(c^2+1\right)f^2}\right)-\frac{2\,\arctan\left(dx+c\right)}{f^2x+ef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(d*(2*(d^2*e - c*d*f)*arctan((d^2*x + c*d)/d)/((d^2*e^2*f - 2*c*d*e*f^2
 + (c^2 + 1)*f^3)*d) - log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*e^2 - 2*c*d*e*
f + (c^2 + 1)*f^2) + 2*log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 + 1)*f^2))
- 2*arctan(d*x + c)/(f^2*x + e*f))*b - a/(f^2*x + e*f)
```

Fricas [A] time = 3.03752, size = 435, normalized size = 2.88

$$-\frac{2\,ad^2e^2-4\,acdef+2\left(ac^2+a\right)f^2-2\left(bcdef-\left(bc^2+b\right)f^2+\left(bd^2ef-bcdf^2\right)x\right)\arctan\left(dx+c\right)+\left(bdf^2x+bdef\right)\log}{2\left(d^2e^3f-2\,cde^2f^2+\left(c^2+1\right)ef^3+\left(d^2e^2f^2-2\,cdef^3+\left(c^2+1\right)f\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d^2*e^2 - 4*a*c*d*e*f + 2*(a*c^2 + a)*f^2 - 2*(b*c*d*e*f - (b*c^2
 + b)*f^2 + (b*d^2*e*f - b*c*d*f^2)*x)*arctan(d*x + c) + (b*d*f^2*x + b*d*e
*f)*log(d^2*x^2 + 2*c*d*x + c^2 + 1) - 2*(b*d*f^2*x + b*d*e*f)*log(f*x + e)
)/(d^2*e^3*f - 2*c*d*e^2*f^2 + (c^2 + 1)*e*f^3 + (d^2*e^2*f^2 - 2*c*d*e*f^3
 + (c^2 + 1)*f^4)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))/(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09362, size = 393, normalized size = 2.6

$$-\frac{1}{2}\Bigg(df^2\Bigg(\frac{\log\Big(d^2+\frac{2\,cdf}{fx+e}+\frac{c^2f^2}{(fx+e)^2}-\frac{2\,d^2e}{fx+e}-\frac{2\,cdfe}{(fx+e)^2}+\frac{d^2e^2}{(fx+e)^2}+\frac{f^2}{(fx+e)^2}\Big)}{c^2f^4-2\,cdf^3e+d^2f^2e^2+f^4}+\frac{2\,(cdf-d^2e)\arctan\Big(-\frac{cdf+\frac{c^2f^2}{fx+e}-d^2e-\frac{2\,cdfe}{fx+e}+\frac{d^2e^2}{fx+e}+\cdots}{df}}{(c^2f^3-2\,cdf^2e+d^2fe^2+f^3)df^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] -1/2*(d*f^2*(log(d^2 + 2*c*d*f/(f*x + e) + c^2*f^2/(f*x + e)^2 - 2*d^2*e/(f
*x + e) - 2*c*d*f*e/(f*x + e)^2 + d^2*e^2/(f*x + e)^2 + f^2/(f*x + e)^2)/(c
^2*f^4 - 2*c*d*f^3*e + d^2*f^2*e^2 + f^4) + 2*(c*d*f - d^2*e)*arctan(-(c*d*
f + c^2*f^2/(f*x + e) - d^2*e - 2*c*d*f*e/(f*x + e) + d^2*e^2/(f*x + e) + f
^2/(f*x + e))/(d*f))/((c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2 + f^3)*d*f^2)) + 2
*arctan(d*x + c)/((f*x + e)*f))*b - a/((f*x + e)*f)
```

3.30 $\int \frac{a+b\tan^{-1}(c+dx)}{(e+fx)^3} dx$

Optimal. Leaf size=227

$$-\frac{a+b\tan^{-1}(c+dx)}{2f(e+fx)^2}-\frac{bd^2(de-cf)\log\left(c^2+2cdx+d^2x^2+1\right)}{2\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)^2}-\frac{bd}{2(e+fx)\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)}+\frac{bd^2(de-}{\left(\left(c^2+1\right)f^2\right.}$$

```
[Out] -(b*d)/(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)*(e + f*x)) + (b*d^2*(d*e +
f - c*f)*(d*e - (1 + c)*f)*ArcTan[c + d*x])/(2*f*(d^2*e^2 - 2*c*d*e*f + (1
+ c^2)*f^2)^2) - (a + b*ArcTan[c + d*x])/(2*f*(e + f*x)^2) + (b*d^2*(d*e -
c*f)*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)^2 - (b*d^2*(d*e -
c*f)*Log[1 + c^2 + 2*c*d*x + d^2*x^2])/(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*
f^2)^2)
```

Rubi [A] time = 0.303089, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5045, 1982, 709, 800, 634, 618, 204, 628}

$$-\frac{a+b\tan^{-1}(c+dx)}{2f(e+fx)^2}-\frac{bd^2(de-cf)\log\left(c^2+2cdx+d^2x^2+1\right)}{2\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)^2}-\frac{bd}{2(e+fx)\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)}+\frac{bd^2(de-}{\left(\left(c^2+1\right)f^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])/(e + f*x)^3,x]
```

```
[Out] -(b*d)/(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)*(e + f*x)) + (b*d^2*(d*e +
f - c*f)*(d*e - (1 + c)*f)*ArcTan[c + d*x])/(2*f*(d^2*e^2 - 2*c*d*e*f + (1
+ c^2)*f^2)^2) - (a + b*ArcTan[c + d*x])/(2*f*(e + f*x)^2) + (b*d^2*(d*e -
c*f)*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)^2 - (b*d^2*(d*e -
c*f)*Log[1 + c^2 + 2*c*d*x + d^2*x^2])/(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*
f^2)^2)
```

Rule 5045

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[((e + f*x)^(m + 1)*(a + b*ArcTan[c + d*x])^p)/(f*(m +
 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcTan[c
 + d*x])^(p - 1))/(1 + (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 1982

```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(c+dx)}{(e+fx)^3}\,dx = -\frac{a+b\tan^{-1}(c+dx)}{2f(e+fx)^2} + \frac{(bd)\int \frac{1}{(e+fx)^2\left(1+(c+dx)^2\right)}\,dx}{2f}$$

$$= -\frac{a+b\tan^{-1}(c+dx)}{2f(e+fx)^2} + \frac{(bd)\int \frac{1}{(e+fx)^2\left(1+c^2+2cdx+d^2x^2\right)}\,dx}{2f}$$

$$= -\frac{bd}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)(e+fx)} - \frac{a+b\tan^{-1}(c+dx)}{2f(e+fx)^2} + \frac{(bd)\int \frac{d(de-2cf)-d^2fx}{(e+fx)\left(1+c^2+2cdx+d^2\right.}}{2f\left(d^2e^2-2cdef+\left(1+\right.\right.}$$

$$= -\frac{bd}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)(e+fx)} - \frac{a+b\tan^{-1}(c+dx)}{2f(e+fx)^2} + \frac{(bd)\int\left(\frac{2df^2(de-cf)}{\left(d^2e^2-2cdef+\left(1+c^2\right)f\right.}\right.}{2f\left(\right.}$$

$$= -\frac{bd}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)(e+fx)} - \frac{a+b\tan^{-1}(c+dx)}{2f(e+fx)^2} + \frac{bd^2(de-cf)\log(e+f}{\left(d^2e^2-2cdef+\left(1+c^2\right)\right.}$$

$$= -\frac{bd}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)(e+fx)} - \frac{a+b\tan^{-1}(c+dx)}{2f(e+fx)^2} + \frac{bd^2(de-cf)\log(e+f}{\left(d^2e^2-2cdef+\left(1+c^2\right)\right.}$$

$$= -\frac{bd}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)(e+fx)} - \frac{a+b\tan^{-1}(c+dx)}{2f(e+fx)^2} + \frac{bd^2(de-cf)\log(e+f}{\left(d^2e^2-2cdef+\left(1+c^2\right)\right.}$$

$$= -\frac{bd}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)(e+fx)} + \frac{bd^2(de-f-cf)(de+f-cf)\tan^{-1}(c+dx)}{2f\left(d^2e^2-2cdef+f^2+c^2f^2\right)^2} -$$

Mathematica [C] time = 0.555227, size = 175, normalized size = 0.77

$$\frac{-\frac{a+b\tan^{-1}(c+dx)}{(e+fx)^2}+\frac{1}{2}bd^2\left(-\frac{2f}{d(e+fx)\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)}-\frac{4f(cf-de)\log(d(e+fx))}{\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)^2}-\frac{i\log(-c-dx+i)}{(de-(c-i)f)^2}+\frac{i\log(c+dx+i)}{(de-(c+i)f)^2}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])/(e + f*x)^3,x]
```

```
[Out] (-((a + b*ArcTan[c + d*x])/(e + f*x)^2) + (b*d^2*((-2*f)/(d*(d^2*e^2 - 2*c*
d*e*f + (1 + c^2)*f^2)*(e + f*x)) - (I*Log[I - c - d*x])/(d*e - (-I + c)*f)
^2 + (I*Log[I + c + d*x])/(d*e - (I + c)*f)^2 - (4*f*(-(d*e) + c*f)*Log[d*(
e + f*x)])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)^2))/2)/(2*f)
```

Maple [A] time = 0.046, size = 438, normalized size = 1.9

$$-\frac{d^2a}{2\left(dfx+de\right)^2f}-\frac{d^2b\arctan\left(dx+c\right)}{2\left(dfx+de\right)^2f}+\frac{d^2bf\arctan\left(dx+c\right)c^2}{2\left(c^2f^2-2\,cdef+d^2e^2+f^2\right)^2}-\frac{bd^3\arctan\left(dx+c\right)ce}{\left(c^2f^2-2\,cdef+d^2e^2+f^2\right)^2}+\frac{d^4b\,\mathrm{ar}}{2\left(c^2f^2-2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))/(f*x+e)^3,x)
```

```
[Out] -1/2*d^2*a/(d*f*x+d*e)^2/f-1/2*d^2*b/(d*f*x+d*e)^2/f*arctan(d*x+c)+1/2*d^2*
b*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)^2*arctan(d*x+c)*c^2-d^3*b/(c^2*f^2-2*c*
d*e*f+d^2*e^2+f^2)^2*arctan(d*x+c)*c*e+1/2*d^4*b/f/(c^2*f^2-2*c*d*e*f+d^2*e
^2+f^2)^2*arctan(d*x+c)*e^2+1/2*d^2*b*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)^2*l
n(1+(d*x+c)^2)*c-1/2*d^3*b/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)^2*ln(1+(d*x+c)^2
)*e-1/2*d^2*b*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)^2*arctan(d*x+c)-1/2*d^2*b/(
c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)/(d*f*x+d*e)-d^2*b*f/(c^2*f^2-2*c*d*e*f+d^2*e
^2+f^2)^2*ln(f*(d*x+c)-c*f+d*e)*c+d^3*b/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)^2*l
n(f*(d*x+c)-c*f+d*e)*e
```

Maxima [A] time = 1.56197, size = 552, normalized size = 2.43

$$-\frac{1}{2}\left(d\left(\frac{\left(d^2e-cdf\right)\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^4e^4-4\,cd^3e^3f+2\left(3\,c^2+1\right)d^2e^2f^2-4\left(c^3+c\right)def^3+\left(c^4+2\,c^2+1\right)f^4}-\frac{2\left(d^2e-cdf\right)\mathrm{lo}}{d^4e^4-4\,cd^3e^3f+2\left(3\,c^2+1\right)d^2e^2f^2-}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(d*((d^2*e - c*d*f)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^4*e^4 - 4*c*d^
3*e^3*f + 2*(3*c^2 + 1)*d^2*e^2*f^2 - 4*(c^3 + c)*d*e*f^3 + (c^4 + 2*c^2 +
1)*f^4) - 2*(d^2*e - c*d*f)*log(f*x + e)/(d^4*e^4 - 4*c*d^3*e^3*f + 2*(3*c^
2 + 1)*d^2*e^2*f^2 - 4*(c^3 + c)*d*e*f^3 + (c^4 + 2*c^2 + 1)*f^4) - (d^4*e^
```

```
2 - 2*c*d^3*e*f + (c^2 - 1)*d^2*f^2)*arctan((d^2*x + c*d)/d)/((d^4*e^4*f -
4*c*d^3*e^3*f^2 + 2*(3*c^2 + 1)*d^2*e^2*f^3 - 4*(c^3 + c)*d*e*f^4 + (c^4 +
2*c^2 + 1)*f^5)*d) + 1/(d^2*e^3 - 2*c*d*e^2*f + (c^2 + 1)*e*f^2 + (d^2*e^2*
f - 2*c*d*e*f^2 + (c^2 + 1)*f^3)*x)) + arctan(d*x + c)/(f^3*x^2 + 2*e*f^2*x
 + e^2*f))*b - 1/2*a/(f^3*x^2 + 2*e*f^2*x + e^2*f)
```

Fricas [B] time = 11.9383, size = 1415, normalized size = 6.23

$$-\frac{ad^4e^4 - (4\,ac - b)d^3e^3f + 2\left(3\,ac^2 - bc + a\right)d^2e^2f^2 - \left(4\,ac^3 - bc^2 + 4\,ac - b\right)def^3 + \left(ac^4 + 2\,ac^2 + a\right)f^4 + \left(bd^3e^2f^2 - 2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a*d^4*e^4 - (4*a*c - b)*d^3*e^3*f + 2*(3*a*c^2 - b*c + a)*d^2*e^2*f^2
 - (4*a*c^3 - b*c^2 + 4*a*c - b)*d*e*f^3 + (a*c^4 + 2*a*c^2 + a)*f^4 + (b*d
^3*e^2*f^2 - 2*b*c*d^2*e*f^3 + (b*c^2 + b)*d*f^4)*x - (2*b*c*d^3*e^3*f - (5
*b*c^2 + 3*b)*d^2*e^2*f^2 + 4*(b*c^3 + b*c)*d*e*f^3 - (b*c^4 + 2*b*c^2 + b)
*f^4 + (b*d^4*e^2*f^2 - 2*b*c*d^3*e*f^3 + (b*c^2 - b)*d^2*f^4)*x^2 + 2*(b*d
^4*e^3*f - 2*b*c*d^3*e^2*f^2 + (b*c^2 - b)*d^2*e*f^3)*x)*arctan(d*x + c) +
(b*d^3*e^3*f - b*c*d^2*e^2*f^2 + (b*d^3*e*f^3 - b*c*d^2*f^4)*x^2 + 2*(b*d^3
*e^2*f^2 - b*c*d^2*e*f^3)*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1) - 2*(b*d^3*e^
3*f - b*c*d^2*e^2*f^2 + (b*d^3*e*f^3 - b*c*d^2*f^4)*x^2 + 2*(b*d^3*e^2*f^2
- b*c*d^2*e*f^3)*x)*log(f*x + e))/(d^4*e^6*f - 4*c*d^3*e^5*f^2 + 2*(3*c^2 +
 1)*d^2*e^4*f^3 - 4*(c^3 + c)*d*e^3*f^4 + (c^4 + 2*c^2 + 1)*e^2*f^5 + (d^4*
e^4*f^3 - 4*c*d^3*e^3*f^4 + 2*(3*c^2 + 1)*d^2*e^2*f^5 - 4*(c^3 + c)*d*e*f^6
 + (c^4 + 2*c^2 + 1)*f^7)*x^2 + 2*(d^4*e^5*f^2 - 4*c*d^3*e^4*f^3 + 2*(3*c^2
 + 1)*d^2*e^3*f^4 - 4*(c^3 + c)*d*e^2*f^5 + (c^4 + 2*c^2 + 1)*e*f^6)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))/(f*x+e)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 6.7621, size = 1723, normalized size = 7.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e)^3,x, algorithm="giac")
```

```
[Out] -1/2*(pi*b*c^2*d^2*f^4*x^2*sgn(d*x + c) - 2*pi*b*c*d^3*f^3*x^2*e*sgn(d*x +
c) - b*c^2*d^2*f^4*x^2*arctan(d*x + c) + 2*b*c*d^3*f^3*x^2*arctan(d*x + c)*
e + pi*b*d^4*f^2*x^2*e^2*sgn(d*x + c) + 2*pi*b*c^2*d^2*f^3*x*e*sgn(d*x + c)
 - b*d^4*f^2*x^2*arctan(d*x + c)*e^2 - 2*b*c^2*d^2*f^3*x*arctan(d*x + c)*e
- b*c*d^2*f^4*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1) + b*d^3*f^3*x^2*e*log(d^
2*x^2 + 2*c*d*x + c^2 + 1) + 2*b*c*d^2*f^4*x^2*log(abs(f*x + e)) - 2*b*d^3*
f^3*x^2*e*log(abs(f*x + e)) - pi*b*d^2*f^4*x^2*sgn(d*x + c) - 4*pi*b*c*d^3*
f^2*x*e^2*sgn(d*x + c) + b*c^4*f^4*arctan(d*x + c) + b*d^2*f^4*x^2*arctan(d
*x + c) + 4*b*c*d^3*f^2*x*arctan(d*x + c)*e^2 - 4*b*c^3*d*f^3*arctan(d*x +
c)*e - 2*b*c*d^2*f^3*x*e*log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 4*b*c*d^2*f^3*x
*e*log(abs(f*x + e)) + 2*pi*b*d^4*f*x*e^3*sgn(d*x + c) + pi*b*c^2*d^2*f^2*e
^2*sgn(d*x + c) - 2*pi*b*d^2*f^3*x*e*sgn(d*x + c) + a*c^4*f^4 + b*c^2*d*f^4
*x - 2*b*d^4*f*x*arctan(d*x + c)*e^3 + 5*b*c^2*d^2*f^2*arctan(d*x + c)*e^2
- 4*a*c^3*d*f^3*e - 2*b*c*d^2*f^3*x*e + 2*b*d^2*f^3*x*arctan(d*x + c)*e + 2
*b*d^3*f^2*x*e^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1) - 4*b*d^3*f^2*x*e^2*log(a
bs(f*x + e)) - 2*pi*b*c*d^3*f*e^3*sgn(d*x + c) + 2*b*c^2*f^4*arctan(d*x + c
) - 2*b*c*d^3*f*arctan(d*x + c)*e^3 + 6*a*c^2*d^2*f^2*e^2 + b*d^3*f^2*x*e^2
 + b*c^2*d*f^3*e - 4*b*c*d*f^3*arctan(d*x + c)*e - b*c*d^2*f^2*e^2*log(d^2*
x^2 + 2*c*d*x + c^2 + 1) + 2*b*c*d^2*f^2*e^2*log(abs(f*x + e)) + pi*b*d^4*e
^4*sgn(d*x + c) - pi*b*d^2*f^2*e^2*sgn(d*x + c) + 2*a*c^2*f^4 + b*d*f^4*x -
 4*a*c*d^3*f*e^3 - 2*b*c*d^2*f^2*e^2 + 3*b*d^2*f^2*arctan(d*x + c)*e^2 - 4*
a*c*d*f^3*e + b*d^3*f*e^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1) - 2*b*d^3*f*e^3*
log(abs(f*x + e)) + b*f^4*arctan(d*x + c) + a*d^4*e^4 + b*d^3*f*e^3 + 2*a*d
^2*f^2*e^2 + b*d*f^3*e + a*f^4)/(c^4*f^7*x^2 - 4*c^3*d*f^6*x^2*e + 6*c^2*d^
2*f^5*x^2*e^2 + 2*c^4*f^6*x*e + 2*c^2*f^7*x^2 - 4*c*d^3*f^4*x^2*e^3 - 8*c^3
*d*f^5*x*e^2 - 4*c*d*f^6*x^2*e + d^4*f^3*x^2*e^4 + 12*c^2*d^2*f^4*x*e^3 + c
^4*f^5*e^2 + 2*d^2*f^5*x^2*e^2 + 4*c^2*f^6*x*e + f^7*x^2 - 8*c*d^3*f^3*x*e^
4 - 4*c^3*d*f^4*e^3 - 8*c*d*f^5*x*e^2 + 2*d^4*f^2*x*e^5 + 6*c^2*d^2*f^3*e^4
 + 4*d^2*f^4*x*e^3 + 2*c^2*f^5*e^2 + 2*f^6*x*e - 4*c*d^3*f^2*e^5 - 4*c*d*f^
4*e^3 + d^4*f*e^6 + 2*d^2*f^3*e^4 + f^5*e^2)
```

3.31 $\int (e+fx)^2 \left(a+b\tan^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=382

$$\frac{ib^2\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{3d^3}+\frac{i\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\left(a+b\tan^{-1}(c+dx\right.}{3d^3}$$

```
[Out] (b^2*f^2*x)/(3*d^2) - (2*a*b*f*(d*e - c*f)*x)/d^2 - (b^2*f^2*ArcTan[c + d*x
])/(3*d^3) - (2*b^2*f*(d*e - c*f)*(c + d*x)*ArcTan[c + d*x])/d^3 - (b*f^2*(
c + d*x)^2*(a + b*ArcTan[c + d*x]))/(3*d^3) + ((I/3)*(3*d^2*e^2 - 6*c*d*e*f
 - (1 - 3*c^2)*f^2)*(a + b*ArcTan[c + d*x])^2)/d^3 - ((d*e - c*f)*(d^2*e^2
- 2*c*d*e*f - (3 - c^2)*f^2)*(a + b*ArcTan[c + d*x])^2)/(3*d^3*f) + ((e + f
*x)^3*(a + b*ArcTan[c + d*x])^2)/(3*f) + (2*b*(3*d^2*e^2 - 6*c*d*e*f - (1 -
 3*c^2)*f^2)*(a + b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))])/(3*d^3) + (b
^2*f*(d*e - c*f)*Log[1 + (c + d*x)^2])/d^3 + ((I/3)*b^2*(3*d^2*e^2 - 6*c*d*
e*f - (1 - 3*c^2)*f^2)*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d^3
```

Rubi [A] time = 0.574053, antiderivative size = 382, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.65, Rules used = {5047, 4864, 4846, 260, 4852, 321, 203, 4984, 4884, 4920, 4854, 2402, 2315}

$$\frac{ib^2\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{3d^3}+\frac{i\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\left(a+b\tan^{-1}(c+dx\right.}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (b^2*f^2*x)/(3*d^2) - (2*a*b*f*(d*e - c*f)*x)/d^2 - (b^2*f^2*ArcTan[c + d*x
])/(3*d^3) - (2*b^2*f*(d*e - c*f)*(c + d*x)*ArcTan[c + d*x])/d^3 - (b*f^2*(
c + d*x)^2*(a + b*ArcTan[c + d*x]))/(3*d^3) + ((I/3)*(3*d^2*e^2 - 6*c*d*e*f
 - (1 - 3*c^2)*f^2)*(a + b*ArcTan[c + d*x])^2)/d^3 - ((d*e - c*f)*(d^2*e^2
- 2*c*d*e*f - (3 - c^2)*f^2)*(a + b*ArcTan[c + d*x])^2)/(3*d^3*f) + ((e + f
*x)^3*(a + b*ArcTan[c + d*x])^2)/(3*f) + (2*b*(3*d^2*e^2 - 6*c*d*e*f - (1 -
 3*c^2)*f^2)*(a + b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))])/(3*d^3) + (b
^2*f*(d*e - c*f)*Log[1 + (c + d*x)^2])/d^3 + ((I/3)*b^2*(3*d^2*e^2 - 6*c*d*
e*f - (1 - 3*c^2)*f^2)*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d^3
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 4984

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int (e+fx)^2 \left(a+b\tan^{-1}(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2 \left(a+b\tan^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^3 \left(a+b\tan^{-1}(c+dx)\right)^2}{3f} - \frac{(2b)\,\text{Subst}\left(\int \left(\frac{3f^2(de-cf)\left(a+b\tan^{-1}(x)\right)}{d^3} + \frac{f^3x(a+\cdots}{\cdots}\right.\right.}{\cdots}$$

$$= \frac{(e+fx)^3 \left(a+b\tan^{-1}(c+dx)\right)^2}{3f} - \frac{(2b)\,\text{Subst}\left(\int \frac{\left((de-cf)\left(d^2e^2-2cdef-3f^2+c^2f^2\right)+f(3d\cdots\right.}{1+\cdots}\right.}{3d\cdots}$$

$$= -\frac{2abf(de-cf)x}{d^2} - \frac{bf^2(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)}{3d^3} + \frac{(e+fx)^3\left(a+b\tan^{-1}(\cdots\right.}{3f}$$

$$= \frac{b^2f^2x}{3d^2} - \frac{2abf(de-cf)x}{d^2} - \frac{2b^2f(de-cf)(c+dx)\tan^{-1}(c+dx)}{d^3} - \frac{bf^2(c+dx)^2(\cdots}{\cdots}$$

$$= \frac{b^2f^2x}{3d^2} - \frac{2abf(de-cf)x}{d^2} - \frac{b^2f^2\tan^{-1}(c+dx)}{3d^3} - \frac{2b^2f(de-cf)(c+dx)\tan^{-1}(c\cdots}{d^3}$$

$$= \frac{b^2f^2x}{3d^2} - \frac{2abf(de-cf)x}{d^2} - \frac{b^2f^2\tan^{-1}(c+dx)}{3d^3} - \frac{2b^2f(de-cf)(c+dx)\tan^{-1}(c\cdots}{d^3}$$

$$= \frac{b^2f^2x}{3d^2} - \frac{2abf(de-cf)x}{d^2} - \frac{b^2f^2\tan^{-1}(c+dx)}{3d^3} - \frac{2b^2f(de-cf)(c+dx)\tan^{-1}(c\cdots}{d^3}$$

$$= \frac{b^2f^2x}{3d^2} - \frac{2abf(de-cf)x}{d^2} - \frac{b^2f^2\tan^{-1}(c+dx)}{3d^3} - \frac{2b^2f(de-cf)(c+dx)\tan^{-1}(c\cdots}{d^3}$$

Mathematica [B] time = 3.65537, size = 801, normalized size = 2.1

$$\frac{1}{3}a^2f^2x^3 + a^2efx^2 + a^2e^2x + \frac{ab\left(-dfx(6de-4cf+dfx)+2\left(f^2c^3-3defc^2+3\left(d^2e^2-f^2\right)c+3def+d^3x\left(3e^2+3fxe\cdots\right.\right.\right.}{3d^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)^2*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] a^2*e^2*x + a^2*e*f*x^2 + (a^2*f^2*x^3)/3 + (a*b*(-(d*f*x*(6*d*e - 4*c*f +
d*f*x)) + 2*(3*d*e*f - 3*c^2*d*e*f + c^3*f^2 + 3*c*(d^2*e^2 - f^2) + d^3*x*
(3*e^2 + 3*e*f*x + f^2*x^2))*ArcTan[c + d*x] + (-3*d^2*e^2 + 6*c*d*e*f + (1
 - 3*c^2)*f^2)*Log[1 + (c + d*x)^2]))/(3*d^3) + (b^2*e^2*(ArcTan[c + d*x]*(
(-I + c + d*x)*ArcTan[c + d*x] + 2*Log[1 + E^((2*I)*ArcTan[c + d*x])]) - I*
PolyLog[2, -E^((2*I)*ArcTan[c + d*x])]))/d + (b^2*e*f*((1 + (2*I)*c - c^2 +
 d^2*x^2)*ArcTan[c + d*x]^2 - 2*ArcTan[c + d*x]*(c + d*x + 2*c*Log[1 + E^((
2*I)*ArcTan[c + d*x])]) + Log[1 + (c + d*x)^2] + (2*I)*c*PolyLog[2, -E^((2*
I)*ArcTan[c + d*x])]))/d^2 + (b^2*f^2*(1 + (c + d*x)^2)^(3/2)*((c + d*x)/Sq
rt[1 + (c + d*x)^2] + (6*c*(c + d*x)*ArcTan[c + d*x])/Sqrt[1 + (c + d*x)^2]
 + (3*(c + d*x)*ArcTan[c + d*x]^2)/Sqrt[1 + (c + d*x)^2] + (3*c^2*(c + d*x)
*ArcTan[c + d*x]^2)/Sqrt[1 + (c + d*x)^2] + I*ArcTan[c + d*x]^2*Cos[3*ArcTa
n[c + d*x]] - (3*I)*c^2*ArcTan[c + d*x]^2*Cos[3*ArcTan[c + d*x]] - 2*ArcTan
[c + d*x]*Cos[3*ArcTan[c + d*x]]*Log[1 + E^((2*I)*ArcTan[c + d*x])] + 6*c^2
*ArcTan[c + d*x]*Cos[3*ArcTan[c + d*x]]*Log[1 + E^((2*I)*ArcTan[c + d*x])]
+ 6*c*Cos[3*ArcTan[c + d*x]]*Log[1/Sqrt[1 + (c + d*x)^2]] + ((3*I - 12*c -
(9*I)*c^2)*ArcTan[c + d*x]^2 + 2*ArcTan[c + d*x]*(-2 + (-3 + 9*c^2)*Log[1 +
 E^((2*I)*ArcTan[c + d*x])]) + 18*c*Log[1/Sqrt[1 + (c + d*x)^2]])/Sqrt[1 +
(c + d*x)^2] - ((4*I)*(-1 + 3*c^2)*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])])/
(1 + (c + d*x)^2)^(3/2) + Sin[3*ArcTan[c + d*x]] + 6*c*ArcTan[c + d*x]*Sin[
3*ArcTan[c + d*x]] - ArcTan[c + d*x]^2*Sin[3*ArcTan[c + d*x]] + 3*c^2*ArcTa
n[c + d*x]^2*Sin[3*ArcTan[c + d*x]]))/(12*d^3)
```

Maple [B] time = 0.133, size = 1622, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(a+b*arctan(d*x+c))^2,x)
```

```
[Out] I/d^2*b^2*ln(d*x+c-I)*ln(1+(d*x+c)^2)*c*e*f+I/d^2*b^2*ln(d*x+c+I)*ln(1/2*I*
(d*x+c-I))*c*e*f-I/d^2*b^2*ln(d*x+c+I)*ln(1+(d*x+c)^2)*c*e*f-I/d^2*b^2*ln(d
*x+c-I)*ln(-1/2*I*(d*x+c+I))*c*e*f-1/2*I/d^3*b^2*dilog(1/2*I*(d*x+c-I))*c^2
*f^2+1/4*I/d^3*b^2*ln(d*x+c-I)^2*c^2*f^2-1/4*I/d^3*b^2*ln(d*x+c+I)^2*c^2*f^
2+1/2*I/d^3*b^2*dilog(-1/2*I*(d*x+c+I))*c^2*f^2+1/2*I/d*b^2*ln(d*x+c+I)*ln(
1+(d*x+c)^2)*e^2+1/2*I/d*b^2*ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I))*e^2-1/2*I/d*b
^2*ln(d*x+c-I)*ln(1+(d*x+c)^2)*e^2-1/2*I/d*b^2*ln(d*x+c+I)*ln(1/2*I*(d*x+c-
I))*e^2-1/6*I/d^3*b^2*ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I))*f^2+1/6*I/d^3*b^2*ln
(d*x+c-I)*ln(1+(d*x+c)^2)*f^2+1/6*I/d^3*b^2*ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))
*f^2-1/6*I/d^3*b^2*ln(d*x+c+I)*ln(1+(d*x+c)^2)*f^2-2/d^3*a*b*f^2*arctan(d*x
+c)*c+2/d^2*a*b*f*arctan(d*x+c)*e+2*a*b*f*arctan(d*x+c)*e*x^2-2/d^2*a*b*f*c
```

```
*e+5/3/d^3*a*b*f^2*c^2+1/2*I/d*b^2*dilog(-1/2*I*(d*x+c+I))*e^2-1/2*I/d*b^2*
dilog(1/2*I*(d*x+c-I))*e^2-1/12*I/d^3*b^2*ln(d*x+c-I)^2*f^2-1/4*I/d*b^2*ln(
d*x+c+I)^2*e^2+1/4*I/d*b^2*ln(d*x+c-I)^2*e^2-1/6*I/d^3*b^2*dilog(-1/2*I*(d*
x+c+I))*f^2-1/d*e^2*a*b*ln(1+(d*x+c)^2)+1/12*I/d^3*b^2*ln(d*x+c+I)^2*f^2+1/
3/d^3*b^2*f^2*arctan(d*x+c)*ln(1+(d*x+c)^2)-1/d*e^2*b^2*arctan(d*x+c)*ln(1+
(d*x+c)^2)-1/3/d*a*b*f^2*x^2+1/d^2*b^2*f*arctan(d*x+c)^2*e+1/d*arctan(d*x+c
)^2*b^2*c*e^2+b^2*f*arctan(d*x+c)^2*e*x^2+1/6*I/d^3*b^2*dilog(1/2*I*(d*x+c-
I))*f^2+1/3/d^3*a*b*f^2*ln(1+(d*x+c)^2)+1/d^2*b^2*f*ln(1+(d*x+c)^2)*e+1/3/d
^3*b^2*f^2*arctan(d*x+c)^2*c^3-1/3/d*b^2*f^2*arctan(d*x+c)*x^2+5/3/d^3*b^2*
f^2*arctan(d*x+c)*c^2-1/d^3*b^2*f^2*arctan(d*x+c)^2*c+2/3*a*b*f^2*arctan(d*
x+c)*x^3-1/d^3*b^2*f^2*ln(1+(d*x+c)^2)*c-1/d^3*a*b*f^2*ln(1+(d*x+c)^2)*c^2+
2/d*arctan(d*x+c)*a*b*c*e^2-1/d^3*b^2*f^2*arctan(d*x+c)*ln(1+(d*x+c)^2)*c^2
-2/d^2*b^2*f*arctan(d*x+c)*e*c+4/3/d^2*b^2*f^2*arctan(d*x+c)*x*c+2/3/d^3*a*
b*f^2*arctan(d*x+c)*c^3-1/d^2*b^2*f*arctan(d*x+c)^2*c^2*e-2/d*b^2*f*arctan(
d*x+c)*e*x+1/3*a^2*f^2*x^3+a^2*x*e^2+4/3*a*b/d^2*f^2*c*x-2*a*b/d*f*e*x+1/3*
a^2/f*e^3+1/3/d^3*b^2*f^2*c+1/3*b^2*f^2*arctan(d*x+c)^2*x^3+arctan(d*x+c)^2
*x*b^2*e^2+a^2*f*x^2*e+2/d^2*a*b*f*ln(1+(d*x+c)^2)*c*e-1/2*I/d^3*b^2*ln(d*x
+c+I)*ln(1/2*I*(d*x+c-I))*c^2*f^2+1/2*I/d^2*b^2*ln(d*x+c+I)^2*c*e*f-1/2*I/d
^2*b^2*ln(d*x+c-I)^2*c*e*f-I/d^2*b^2*dilog(-1/2*I*(d*x+c+I))*c*e*f+1/2*I/d^
3*b^2*ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I))*c^2*f^2-1/2*I/d^3*b^2*ln(d*x+c-I)*ln
(1+(d*x+c)^2)*c^2*f^2+1/2*I/d^3*b^2*ln(d*x+c+I)*ln(1+(d*x+c)^2)*c^2*f^2+2/d
^2*b^2*f*arctan(d*x+c)*ln(1+(d*x+c)^2)*c*e+I/d^2*b^2*dilog(1/2*I*(d*x+c-I))
*c*e*f-2/d^2*a*b*f*arctan(d*x+c)*c^2*e+2*arctan(d*x+c)*x*a*b*e^2+1/3*b^2*f^
2*x/d^2-1/3*b^2*f^2*arctan(d*x+c)/d^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 3/4*b^2*c^2*e^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 1/4*(3*arctan
(d*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*b^2*c^
2*e^2 + 1/3*a^2*f^2*x^3 + 36*b^2*d^2*f^2*integrate(1/48*x^4*arctan(d*x + c)
^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*d^2*f^2*integrate(1/48*x^4*log
(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 72*b^2*
d^2*e*f*integrate(1/48*x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1),
 x) + 72*b^2*c*d*f^2*integrate(1/48*x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*
x + c^2 + 1), x) + 4*b^2*d^2*f^2*integrate(1/48*x^4*log(d^2*x^2 + 2*c*d*x +
 c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^2*d^2*e*f*integrate(1/48*
```

```
x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) +
6*b^2*c*d*f^2*integrate(1/48*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^
2 + 2*c*d*x + c^2 + 1), x) + 36*b^2*d^2*e^2*integrate(1/48*x^2*arctan(d*x +
 c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 144*b^2*c*d*e*f*integrate(1/48*x^
2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 36*b^2*c^2*f^2*inte
grate(1/48*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^2
*d^2*e*f*integrate(1/48*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c
*d*x + c^2 + 1), x) + 4*b^2*c*d*f^2*integrate(1/48*x^3*log(d^2*x^2 + 2*c*d*
x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*d^2*e^2*integrate(1/
48*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
 + 12*b^2*c*d*e*f*integrate(1/48*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^
2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*c^2*f^2*integrate(1/48*x^2*log(d^2*x
^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 72*b^2*c*d*e^
2*integrate(1/48*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 72
*b^2*c^2*e*f*integrate(1/48*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 +
1), x) + 12*b^2*d^2*e^2*integrate(1/48*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)
/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^2*c*d*e*f*integrate(1/48*x^2*log(
d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^2*c*d*
e^2*integrate(1/48*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x
+ c^2 + 1), x) + 6*b^2*c^2*e*f*integrate(1/48*x*log(d^2*x^2 + 2*c*d*x + c^2
 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^2*c*d*e^2*integrate(1/48*x
*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2
*c^2*e^2*integrate(1/48*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d
*x + c^2 + 1), x) + a^2*e*f*x^2 + 3/4*b^2*e^2*arctan(d*x + c)^2*arctan((d^2
*x + c*d)/d)/d - 8*b^2*d*f^2*integrate(1/48*x^3*arctan(d*x + c)/(d^2*x^2 +
2*c*d*x + c^2 + 1), x) - 24*b^2*d*e*f*integrate(1/48*x^2*arctan(d*x + c)/(d
^2*x^2 + 2*c*d*x + c^2 + 1), x) - 24*b^2*d*e^2*integrate(1/48*x*arctan(d*x
+ c)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 1/4*(3*arctan(d*x + c)*arctan((d^2
*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*b^2*e^2 + 2*(x^2*arctan(d*x
 + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 +
2*c*d*x + c^2 + 1)/d^3))*a*b*e*f + 1/3*(2*x^3*arctan(d*x + c) - d*((d*x^2 -
 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d
^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*a*b*f^2 + a^2*e^2*x + 36*b^2*f^2*integrat
e(1/48*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*f^2*
integrate(1/48*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x +
c^2 + 1), x) + 72*b^2*e*f*integrate(1/48*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c
*d*x + c^2 + 1), x) + 6*b^2*e*f*integrate(1/48*x*log(d^2*x^2 + 2*c*d*x + c^
2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*e^2*integrate(1/48*log(d
^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + (2*(d*x +
 c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*a*b*e^2/d + 1/12*(b^2*f^2*x^3 +
 3*b^2*e*f*x^2 + 3*b^2*e^2*x)*arctan(d*x + c)^2 - 1/48*(b^2*f^2*x^3 + 3*b^2
*e*f*x^2 + 3*b^2*e^2*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(a^2f^2x^2+2\,a^2efx+a^2e^2+\left(b^2f^2x^2+2\,b^2efx+b^2e^2\right)\arctan\left(dx+c\right)^2+2\left(abf^2x^2+2\,abefx+abe^2\right)\arctan\left(a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*f^2*x^2 + 2*a^2*e*f*x + a^2*e^2 + (b^2*f^2*x^2 + 2*b^2*e*f*x +
 b^2*e^2)*arctan(d*x + c)^2 + 2*(a*b*f^2*x^2 + 2*a*b*e*f*x + a*b*e^2)*arcta
n(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\,\text{atan}\,(c+dx))^2\left(e+fx\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(a+b*atan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*atan(c + d*x))**2*(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx+e\right)^2(b\arctan\left(dx+c\right)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*(b*arctan(d*x + c) + a)^2, x)
```

3.32 $\int (e+fx)\left(a+b\tan^{-1}(c+dx)\right)^2\,dx$

Optimal. Leaf size=222

$$\frac{ib^2(de-cf)\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2}+\frac{i(de-cf)\left(a+b\tan^{-1}(c+dx)\right)^2}{d^2}-\frac{(-cf+de+f)(de-(c+1)f)\left(a+b\tan^{-1}\right.}{2d^2f}$$

```
[Out] -((a*b*f*x)/d) - (b^2*f*(c + d*x)*ArcTan[c + d*x])/d^2 + (I*(d*e - c*f)*(a
+ b*ArcTan[c + d*x])^2)/d^2 - ((d*e + f - c*f)*(d*e - (1 + c)*f)*(a + b*Arc
Tan[c + d*x])^2)/(2*d^2*f) + ((e + f*x)^2*(a + b*ArcTan[c + d*x])^2)/(2*f)
+ (2*b*(d*e - c*f)*(a + b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))])/d^2 +
(b^2*f*Log[1 + (c + d*x)^2])/(2*d^2) + (I*b^2*(d*e - c*f)*PolyLog[2, 1 - 2/
(1 + I*(c + d*x))])/d^2
```

Rubi [A] time = 0.373148, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {5047, 4864, 4846, 260, 4984, 4884, 4920, 4854, 2402, 2315}

$$\frac{ib^2(de-cf)\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2}+\frac{i(de-cf)\left(a+b\tan^{-1}(c+dx)\right)^2}{d^2}-\frac{(-cf+de+f)(de-(c+1)f)\left(a+b\tan^{-1}\right.}{2d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] -((a*b*f*x)/d) - (b^2*f*(c + d*x)*ArcTan[c + d*x])/d^2 + (I*(d*e - c*f)*(a
+ b*ArcTan[c + d*x])^2)/d^2 - ((d*e + f - c*f)*(d*e - (1 + c)*f)*(a + b*Arc
Tan[c + d*x])^2)/(2*d^2*f) + ((e + f*x)^2*(a + b*ArcTan[c + d*x])^2)/(2*f)
+ (2*b*(d*e - c*f)*(a + b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))])/d^2 +
(b^2*f*Log[1 + (c + d*x)^2])/(2*d^2) + (I*b^2*(d*e - c*f)*PolyLog[2, 1 - 2/
(1 + I*(c + d*x))])/d^2
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4984

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int (e+fx)\left(a+b\tan^{-1}(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int\left(\frac{de-cf}{d}+\frac{fx}{d}\right)\left(a+b\tan^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$=\frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2f}-\frac{b\,\text{Subst}\left(\int\left(\frac{f^2\left(a+b\tan^{-1}(x)\right)}{d^2}+\frac{((de-f-cf)(de+f-c}{}\right.\right.}{f}$$

$$=\frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2f}-\frac{b\,\text{Subst}\left(\int\frac{((de-f-cf)(de+f-cf)+2f(de-cf)x)\left(a+b\,\text{ta}\right.}{1+x^2}\right.}{d^2f}$$

$$=-\frac{abfx}{d}+\frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2f}-\frac{b\,\text{Subst}\left(\int\left(\frac{(de+f-cf)(de-(1+c)f)\left(a+b\right.}{1+x^2}\right.\right.}{}$$

$$=-\frac{abfx}{d}-\frac{b^2f(c+dx)\tan^{-1}(c+dx)}{d^2}+\frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2f}+\frac{\left(b^2f\right)}{}$$

$$=-\frac{abfx}{d}-\frac{b^2f(c+dx)\tan^{-1}(c+dx)}{d^2}+\frac{i(de-cf)\left(a+b\tan^{-1}(c+dx)\right)^2}{d^2}-\frac{(de+}{}$$

$$=-\frac{abfx}{d}-\frac{b^2f(c+dx)\tan^{-1}(c+dx)}{d^2}+\frac{i(de-cf)\left(a+b\tan^{-1}(c+dx)\right)^2}{d^2}-\frac{(de+}{}$$

$$=-\frac{abfx}{d}-\frac{b^2f(c+dx)\tan^{-1}(c+dx)}{d^2}+\frac{i(de-cf)\left(a+b\tan^{-1}(c+dx)\right)^2}{d^2}-\frac{(de+}{}$$

$$=-\frac{abfx}{d}-\frac{b^2f(c+dx)\tan^{-1}(c+dx)}{d^2}+\frac{i(de-cf)\left(a+b\tan^{-1}(c+dx)\right)^2}{d^2}-\frac{(de+}{}$$

Mathematica [A] time = 0.379037, size = 264, normalized size = 1.19

$$-2ib^2(de-cf)\text{PolyLog}\left(2,-e^{2i\tan^{-1}(c+dx)}\right)-a^2c^2f+2a^2cde+2a^2d^2ex+a^2d^2fx^2-2b\tan^{-1}(c+dx)\left(a\left(c^2f-2cde-2a\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (2*a^2*c*d*e - 2*a*b*c*f - a^2*c^2*f + 2*a^2*d^2*e*x - 2*a*b*d*f*x + a^2*d^
2*f*x^2 + b^2*(-I + c + d*x)*(2*d*e + I*f - c*f + d*f*x)*ArcTan[c + d*x]^2
- 2*b*ArcTan[c + d*x]*(b*f*(c + d*x) + a*(-2*c*d*e + c^2*f - 2*d^2*e*x - f*
(1 + d^2*x^2)) - 2*b*(d*e - c*f)*Log[1 + E^((2*I)*ArcTan[c + d*x])]) + 4*a*
b*d*e*Log[1/Sqrt[1 + (c + d*x)^2]] - 2*b^2*f*Log[1/Sqrt[1 + (c + d*x)^2]] -
 4*a*b*c*f*Log[1/Sqrt[1 + (c + d*x)^2]] - (2*I)*b^2*(d*e - c*f)*PolyLog[2,
-E^((2*I)*ArcTan[c + d*x])])/(2*d^2)
```

Maple [B] time = 0.119, size = 748, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(a+b*arctan(d*x+c))^2,x)
```

```
[Out] -1/d^2*b^2*arctan(d*x+c)*f*c-1/2/d^2*b^2*arctan(d*x+c)^2*f*c^2+1/d^2*a*b*f*
arctan(d*x+c)+a*b*f*arctan(d*x+c)*x^2+2*arctan(d*x+c)*x*a*b*e-1/2*I/d*b^2*d
ilog(1/2*I*(d*x+c-I))*e+1/4*I/d*b^2*ln(d*x+c-I)^2*e-1/4*I/d*b^2*ln(d*x+c+I)
^2*e+1/2*I/d*b^2*dilog(-1/2*I*(d*x+c+I))*e-1/d*a*b*ln(1+(d*x+c)^2)*e+1/d*ar
ctan(d*x+c)^2*b^2*c*e-1/d*b^2*arctan(d*x+c)*ln(1+(d*x+c)^2)*e-1/d^2*a*b*f*c
-1/d*b^2*arctan(d*x+c)*f*x+a^2*e*x+1/2*a^2*x^2*f-1/2*I/d^2*b^2*ln(d*x+c-I)*
ln(-1/2*I*(d*x+c+I))*c*f-1/2*I/d^2*b^2*ln(1+(d*x+c)^2)*ln(d*x+c+I)*c*f+1/2*
I/d^2*b^2*ln(1+(d*x+c)^2)*ln(d*x+c-I)*c*f+1/2*I/d^2*b^2*ln(d*x+c+I)*ln(1/2*
I*(d*x+c-I))*c*f-1/2/d^2*a^2*f*c^2+1/d*a^2*c*e+arctan(d*x+c)^2*x*b^2*e+1/2*
b^2*arctan(d*x+c)^2*f*x^2+1/2/d^2*b^2*arctan(d*x+c)^2*f-1/2*I/d*b^2*ln(d*x+
c+I)*ln(1/2*I*(d*x+c-I))*e+1/2*I/d*b^2*ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I))*e+1
/2*I/d*b^2*ln(1+(d*x+c)^2)*ln(d*x+c+I)*e-1/4*I/d^2*b^2*ln(d*x+c-I)^2*c*f+1/
2*I/d^2*b^2*dilog(1/2*I*(d*x+c-I))*c*f+2/d*arctan(d*x+c)*a*b*c*e+1/d^2*b^2*
arctan(d*x+c)*ln(1+(d*x+c)^2)*c*f-1/d^2*a*b*arctan(d*x+c)*c^2*f-1/2*I/d*b^2
*ln(1+(d*x+c)^2)*ln(d*x+c-I)*e-1/2*I/d^2*b^2*dilog(-1/2*I*(d*x+c+I))*c*f+1/
4*I/d^2*b^2*ln(d*x+c+I)^2*c*f+1/d^2*a*b*ln(1+(d*x+c)^2)*c*f-a*b*f*x/d+1/2*b
```

```
^2*f*ln(1+(d*x+c)^2)/d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 3/4*b^2*c^2*e*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 1/4*(3*arctan(d
*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*b^2*c^2*
e + 12*b^2*d^2*f*integrate(1/16*x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x +
c^2 + 1), x) + b^2*d^2*f*integrate(1/16*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1
)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^2*d^2*e*integrate(1/16*x^2*arc
tan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 24*b^2*c*d*f*integrate(1
/16*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 2*b^2*d^2*f*i
ntegrate(1/16*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2
 + 1), x) + b^2*d^2*e*integrate(1/16*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2
/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 2*b^2*c*d*f*integrate(1/16*x^2*log(d^2
*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 24*b^2*c*d*
e*integrate(1/16*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12
*b^2*c^2*f*integrate(1/16*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1)
, x) + 4*b^2*d^2*e*integrate(1/16*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2
*x^2 + 2*c*d*x + c^2 + 1), x) + 2*b^2*c*d*f*integrate(1/16*x^2*log(d^2*x^2
+ 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 2*b^2*c*d*e*integr
ate(1/16*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1)
, x) + b^2*c^2*f*integrate(1/16*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x
^2 + 2*c*d*x + c^2 + 1), x) + 4*b^2*c*d*e*integrate(1/16*x*log(d^2*x^2 + 2*
c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + b^2*c^2*e*integrate(1/
16*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 1
/2*a^2*f*x^2 + 3/4*b^2*e*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 4*b^
2*d*f*integrate(1/16*x^2*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
- 8*b^2*d*e*integrate(1/16*x*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1),
 x) - 1/4*(3*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x +
c*d)/d)^3/d)*b^2*e + (x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^
2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a*b*f + a^2*e*
x + 12*b^2*f*integrate(1/16*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 +
1), x) + b^2*f*integrate(1/16*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2
 + 2*c*d*x + c^2 + 1), x) + b^2*e*integrate(1/16*log(d^2*x^2 + 2*c*d*x + c^
2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + (2*(d*x + c)*arctan(d*x + c) -
 log((d*x + c)^2 + 1))*a*b*e/d + 1/8*(b^2*f*x^2 + 2*b^2*e*x)*arctan(d*x + c
```

```
)^2 - 1/32*(b^2*f*x^2 + 2*b^2*e*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(a^2fx + a^2e + \left(b^2fx + b^2e\right)\arctan\left(dx + c\right)^2 + 2\left(abfx + abe\right)\arctan\left(dx + c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*f*x + a^2*e + (b^2*f*x + b^2*e)*arctan(d*x + c)^2 + 2*(a*b*f*x
 + a*b*e)*arctan(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\operatorname{atan}\left(c + dx\right)\right)^2 \left(e + fx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*atan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*atan(c + d*x))**2*(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx + e\right)\left(b\arctan\left(dx + c\right) + a\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(b*arctan(d*x + c) + a)^2, x)
```

3.33 $\int \left(a+b\tan^{-1}(c+dx)\right)^2\,dx$

Optimal. Leaf size=102

$$\frac{ib^2\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d}+\frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{d}+\frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{d}+\frac{2b\log\left(\frac{2}{1+i(c+dx)}\right)\left(a+b\,\text{ta}\right.}{d}$$

```
[Out] (I*(a + b*ArcTan[c + d*x])^2)/d + ((c + d*x)*(a + b*ArcTan[c + d*x])^2)/d +
 (2*b*(a + b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))])/d + (I*b^2*PolyLog[
2, 1 - 2/(1 + I*(c + d*x))])/d
```

Rubi [A] time = 0.106977, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5039, 4846, 4920, 4854, 2402, 2315}

$$\frac{ib^2\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d}+\frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{d}+\frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{d}+\frac{2b\log\left(\frac{2}{1+i(c+dx)}\right)\left(a+b\,\text{ta}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (I*(a + b*ArcTan[c + d*x])^2)/d + ((c + d*x)*(a + b*ArcTan[c + d*x])^2)/d +
 (2*b*(a + b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))])/d + (I*b^2*PolyLog[
2, 1 - 2/(1 + I*(c + d*x))])/d
```

Rule 5039

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \left(a+b\tan^{-1}(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \left(a+b\tan^{-1}(x)\right)^2\,dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{d} - \frac{(2b)\,\text{Subst}\left(\int \frac{x\left(a+b\tan^{-1}(x)\right)}{1+x^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{d} + \frac{(2b)\,\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{i-x}\,dx\right.}{d}$$

$$= \frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{d} + \frac{2b\left(a+b\tan^{-1}(c+dx)\right)\log}{d}$$

$$= \frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{d} + \frac{2b\left(a+b\tan^{-1}(c+dx)\right)\log}{d}$$

$$= \frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{d} + \frac{2b\left(a+b\tan^{-1}(c+dx)\right)\log}{d}$$

Mathematica [A] time = 0.0730931, size = 109, normalized size = 1.07

$$\frac{-ib^2\text{PolyLog}\left(2,-e^{2i\tan^{-1}(c+dx)}\right)+a\left(ac+adx+2b\log\left(\frac{1}{\sqrt{(c+dx)^2+1}}\right)\right)+2b\tan^{-1}(c+dx)\left(ac+adx+b\log\left(1+e^{2i\tan^{-1}\right.\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (b^2*(-I + c + d*x)*ArcTan[c + d*x]^2 + 2*b*ArcTan[c + d*x]*(a*c + a*d*x +
b*Log[1 + E^((2*I)*ArcTan[c + d*x])]) + a*(a*c + a*d*x + 2*b*Log[1/Sqrt[1 +
 (c + d*x)^2]]) - I*b^2*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])])/d
```

Maple [A] time = 0.105, size = 180, normalized size = 1.8

$$(\arctan(dx+c))^2 xb^2 - \frac{i(\arctan(dx+c))^2 b^2}{d} + \frac{(\arctan(dx+c))^2 b^2 c}{d} + 2\arctan(dx+c)xab + 2\frac{\arctan(dx+c)b^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^2,x)
```

```
[Out] arctan(d*x+c)^2*x*b^2-I/d*arctan(d*x+c)^2*b^2+1/d*arctan(d*x+c)^2*b^2*c+2*a
rctan(d*x+c)*x*a*b+2/d*arctan(d*x+c)*ln((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)*b^
2+2/d*arctan(d*x+c)*a*b*c-I/d*polylog(2,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))*b^2
+a^2*x-1/d*a*b*ln(1+(d*x+c)^2)+a^2*c/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/16*(12*c^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 4*(3*arctan(d*x
+ c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*c^2 + 4*x*a
rctan(d*x + c)^2 + 192*d^2*integrate(1/16*x^2*arctan(d*x + c)^2/(d^2*x^2 +
```

```
2*c*d*x + c^2 + 1), x) + 16*d^2*integrate(1/16*x^2*log(d^2*x^2 + 2*c*d*x +
c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 384*c*d*integrate(1/16*x*arc
tan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 64*d^2*integrate(1/16*x^
2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 32*c
*d*integrate(1/16*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x +
 c^2 + 1), x) + 64*c*d*integrate(1/16*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d
^2*x^2 + 2*c*d*x + c^2 + 1), x) + 16*c^2*integrate(1/16*log(d^2*x^2 + 2*c*d
*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - x*log(d^2*x^2 + 2*c*d*x
 + c^2 + 1)^2 + 12*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 12*arctan(
d*x + c)*arctan((d^2*x + c*d)/d)^2/d + 4*arctan((d^2*x + c*d)/d)^3/d - 128*
d*integrate(1/16*x*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 16*i
ntegrate(1/16*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 +
 1), x))*b^2 + a^2*x + (2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))
*a*b/d
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^2 \arctan(dx+c)^2 + 2\,ab \arctan(dx+c) + a^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\,\text{atan}(c+dx))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*atan(c + d*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \arctan(dx+c) + a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(d*x + c) + a)^2, x)
```

3.34 $\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{e+fx}\,dx$

Optimal. Leaf size=261

$$-\frac{ib\left(a+b\tan^{-1}(c+dx)\right)\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{f}+\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{f}+\frac{b^2\cdots}{\cdots}$$

```
[Out] -(((a + b*ArcTan[c + d*x])^2*Log[2/(1 - I*(c + d*x))])/f) + ((a + b*ArcTan[
c + d*x])^2*Log[(2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f +
 (I*b*(a + b*ArcTan[c + d*x])*PolyLog[2, 1 - 2/(1 - I*(c + d*x))])/f - (I*b
*(a + b*ArcTan[c + d*x])*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*
(1 - I*(c + d*x)))])/f - (b^2*PolyLog[3, 1 - 2/(1 - I*(c + d*x))])/(2*f) +
(b^2*PolyLog[3, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])
/(2*f)
```

Rubi [A] time = 0.164644, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {5047, 4858}

$$-\frac{ib\left(a+b\tan^{-1}(c+dx)\right)\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{f}+\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{f}+\frac{b^2\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^2/(e + f*x),x]
```

```
[Out] -(((a + b*ArcTan[c + d*x])^2*Log[2/(1 - I*(c + d*x))])/f) + ((a + b*ArcTan[
c + d*x])^2*Log[(2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f +
 (I*b*(a + b*ArcTan[c + d*x])*PolyLog[2, 1 - 2/(1 - I*(c + d*x))])/f - (I*b
*(a + b*ArcTan[c + d*x])*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*
(1 - I*(c + d*x)))])/f - (b^2*PolyLog[3, 1 - 2/(1 - I*(c + d*x))])/(2*f) +
(b^2*PolyLog[3, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])
/(2*f)
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
```

```
tQ[p, 0]
```

Rule 4858

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
-Simp[((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e, x] + (Simp[((a + b*ArcT
an[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x] + Simp[(I*
b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/e, x] - Simp[(I*b*(a +
 b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/
e, x] - Simp[(b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e), x] + Simp[(b^2*Poly
Log[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e), x]) /; FreeQ[
{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{e+fx}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{\frac{de-cf}{d}+\frac{fx}{d}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2\log\left(\frac{2}{1-i(c+dx)}\right)}{f} + \frac{\left(a+b\tan^{-1}(c+dx)\right)^2\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f}$$

Mathematica [F] time = 5.44485, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^2/(e + f*x),x]

[Out] Integrate[(a + b*ArcTan[c + d*x])^2/(e + f*x), x]
```

Maple [C] time = 1.339, size = 2149, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^2/(f*x+e),x)
```

```
[Out] -1/2*d*b^2/f*e/(I*f+c*f-d*e)*polylog(3,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*
x+c)^2)/(d*e+I*f-c*f))+I*b^2/f*arctan(d*x+c)^2*csgn(I*(I*f*(1+I*(d*x+c))^2/
(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x
+c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*Pi+I*a*b*ln(f*(d*x
+c)-c*f+d*e)/f*ln((I*f-f*(d*x+c))/(d*e+I*f-c*f))-1/2*I*b^2/f*arctan(d*x+c)^
2*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^
2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+
c)^2)+1))^3*Pi-I*b^2*c/(I*f+c*f-d*e)*arctan(d*x+c)*polylog(2,(I*f+c*f-d*e)*
(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-I*a*b*ln(f*(d*x+c)-c*f+d*e)/f*
ln((I*f+f*(d*x+c))/(I*f+c*f-d*e))-1/2*b^2/f*polylog(3,-(1+I*(d*x+c))^2/(1+(
d*x+c)^2))+a^2*ln(f*(d*x+c)-c*f+d*e)/f-b^2/f*arctan(d*x+c)^2*ln(I*f*(1+I*(d
*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^
2/(1+(d*x+c)^2)-I*f+c*f-d*e)+b^2/(I*f+c*f-d*e)*arctan(d*x+c)*polylog(2,(I*f
+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+b^2*ln(f*(d*x+c)-c*f
+d*e)/f*arctan(d*x+c)^2+1/2*b^2*c/(I*f+c*f-d*e)*polylog(3,(I*f+c*f-d*e)*(1+
I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+1/2*I*b^2/(I*f+c*f-d*e)*polylog(3
,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+2*I*d*b^2/f*e*a
rctan(d*x+c)*polylog(2,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f
-c*f))/(2*I*f+2*c*f-2*d*e)+1/2*I*b^2/f*arctan(d*x+c)^2*csgn(I/((1+I*(d*x+c)
)^2/(1+(d*x+c)^2)+1))*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d
*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e))*csgn
(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e
*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+
1))*Pi-d*b^2/f*e/(I*f+c*f-d*e)*arctan(d*x+c)^2*ln(1-(I*f+c*f-d*e)*(1+I*(d*x
+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-1/2*I*b^2/f*arctan(d*x+c)^2*csgn(I*(I*f
*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(
d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e))*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c
)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*
f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*Pi-1/2*I*b^2/f*arctan(d*x+c
)^2*csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(I*(I*f*(1+I*(d*x+c))^2/(
1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+
c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*Pi+b^2*c/(I*f+c*f-d
*e)*arctan(d*x+c)^2*ln(1-(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I
*f-c*f))+I*a*b/f*dilog((I*f-f*(d*x+c))/(d*e+I*f-c*f))-I*b^2/f*Pi*arctan(d*x
+c)^2-I*a*b/f*dilog((I*f+f*(d*x+c))/(I*f+c*f-d*e))+2*a*b*ln(f*(d*x+c)-c*f+d
*e)/f*arctan(d*x+c)+I*b^2/f*arctan(d*x+c)*polylog(2,-(1+I*(d*x+c))^2/(1+(d*
x+c)^2))+I*b^2/(I*f+c*f-d*e)*arctan(d*x+c)^2*ln(1-(I*f+c*f-d*e)*(1+I*(d*x+c
))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^{2}\log\left(fx+e\right)}{f}+\int\frac{12\,b^{2}\arctan\left(dx+c\right)^{2}+b^{2}\log\left(d^{2}x^{2}+2\,cdx+c^{2}+1\right)^{2}+32\,ab\arctan\left(dx+c\right)}{16\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(f*x+e),x, algorithm="maxima")
```

```
[Out] a^2*log(f*x + e)/f + integrate(1/16*(12*b^2*arctan(d*x + c)^2 + b^2*log(d^2
*x^2 + 2*c*d*x + c^2 + 1)^2 + 32*a*b*arctan(d*x + c))/(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^{2}\arctan\left(dx+c\right)^{2}+2\,ab\arctan\left(dx+c\right)+a^{2}}{fx+e},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2)/(f*x + e), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\,\mathrm{atan}\left(c+dx\right)\right)^{2}}{e+fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**2/(f*x+e),x)
```

```
[Out] Integral((a + b*atan(c + d*x))**2/(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan (dx + c) + a)^2}{fx + e} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(d*x + c) + a)^2/(f*x + e), x)
```

3.35 $\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{(e+fx)^2}\,dx$

Optimal. Leaf size=568

$$\frac{ib^2d\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{\left(c^2+1\right)f^2-2cdef+d^2e^2}-\frac{ib^2d\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{\left(c^2+1\right)f^2-2cdef+d^2e^2}+\frac{ib^2d\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{\left(c^2+1\right)f^2-2cdef+d^2e^2}+\frac{2abd\log}{(de-c}$$

```
[Out] (2*a*b*d*(d*e - c*f)*ArcTan[c + d*x])/(f*(f^2 + (d*e - c*f)^2)) + (I*b^2*d*
ArcTan[c + d*x]^2)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (b^2*d*(d*e - c*
f)*ArcTan[c + d*x]^2)/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)) - (a + b*Ar
cTan[c + d*x])^2/(f*(e + f*x)) + (2*a*b*d*Log[e + f*x])/(f^2 + (d*e - c*f)^
2) - (2*b^2*d*ArcTan[c + d*x]*Log[2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*
f + (1 + c^2)*f^2) + (2*b^2*d*ArcTan[c + d*x]*Log[(2*d*(e + f*x))/((d*e + I
*f - c*f)*(1 - I*(c + d*x)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (2*b
^2*d*ArcTan[c + d*x]*Log[2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 +
c^2)*f^2) - (a*b*d*Log[1 + (c + d*x)^2])/(f^2 + (d*e - c*f)^2) + (I*b^2*d*P
olyLog[2, 1 - 2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) -
 (I*b^2*d*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x
)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (I*b^2*d*PolyLog[2, 1 - 2/(1
+ I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)
```

Rubi [A] time = 1.35103, antiderivative size = 568, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 25, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.25, Rules used = {5045, 1982, 705, 31, 634, 618, 204, 628, 6741, 5057, 706, 635, 203, 260, 6688, 12, 6725, 4856, 2402, 2315, 2447, 4984, 4884, 4920, 4854}

$$\frac{ib^2d\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{\left(c^2+1\right)f^2-2cdef+d^2e^2}-\frac{ib^2d\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(1-i(c+dx))(de+(-c+i)f)}\right)}{\left(c^2+1\right)f^2-2cdef+d^2e^2}+\frac{ib^2d\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{\left(c^2+1\right)f^2-2cdef+d^2e^2}+\frac{2abd\log}{(de-c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^2/(e + f*x)^2,x]
```

```
[Out] (2*a*b*d*(d*e - c*f)*ArcTan[c + d*x])/(f*(f^2 + (d*e - c*f)^2)) + (I*b^2*d*
ArcTan[c + d*x]^2)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (b^2*d*(d*e - c*
f)*ArcTan[c + d*x]^2)/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)) - (a + b*Ar
cTan[c + d*x])^2/(f*(e + f*x)) + (2*a*b*d*Log[e + f*x])/(f^2 + (d*e - c*f)^
2) - (2*b^2*d*ArcTan[c + d*x]*Log[2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*
f + (1 + c^2)*f^2) + (2*b^2*d*ArcTan[c + d*x]*Log[(2*d*(e + f*x))/((d*e + (
```

```
I - c)*f)*(1 - I*(c + d*x)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (2*b
^2*d*ArcTan[c + d*x]*Log[2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 +
c^2)*f^2) - (a*b*d*Log[1 + (c + d*x)^2])/(f^2 + (d*e - c*f)^2) + (I*b^2*d*P
olyLog[2, 1 - 2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) -
 (I*b^2*d*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + (I - c)*f)*(1 - I*(c + d*x
)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (I*b^2*d*PolyLog[2, 1 - 2/(1
+ I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)
```

Rule 5045

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[((e + f*x)^(m + 1)*(a + b*ArcTan[c + d*x])^p)/(f*(m +
 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcTan[c
 + d*x])^(p - 1))/(1 + (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 1982

```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 5057

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst
[Int[((d*e - c*f)/d + (f*x)/d)^m*(C/d^2 + (C*x^2)/d^2)^q*(a + b*ArcTan[x])^
p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &&
 EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4984

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{(e+fx)^2}\,dx = -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2bd)\int \frac{a+b\tan^{-1}(c+dx)}{(e+fx)\left(1+(c+dx)^2\right)}\,dx}{f}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2bd)\int \frac{a+b\tan^{-1}(c+dx)}{(e+fx)\left(1+c^2+2cdx+d^2x^2\right)}\,dx}{f}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2b)\,\mathrm{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)(1+x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2b)\,\mathrm{Subst}\left(\int \frac{d\left(a+b\tan^{-1}(x)\right)}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2bd)\,\mathrm{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2bd)\,\mathrm{Subst}\left(\int \left(\frac{a}{(de-cf+fx)(1+x^2)} + \frac{b\tan^{-1}(x)}{(de-cf+fx)(1+x^2)}\right) dx, x, c+d\right.}{f}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2abd)\,\mathrm{Subst}\left(\int \frac{1}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f} + \frac{\left(2b^2d\right)\mathrm{Subst}}{}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{\left(2b^2d\right)\mathrm{Subst}\left(\int \left(\frac{f^2\tan^{-1}(x)}{\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)(de-cf+fx)} + \frac{(de-cf-fx)}{\left(d^2e^2-2cdef+(1\right.}\right.\right.}{f}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{2abd\log(e+fx)}{f^2+(de-cf)^2} + \frac{\left(2b^2d\right)\mathrm{Subst}\left(\int \frac{(de-cf-fx)\tan^{-1}(x)}{1+x^2}\,dx, x, c\right.}{f\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)}$$

$$= \frac{2abd(de-cf)\tan^{-1}(c+dx)}{f\left(f^2+(de-cf)^2\right)} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{2abd\log(e+fx)}{f^2+(de-cf)^2} - \frac{2b^2d\tan^{-1}(}{d^2e^2-2c}$$

$$= \frac{2abd(de-cf)\tan^{-1}(c+dx)}{f\left(f^2+(de-cf)^2\right)} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{2abd\log(e+fx)}{f^2+(de-cf)^2} - \frac{2b^2d\tan^{-1}(}{d^2e^2-2c}$$

$$= \frac{2abd(de-cf)\tan^{-1}(c+dx)}{f\left(f^2+(de-cf)^2\right)} + \frac{ib^2d\tan^{-1}(c+dx)^2}{d^2e^2-2cdef+\left(1+c^2\right)f^2} + \frac{b^2d(de-cf)\tan^{-1}(c+dx)}{f\left(d^2e^2-2cdef+\left(1+c^2\right)f\right.}$$

$$= \frac{2abd(de-cf)\tan^{-1}(c+dx)}{f\left(f^2+(de-cf)^2\right)} + \frac{ib^2d\tan^{-1}(c+dx)^2}{d^2e^2-2cdef+\left(1+c^2\right)f^2} + \frac{b^2d(de-cf)\tan^{-1}(c+dx)}{f\left(d^2e^2-2cdef+\left(1+c^2\right)f\right.}$$

$$= \frac{2abd(de-cf)\tan^{-1}(c+dx)}{f\left(f^2+(de-cf)^2\right)} + \frac{ib^2d\tan^{-1}(c+dx)^2}{d^2e^2-2cdef+\left(1+c^2\right)f^2} + \frac{b^2d(de-cf)\tan^{-1}(c+dx)}{f\left(d^2e^2-2cdef+\left(1+c^2\right)f\right.}$$

Mathematica [A] time = 6.31543, size = 419, normalized size = 0.74

$$\frac{b^2 d(e+fx)\left(-\frac{(de-cf)\left(i\text{PolyLog}\left(2,\exp\left(2i\left(\tan^{-1}\left(\frac{de-cf}{f}\right)+\tan^{-1}(c+dx)\right)\right)\right)-2\left(\tan^{-1}\left(\frac{de-cf}{f}\right)+\tan^{-1}(c+dx)\right)\log\left(1-\exp\left(2i\left(\tan^{-1}\left(\frac{de-cf}{f}\right)+\tan^{-1}(c+dx)\right)\right)\right)-i\tan^{-1}(c+dx)\left(\pi-2\tan\right.}{f^2\left(\frac{(de-cf)^2}{f^2}+1\right)}\right.}{de}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^2/(e + f*x)^2,x]
```

```
[Out] (-(a^2/f) + (2*a*b*(-((-(c*d*e) + f + c^2*f - d^2*e*x + c*d*f*x)*ArcTan[c +
 d*x]) + d*(e + f*x)*Log[(d*(e + f*x))/Sqrt[1 + (c + d*x)^2]]))/(d^2*e^2 -
2*c*d*e*f + (1 + c^2)*f^2) + (b^2*d*(e + f*x)*(-((E^(I*ArcTan[(d*e - c*f)/f
])*ArcTan[c + d*x]^2)/(f*Sqrt[1 + (d*e - c*f)^2/f^2])) + ((c + d*x)*ArcTan[
c + d*x]^2)/(d*(e + f*x)) - ((d*e - c*f)*((-I)*(Pi - 2*ArcTan[(d*e - c*f)/f
])*ArcTan[c + d*x] - Pi*Log[1 + E^((-2*I)*ArcTan[c + d*x])] - 2*(ArcTan[(d*
e - c*f)/f] + ArcTan[c + d*x])*Log[1 - E^((2*I)*(ArcTan[(d*e - c*f)/f] + Ar
cTan[c + d*x]))] + Pi*Log[1/Sqrt[1 + (c + d*x)^2]] + 2*ArcTan[(d*e - c*f)/f
]*Log[Sin[ArcTan[(d*e - c*f)/f] + ArcTan[c + d*x]]] + I*PolyLog[2, E^((2*I)
*(ArcTan[(d*e - c*f)/f] + ArcTan[c + d*x]))]))/(f^2*(1 + (d*e - c*f)^2/f^2)
)))/(d*e - c*f))/(e + f*x)
```

Maple [A] time = 0.132, size = 1087, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^2/(f*x+e)^2,x)
```

```
[Out] -d*a^2/(d*f*x+d*e)/f-d*b^2/(d*f*x+d*e)/f*arctan(d*x+c)^2-d*b^2*arctan(d*x+c
)/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(1+(d*x+c)^2)-d*b^2/(c^2*f^2-2*c*d*e*f+
d^2*e^2+f^2)*arctan(d*x+c)^2*c+d^2*b^2/f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ar
ctan(d*x+c)^2*e+2*d*b^2*arctan(d*x+c)/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(f*
(d*x+c)-c*f+d*e)+I*d*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*dilog((I*f-f*(d*x+
c))/(d*e+I*f-c*f))-I*d*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*dilog((I*f+f*(d*
x+c))/(I*f+c*f-d*e))-1/4*I*d*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(d*x+c+I
)^2-1/2*I*d*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*dilog(1/2*I*(d*x+c-I))+1/2*
I*d*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(d*x+c+I)*ln(1+(d*x+c)^2)+1/4*I*d
```

```
*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(d*x+c-I)^2+I*d*b^2/(c^2*f^2-2*c*d*e
*f+d^2*e^2+f^2)*ln(f*(d*x+c)-c*f+d*e)*ln((I*f-f*(d*x+c))/(d*e+I*f-c*f))+1/2
*I*d*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I))-I
*d*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(f*(d*x+c)-c*f+d*e)*ln((I*f+f*(d*x
+c))/(I*f+c*f-d*e))-1/2*I*d*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(d*x+c-I)
*ln(1+(d*x+c)^2)-1/2*I*d*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(d*x+c+I)*ln
(1/2*I*(d*x+c-I))+1/2*I*d*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*dilog(-1/2*I*
(d*x+c+I))-2*d*a*b/(d*f*x+d*e)/f*arctan(d*x+c)-d*a*b/(c^2*f^2-2*c*d*e*f+d^2
*e^2+f^2)*ln(1+(d*x+c)^2)-2*d*a*b/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*
x+c)*c+2*d^2*a*b/f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)*e+2*d*a*b/
(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(f*(d*x+c)-c*f+d*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(d\left(\frac{2\left(d^{2}e-cdf\right)\arctan\left(\frac{d^{2}x+cd}{d}\right)}{\left(d^{2}e^{2}f-2\,cdef^{2}+\left(c^{2}+1\right)f^{3}\right)d}-\frac{\log\left(d^{2}x^{2}+2\,cdx+c^{2}+1\right)}{d^{2}e^{2}-2\,cdef+\left(c^{2}+1\right)f^{2}}+\frac{2\,\log\left(fx+e\right)}{d^{2}e^{2}-2\,cdef+\left(c^{2}+1\right)f^{2}}\right)-\frac{2\arctan\left(dx+c\right)}{f^{2}x+ef}\right)ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] (d*(2*(d^2*e - c*d*f)*arctan((d^2*x + c*d)/d)/((d^2*e^2*f - 2*c*d*e*f^2 + (
c^2 + 1)*f^3)*d) - log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*e^2 - 2*c*d*e*f +
(c^2 + 1)*f^2) + 2*log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 + 1)*f^2)) - 2*
arctan(d*x + c)/(f^2*x + e*f))*a*b - 1/16*(4*arctan(d*x + c)^2 - 16*(f^2*x
+ e*f)*integrate(1/16*(12*(d^2*f*x^2 + 2*c*d*f*x + (c^2 + 1)*f)*arctan(d*x
+ c)^2 + (d^2*f*x^2 + 2*c*d*f*x + (c^2 + 1)*f)*log(d^2*x^2 + 2*c*d*x + c^2
+ 1)^2 + 8*(d*f*x + d*e)*arctan(d*x + c) - 4*(d^2*f*x^2 + c*d*e + (d^2*e +
c*d*f)*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^2*f^3*x^4 + (c^2 + 1)*e^2*f
+ 2*(d^2*e*f^2 + c*d*f^3)*x^3 + (d^2*e^2*f + 4*c*d*e*f^2 + (c^2 + 1)*f^3)*x
^2 + 2*(c*d*e^2*f + (c^2 + 1)*e*f^2)*x), x) - log(d^2*x^2 + 2*c*d*x + c^2 +
 1)^2)*b^2/(f^2*x + e*f) - a^2/(f^2*x + e*f)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^{2}\arctan\left(dx+c\right)^{2}+2\,ab\arctan\left(dx+c\right)+a^{2}}{f^{2}x^{2}+2\,efx+e^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2)/(f^2*x^2 + 2
*e*f*x + e^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**2/(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan (dx + c) + a)^2}{(fx + e)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(d*x + c) + a)^2/(f*x + e)^2, x)
```

3.36 $\int (e+fx)^2 \left(a+b\tan^{-1}(c+dx)\right)^3 dx$

Optimal. Leaf size=564

$$\frac{ib^2\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{d^3}+\frac{b^3\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e\right.}{2d^3}$$

```
[Out] (a*b^2*f^2*x)/d^2 + (b^3*f^2*(c + d*x)*ArcTan[c + d*x])/d^3 - (b*f^2*(a + b
*ArcTan[c + d*x])^2)/(2*d^3) - ((3*I)*b*f*(d*e - c*f)*(a + b*ArcTan[c + d*x
])^2)/d^3 - (3*b*f*(d*e - c*f)*(c + d*x)*(a + b*ArcTan[c + d*x])^2)/d^3 - (
b*f^2*(c + d*x)^2*(a + b*ArcTan[c + d*x])^2)/(2*d^3) + ((I/3)*(3*d^2*e^2 -
6*c*d*e*f - (1 - 3*c^2)*f^2)*(a + b*ArcTan[c + d*x])^3)/d^3 - ((d*e - c*f)*
(d^2*e^2 - 2*c*d*e*f - (3 - c^2)*f^2)*(a + b*ArcTan[c + d*x])^3)/(3*d^3*f)
+ ((e + f*x)^3*(a + b*ArcTan[c + d*x])^3)/(3*f) - (6*b^2*f*(d*e - c*f)*(a +
 b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))])/d^3 + (b*(3*d^2*e^2 - 6*c*d*e
*f - (1 - 3*c^2)*f^2)*(a + b*ArcTan[c + d*x])^2*Log[2/(1 + I*(c + d*x))])/d
^3 - (b^3*f^2*Log[1 + (c + d*x)^2])/(2*d^3) - ((3*I)*b^3*f*(d*e - c*f)*Poly
Log[2, 1 - 2/(1 + I*(c + d*x))])/d^3 + (I*b^2*(3*d^2*e^2 - 6*c*d*e*f - (1 -
 3*c^2)*f^2)*(a + b*ArcTan[c + d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d
^3 + (b^3*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*c^2)*f^2)*PolyLog[3, 1 - 2/(1 + I
*(c + d*x))])/(2*d^3)
```

Rubi [A] time = 0.937082, antiderivative size = 564, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5047, 4864, 4846, 4920, 4854, 2402, 2315, 4852, 4916, 260, 4884, 4984, 4994, 6610}

$$\frac{ib^2\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{d^3}+\frac{b^3\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e\right.}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*(a + b*ArcTan[c + d*x])^3,x]
```



```
[Out] (a*b^2*f^2*x)/d^2 + (b^3*f^2*(c + d*x)*ArcTan[c + d*x])/d^3 - (b*f^2*(a + b
*ArcTan[c + d*x])^2)/(2*d^3) - ((3*I)*b*f*(d*e - c*f)*(a + b*ArcTan[c + d*x
])^2)/d^3 - (3*b*f*(d*e - c*f)*(c + d*x)*(a + b*ArcTan[c + d*x])^2)/d^3 - (
b*f^2*(c + d*x)^2*(a + b*ArcTan[c + d*x])^2)/(2*d^3) + ((I/3)*(3*d^2*e^2 -
6*c*d*e*f - (1 - 3*c^2)*f^2)*(a + b*ArcTan[c + d*x])^3)/d^3 - ((d*e - c*f)*
(d^2*e^2 - 2*c*d*e*f - (3 - c^2)*f^2)*(a + b*ArcTan[c + d*x])^3)/(3*d^3*f)
```

```
+ ((e + f*x)^3*(a + b*ArcTan[c + d*x])^3)/(3*f) - (6*b^2*f*(d*e - c*f)*(a +
 b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))])/d^3 + (b*(3*d^2*e^2 - 6*c*d*e
*f - (1 - 3*c^2)*f^2)*(a + b*ArcTan[c + d*x])^2*Log[2/(1 + I*(c + d*x))])/d
^3 - (b^3*f^2*Log[1 + (c + d*x)^2])/(2*d^3) - ((3*I)*b^3*f*(d*e - c*f)*Poly
Log[2, 1 - 2/(1 + I*(c + d*x))])/d^3 + (I*b^2*(3*d^2*e^2 - 6*c*d*e*f - (1 -
 3*c^2)*f^2)*(a + b*ArcTan[c + d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d
^3 + (b^3*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*c^2)*f^2)*PolyLog[3, 1 - 2/(1 + I
*(c + d*x))])/(2*d^3)
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4984

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2\left(a+b\tan^{-1}(x)\right)^3\,dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^3\left(a+b\tan^{-1}(c+dx)\right)^3}{3f} - \frac{b\,\text{Subst}\left(\int \left(\frac{3f^2(de-cf)\left(a+b\tan^{-1}(x)\right)^2}{d^3}+\frac{f^3x(a+b\tan\ldots}{d\ldots}\right.\right.}{\ldots}$$

$$= \frac{(e+fx)^3\left(a+b\tan^{-1}(c+dx)\right)^3}{3f} - \frac{b\,\text{Subst}\left(\int \frac{\left((de-cf)\left(d^2e^2-2cdef-3f^2+c^2f^2\right)+f\left(3d^2e^2-\ldots\right.\right.}{1+x^2}\ldots\right.}{d^3f\ldots}$$

$$= -\frac{3bf(de-cf)(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{d^3} - \frac{bf^2(c+dx)^2\left(a+b\tan^{-1}(c+d\ldots\right.}{2d^3}\ldots$$

$$= -\frac{3ibf(de-cf)\left(a+b\tan^{-1}(c+dx)\right)^2}{d^3} - \frac{3bf(de-cf)(c+dx)\left(a+b\tan^{-1}(c+\ldots\right.}{d^3}\ldots$$

$$= \frac{ab^2f^2x}{d^2} - \frac{bf^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^3} - \frac{3ibf(de-cf)\left(a+b\tan^{-1}(c+dx)\right)^2}{d^3} - \ldots$$

$$= \frac{ab^2f^2x}{d^2} + \frac{b^3f^2(c+dx)\tan^{-1}(c+dx)}{d^3} - \frac{bf^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^3} - \frac{3ibf(de-\ldots}{\ldots}$$

$$= \frac{ab^2f^2x}{d^2} + \frac{b^3f^2(c+dx)\tan^{-1}(c+dx)}{d^3} - \frac{bf^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^3} - \frac{3ibf(de-\ldots}{\ldots}$$

$$= \frac{ab^2f^2x}{d^2} + \frac{b^3f^2(c+dx)\tan^{-1}(c+dx)}{d^3} - \frac{bf^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^3} - \frac{3ibf(de-\ldots}{\ldots}$$

Mathematica [B] time = 8.96162, size = 1844, normalized size = 3.27

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)^2*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (a^2*(a*d^2*e^2 - 3*b*d*e*f + 2*b*c*f^2)*x)/d^2 - (a^2*f*(-2*a*d*e + b*f)*x
^2)/(2*d) + (a^3*f^2*x^3)/3 + ((3*a^2*b*c*d^2*e^2 + 3*a^2*b*d*e*f - 3*a^2*b
```

```
*c^2*d*e*f - 3*a^2*b*c*f^2 + a^2*b*c^3*f^2)*ArcTan[c + d*x])/d^3 + a^2*b*x*
(3*e^2 + 3*e*f*x + f^2*x^2)*ArcTan[c + d*x] + ((-3*a^2*b*d^2*e^2 + 6*a^2*b*
c*d*e*f + a^2*b*f^2 - 3*a^2*b*c^2*f^2)*Log[1 + c^2 + 2*c*d*x + d^2*x^2])/(2
*d^3) + (3*a*b^2*e^2*((-I)*ArcTan[c + d*x]^2 + (c + d*x)*ArcTan[c + d*x]^2
+ 2*ArcTan[c + d*x]*Log[1 + E^((2*I)*ArcTan[c + d*x])] - I*PolyLog[2, -E^((
2*I)*ArcTan[c + d*x])]))/d + 6*a*b^2*e*f*(-(((c + d*x)*ArcTan[c + d*x])/d^2
) + (I*c*ArcTan[c + d*x]^2)/d^2 - (c*(c + d*x)*ArcTan[c + d*x]^2)/d^2 + ((1
 + (c + d*x)^2)*ArcTan[c + d*x]^2)/(2*d^2) - (2*c*ArcTan[c + d*x]*Log[1 + E
^((2*I)*ArcTan[c + d*x])])/d^2 - Log[1/Sqrt[1 + (c + d*x)^2]]/d^2 + (I*c*Po
lyLog[2, -E^((2*I)*ArcTan[c + d*x])])/d^2) + (b^3*e^2*((-I)*ArcTan[c + d*x]
^3 + (c + d*x)*ArcTan[c + d*x]^3 + 3*ArcTan[c + d*x]^2*Log[1 + E^((2*I)*Arc
Tan[c + d*x])] - (3*I)*ArcTan[c + d*x]*PolyLog[2, -E^((2*I)*ArcTan[c + d*x]
)] + (3*PolyLog[3, -E^((2*I)*ArcTan[c + d*x])])/2))/d + (b^3*e*f*(ArcTan[c
+ d*x]*((3*I)*ArcTan[c + d*x] + (2*I)*c*ArcTan[c + d*x]^2 + (1 + (c + d*x)^
2)*ArcTan[c + d*x]^2 - (c + d*x)*ArcTan[c + d*x]*(3 + 2*c*ArcTan[c + d*x])
- 6*Log[1 + E^((2*I)*ArcTan[c + d*x])] - 6*c*ArcTan[c + d*x]*Log[1 + E^((2*
I)*ArcTan[c + d*x])]) + (3*I)*(1 + 2*c*ArcTan[c + d*x])*PolyLog[2, -E^((2*I
)*ArcTan[c + d*x])] - 3*c*PolyLog[3, -E^((2*I)*ArcTan[c + d*x])]))/d^2 + (a
*b^2*f^2*(1 + (c + d*x)^2)^(3/2)*((c + d*x)/Sqrt[1 + (c + d*x)^2] + (6*c*(c
 + d*x)*ArcTan[c + d*x])/Sqrt[1 + (c + d*x)^2] + (3*(c + d*x)*ArcTan[c + d*
x]^2)/Sqrt[1 + (c + d*x)^2] + (3*c^2*(c + d*x)*ArcTan[c + d*x]^2)/Sqrt[1 +
(c + d*x)^2] + I*ArcTan[c + d*x]^2*Cos[3*ArcTan[c + d*x]] - (3*I)*c^2*ArcTa
n[c + d*x]^2*Cos[3*ArcTan[c + d*x]] - 2*ArcTan[c + d*x]*Cos[3*ArcTan[c + d*
x]]*Log[1 + E^((2*I)*ArcTan[c + d*x])] + 6*c^2*ArcTan[c + d*x]*Cos[3*ArcTan
[c + d*x]]*Log[1 + E^((2*I)*ArcTan[c + d*x])] + 6*c*Cos[3*ArcTan[c + d*x]]*
Log[1/Sqrt[1 + (c + d*x)^2]] + (ArcTan[c + d*x]*(-4 + (3*I - 12*c - (9*I)*c
^2)*ArcTan[c + d*x]) + 6*(-1 + 3*c^2)*ArcTan[c + d*x]*Log[1 + E^((2*I)*ArcT
an[c + d*x])] + 18*c*Log[1/Sqrt[1 + (c + d*x)^2]])/Sqrt[1 + (c + d*x)^2] -
((4*I)*(-1 + 3*c^2)*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])])/(1 + (c + d*x)^
2)^(3/2) + Sin[3*ArcTan[c + d*x]] + 6*c*ArcTan[c + d*x]*Sin[3*ArcTan[c + d*
x]] - ArcTan[c + d*x]^2*Sin[3*ArcTan[c + d*x]] + 3*c^2*ArcTan[c + d*x]^2*Si
n[3*ArcTan[c + d*x]]))/(4*d^3) + (b^3*f^2*((-I)*(3*c - ArcTan[c + d*x] + 3*
c^2*ArcTan[c + d*x])*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])] + ((1 + (c + d*
x)^2)^(3/2)*((3*(c + d*x)*ArcTan[c + d*x])/Sqrt[1 + (c + d*x)^2] + (9*c*(c
+ d*x)*ArcTan[c + d*x]^2)/Sqrt[1 + (c + d*x)^2] + (3*(c + d*x)*ArcTan[c + d
*x]^3)/Sqrt[1 + (c + d*x)^2] + (3*c^2*(c + d*x)*ArcTan[c + d*x]^3)/Sqrt[1 +
 (c + d*x)^2] - (9*I)*c*ArcTan[c + d*x]^2*Cos[3*ArcTan[c + d*x]] + I*ArcTan
[c + d*x]^3*Cos[3*ArcTan[c + d*x]] - (3*I)*c^2*ArcTan[c + d*x]^3*Cos[3*ArcT
an[c + d*x]] + 18*c*ArcTan[c + d*x]*Cos[3*ArcTan[c + d*x]]*Log[1 + E^((2*I)
*ArcTan[c + d*x])] - 3*ArcTan[c + d*x]^2*Cos[3*ArcTan[c + d*x]]*Log[1 + E^(
(2*I)*ArcTan[c + d*x])] + 9*c^2*ArcTan[c + d*x]^2*Cos[3*ArcTan[c + d*x]]*Lo
g[1 + E^((2*I)*ArcTan[c + d*x])] + 3*Cos[3*ArcTan[c + d*x]]*Log[1/Sqrt[1 +
(c + d*x)^2]] + (3*(ArcTan[c + d*x]^2*(-2 - (9*I)*c + I*ArcTan[c + d*x] - 4
*c*ArcTan[c + d*x] - (3*I)*c^2*ArcTan[c + d*x]) + 3*ArcTan[c + d*x]*(6*c -
ArcTan[c + d*x] + 3*c^2*ArcTan[c + d*x])*Log[1 + E^((2*I)*ArcTan[c + d*x])]
```

```
 + 3*Log[1/Sqrt[1 + (c + d*x)^2]]))/Sqrt[1 + (c + d*x)^2] + (6*(-1 + 3*c^2)
*PolyLog[3, -E^((2*I)*ArcTan[c + d*x])])/(1 + (c + d*x)^2)^(3/2) + 3*ArcTan
[c + d*x]*Sin[3*ArcTan[c + d*x]] + 9*c*ArcTan[c + d*x]^2*Sin[3*ArcTan[c + d
*x]] - ArcTan[c + d*x]^3*Sin[3*ArcTan[c + d*x]] + 3*c^2*ArcTan[c + d*x]^3*S
in[3*ArcTan[c + d*x]]))/12))/d^3
```

Maple [C] time = 2.95, size = 6682, normalized size = 11.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(a+b*arctan(d*x+c))^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 7/8*b^3*c^2*e^2*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 3*a*b^2*c^2*e
^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - (3*arctan(d*x + c)*arctan(
(d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*c^2*e^2 - 7/32*(6
*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*arctan((
d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3*c^2*e^2 + 1/3*a^3*f^
2*x^3 + 7/8*b^3*e^2*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 28*b^3*d^
2*f^2*integrate(1/32*x^4*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x
) + 3*b^3*d^2*f^2*integrate(1/32*x^4*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x
+ c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 96*a*b^2*d^2*f^2*integrate
(1/32*x^4*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b^3*d^2*
e*f*integrate(1/32*x^3*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
+ 56*b^3*c*d*f^2*integrate(1/32*x^3*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x +
c^2 + 1), x) + 4*b^3*d^2*f^2*integrate(1/32*x^4*arctan(d*x + c)*log(d^2*x^2
 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*d^2*e*f*int
egrate(1/32*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2
```

+ 2*c*d*x + c^2 + 1), x) + 6*b^3*c*d*f^2*integrate(1/32*x^3*arctan(d*x + c
)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 19
2*a*b^2*d^2*e*f*integrate(1/32*x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c
^2 + 1), x) + 192*a*b^2*c*d*f^2*integrate(1/32*x^3*arctan(d*x + c)^2/(d^2*x
^2 + 2*c*d*x + c^2 + 1), x) + 28*b^3*d^2*e^2*integrate(1/32*x^2*arctan(d*x
+ c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 112*b^3*c*d*e*f*integrate(1/32*x
^2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 28*b^3*c^2*f^2*int
egrate(1/32*x^2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^
3*d^2*e*f*integrate(1/32*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 +
1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 4*b^3*c*d*f^2*integrate(1/32*x^3*arc
tan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1)
, x) + 3*b^3*d^2*e^2*integrate(1/32*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d
*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d*e*f*integrat
e(1/32*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*
c*d*x + c^2 + 1), x) + 3*b^3*c^2*f^2*integrate(1/32*x^2*arctan(d*x + c)*log
(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 96*a*b^
2*d^2*e^2*integrate(1/32*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1
), x) + 384*a*b^2*c*d*e*f*integrate(1/32*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2
*c*d*x + c^2 + 1), x) + 96*a*b^2*c^2*f^2*integrate(1/32*x^2*arctan(d*x + c)
^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b^3*c*d*e^2*integrate(1/32*x*arct
an(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b^3*c^2*e*f*integrate(
1/32*x*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*d^2*e^2
*integrate(1/32*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x
^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d*e*f*integrate(1/32*x^2*arctan(d*x
+ c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6
*b^3*c*d*e^2*integrate(1/32*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 +
1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*c^2*e*f*integrate(1/32*x*ar
ctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 +
1), x) + 192*a*b^2*c*d*e^2*integrate(1/32*x*arctan(d*x + c)^2/(d^2*x^2 + 2
*c*d*x + c^2 + 1), x) + 192*a*b^2*c^2*e*f*integrate(1/32*x*arctan(d*x + c)^
2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d*e^2*integrate(1/32*x*arcta
n(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1),
x) + 3*b^3*c^2*e^2*integrate(1/32*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c
^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + a^3*e*f*x^2 + 3*a*b^2*e^2*arc
tan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 4*b^3*d*f^2*integrate(1/32*x^3*a
rctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + b^3*d*f^2*integrate(1/
32*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
- 12*b^3*d*e*f*integrate(1/32*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c
^2 + 1), x) + 3*b^3*d*e*f*integrate(1/32*x^2*log(d^2*x^2 + 2*c*d*x + c^2 +
1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 12*b^3*d*e^2*integrate(1/32*x*arct
an(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*d*e^2*integrate(1/3
2*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) -
(3*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/
d)*a*b^2*e^2 - 7/32*(6*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*ar
ctan(d*x + c)*arctan((d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^

```
3*e^2 + 3*(x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/
d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a^2*b*e*f + 1/2*(2*x^3*ar
ctan(d*x + c) - d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)
/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*a^2*b*f^2 + a^
3*e^2*x + 28*b^3*f^2*integrate(1/32*x^2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*
x + c^2 + 1), x) + 3*b^3*f^2*integrate(1/32*x^2*arctan(d*x + c)*log(d^2*x^2
 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 96*a*b^2*f^2*in
tegrate(1/32*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b
^3*e*f*integrate(1/32*x*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
 + 6*b^3*e*f*integrate(1/32*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 +
 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*a*b^2*e*f*integrate(1/32*x*ar
ctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*e^2*integrate(1/3
2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c
^2 + 1), x) + 3/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*a^2*
b*e^2/d + 1/24*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x)*arctan(d*x + c)^
3 - 1/32*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x)*arctan(d*x + c)*log(d^
2*x^2 + 2*c*d*x + c^2 + 1)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(a^3f^2x^2 + 2\,a^3efx + a^3e^2 + \left(b^3f^2x^2 + 2\,b^3efx + b^3e^2\right)\arctan\left(dx+c\right)^3 + 3\left(ab^2f^2x^2 + 2\,ab^2efx + ab^2e^2\right)\text{arctan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*f^2*x^2 + 2*a^3*e*f*x + a^3*e^2 + (b^3*f^2*x^2 + 2*b^3*e*f*x +
 b^3*e^2)*arctan(d*x + c)^3 + 3*(a*b^2*f^2*x^2 + 2*a*b^2*e*f*x + a*b^2*e^2)
*arctan(d*x + c)^2 + 3*(a^2*b*f^2*x^2 + 2*a^2*b*e*f*x + a^2*b*e^2)*arctan(d
*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\,\text{atan}\left(c + dx\right)\right)^3 \left(e + fx\right)^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(a+b*atan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*atan(c + d*x))**3*(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx+e\right)^2 (b\arctan\left(dx+c\right)+a)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*(b*arctan(d*x + c) + a)^3, x)
```

3.37 $\int (e+fx)\left(a+b\tan^{-1}(c+dx)\right)^3\,dx$

Optimal. Leaf size=337

$$\frac{3ib^2(de-cf)\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{d^2}+\frac{3b^3(de-cf)\text{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d^2}-\frac{3ib^3f\text{PolyLog}}{2}$$

```
[Out] (((-3*I)/2)*b*f*(a + b*ArcTan[c + d*x])^2)/d^2 - (3*b*f*(c + d*x)*(a + b*Ar
cTan[c + d*x])^2)/(2*d^2) + (I*(d*e - c*f)*(a + b*ArcTan[c + d*x])^3)/d^2 -
 ((d*e + f - c*f)*(d*e - (1 + c)*f)*(a + b*ArcTan[c + d*x])^3)/(2*d^2*f) +
((e + f*x)^2*(a + b*ArcTan[c + d*x])^3)/(2*f) - (3*b^2*f*(a + b*ArcTan[c +
d*x])*Log[2/(1 + I*(c + d*x))])/d^2 + (3*b*(d*e - c*f)*(a + b*ArcTan[c + d*
x])^2*Log[2/(1 + I*(c + d*x))])/d^2 - (((3*I)/2)*b^3*f*PolyLog[2, 1 - 2/(1
+ I*(c + d*x))])/d^2 + ((3*I)*b^2*(d*e - c*f)*(a + b*ArcTan[c + d*x])*PolyL
og[2, 1 - 2/(1 + I*(c + d*x))])/d^2 + (3*b^3*(d*e - c*f)*PolyLog[3, 1 - 2/(
1 + I*(c + d*x))])/(2*d^2)
```

Rubi [A] time = 0.631683, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {5047, 4864, 4846, 4920, 4854, 2402, 2315, 4984, 4884, 4994, 6610}

$$\frac{3ib^2(de-cf)\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{d^2}+\frac{3b^3(de-cf)\text{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d^2}-\frac{3ib^3f\text{PolyLog}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*(a + b*ArcTan[c + d*x])^3,x]
```



```
[Out] (((-3*I)/2)*b*f*(a + b*ArcTan[c + d*x])^2)/d^2 - (3*b*f*(c + d*x)*(a + b*Ar
cTan[c + d*x])^2)/(2*d^2) + (I*(d*e - c*f)*(a + b*ArcTan[c + d*x])^3)/d^2 -
 ((d*e + f - c*f)*(d*e - (1 + c)*f)*(a + b*ArcTan[c + d*x])^3)/(2*d^2*f) +
((e + f*x)^2*(a + b*ArcTan[c + d*x])^3)/(2*f) - (3*b^2*f*(a + b*ArcTan[c +
d*x])*Log[2/(1 + I*(c + d*x))])/d^2 + (3*b*(d*e - c*f)*(a + b*ArcTan[c + d*
x])^2*Log[2/(1 + I*(c + d*x))])/d^2 - (((3*I)/2)*b^3*f*PolyLog[2, 1 - 2/(1
+ I*(c + d*x))])/d^2 + ((3*I)*b^2*(d*e - c*f)*(a + b*ArcTan[c + d*x])*PolyL
og[2, 1 - 2/(1 + I*(c + d*x))])/d^2 + (3*b^3*(d*e - c*f)*PolyLog[3, 1 - 2/(
1 + I*(c + d*x))])/(2*d^2)
```


Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4984

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (e+fx)\left(a+b\tan^{-1}(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)\left(a+b\tan^{-1}(x)\right)^3\,dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)^3}{2f} - \frac{(3b)\,\text{Subst}\left(\int \left(\frac{f^2\left(a+b\tan^{-1}(x)\right)^2}{d^2}+\frac{((de-f-cf)(de\cdots\right.\right.}{2\cdots}$$

$$= \frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)^3}{2f} - \frac{(3b)\,\text{Subst}\left(\int \frac{((de-f-cf)(de+f-cf)+2f(de-cf)x)\left(a+\cdots\right.}{1+x^2}\right.}{2d^2f}$$

$$= -\frac{3bf(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2} + \frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)^3}{2f} - \frac{(3b)\,\cdots}{\cdots}$$

$$= -\frac{3ibf\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2} - \frac{3bf(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2} + \frac{(e+fx)^2\cdots}{\cdots}$$

$$= -\frac{3ibf\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2} - \frac{3bf(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2} + \frac{i(de-cf)\cdots}{\cdots}$$

$$= -\frac{3ibf\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2} - \frac{3bf(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2} + \frac{i(de-cf)\cdots}{\cdots}$$

$$= -\frac{3ibf\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2} - \frac{3bf(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2} + \frac{i(de-cf)\cdots}{\cdots}$$

$$= -\frac{3ibf\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2} - \frac{3bf(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2} + \frac{i(de-cf)\cdots}{\cdots}$$

Mathematica [A] time = 0.678918, size = 592, normalized size = 1.76

$$\frac{6ab^2de\left(\tan^{-1}(c+dx)\left((c+dx-i)\tan^{-1}(c+dx)+2\log\left(1+e^{2i\tan^{-1}(c+dx)}\right)\right)-i\text{PolyLog}\left(2,-e^{2i\tan^{-1}(c+dx)}\right)\right)-6ab^2cf\cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (a^2*(2*a*d*e - 3*b*f - 2*a*c*f)*(c + d*x) + a^3*f*(c + d*x)^2 + 3*a^2*b*f*
ArcTan[c + d*x] - 3*a^2*b*(c + d*x)*(c*f - d*(2*e + f*x))*ArcTan[c + d*x] +
 6*a*b^2*f*(-((c + d*x)*ArcTan[c + d*x]) + ((1 + (c + d*x)^2)*ArcTan[c + d*
x]^2)/2 - Log[1/Sqrt[1 + (c + d*x)^2]]) - 3*a^2*b*(d*e - c*f)*Log[1 + (c +
d*x)^2] + 6*a*b^2*d*e*(ArcTan[c + d*x]*((-I + c + d*x)*ArcTan[c + d*x] + 2*
Log[1 + E^((2*I)*ArcTan[c + d*x])]) - I*PolyLog[2, -E^((2*I)*ArcTan[c + d*x
])]) - 6*a*b^2*c*f*(ArcTan[c + d*x]*((-I + c + d*x)*ArcTan[c + d*x] + 2*Log
[1 + E^((2*I)*ArcTan[c + d*x])]) - I*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])]
) + b^3*f*(ArcTan[c + d*x]*((3*I)*ArcTan[c + d*x] - 3*(c + d*x)*ArcTan[c +
d*x] + (1 + (c + d*x)^2)*ArcTan[c + d*x]^2 - 6*Log[1 + E^((2*I)*ArcTan[c +
d*x])]) + (3*I)*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])]) + 2*b^3*d*e*(ArcTan
[c + d*x]^2*((-I + c + d*x)*ArcTan[c + d*x] + 3*Log[1 + E^((2*I)*ArcTan[c +
 d*x])]) - (3*I)*ArcTan[c + d*x]*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])] + (
3*PolyLog[3, -E^((2*I)*ArcTan[c + d*x])])/2) - 2*b^3*c*f*(ArcTan[c + d*x]^2
*((-I + c + d*x)*ArcTan[c + d*x] + 3*Log[1 + E^((2*I)*ArcTan[c + d*x])]) -
(3*I)*ArcTan[c + d*x]*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])] + (3*PolyLog[3
, -E^((2*I)*ArcTan[c + d*x])])/2))/(2*d^2)
```

Maple [C] time = 1.261, size = 16362, normalized size = 48.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(a+b*arctan(d*x+c))^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 7/8*b^3*c^2*e*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 3*a*b^2*c^2*e*a
rctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - (3*arctan(d*x + c)*arctan((d^2
*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*c^2*e - 7/32*(6*arcta
```

n(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3*c^2*e + 7/8*b^3*e*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 56*b^3*d^2*f*integrate(1/64*x^3*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*d^2*f*integrate(1/64*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*a*b^2*d^2*f*integrate(1/64*x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b^3*d^2*e*integrate(1/64*x^2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 112*b^3*c*d*f*integrate(1/64*x^2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*d^2*f*integrate(1/64*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*d^2*e*integrate(1/64*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d*f*integrate(1/64*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*a*b^2*d^2*e*integrate(1/64*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 384*a*b^2*c*d*f*integrate(1/64*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 112*b^3*c*d*e*integrate(1/64*x*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b^3*c^2*f*integrate(1/64*x*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 24*b^3*d^2*e*integrate(1/64*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d*f*integrate(1/64*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d*e*integrate(1/64*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*c^2*f*integrate(1/64*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 384*a*b^2*c*d*e*integrate(1/64*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*a*b^2*c^2*f*integrate(1/64*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 24*b^3*c*d*e*integrate(1/64*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*c^2*e*integrate(1/64*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 1/2*a^3*f*x^2 + 3*a*b^2*e*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 12*b^3*d*f*integrate(1/64*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*d*f*integrate(1/64*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 24*b^3*d*e*integrate(1/64*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*d*e*integrate(1/64*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - (3*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*e - 7/32*(6*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3*e + 3/2*(x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a^2*b*f + a^3*e*x + 56*b^3*f*integrate(1/64*x*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*f*integrate(1/64*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*a*b^2*f*integrate(1/64*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*e*integrate(1/64*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)

```
^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3/2*(2*(d*x + c)*arctan(d*x + c) - l
og((d*x + c)^2 + 1))*a^2*b*e/d + 1/16*(b^3*f*x^2 + 2*b^3*e*x)*arctan(d*x +
c)^3 - 3/64*(b^3*f*x^2 + 2*b^3*e*x)*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x +
 c^2 + 1)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(a^3fx + a^3e + \left(b^3fx + b^3e\right)\arctan\left(dx+c\right)^3 + 3\left(ab^2fx + ab^2e\right)\arctan\left(dx+c\right)^2 + 3\left(a^2bfx + a^2be\right)\arctan\left(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*f*x + a^3*e + (b^3*f*x + b^3*e)*arctan(d*x + c)^3 + 3*(a*b^2*f
*x + a*b^2*e)*arctan(d*x + c)^2 + 3*(a^2*b*f*x + a^2*b*e)*arctan(d*x + c),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\operatorname{atan}\left(c + dx\right)\right)^3\left(e + fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*atan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*atan(c + d*x))**3*(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx + e\right)\left(b\arctan\left(dx + c\right) + a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(b*arctan(d*x + c) + a)^3, x)
```

3.38 $\int \left(a+b\tan^{-1}(c+dx)\right)^3\,dx$

Optimal. Leaf size=143

$$\frac{3ib^2\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{d}+\frac{3b^3\text{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d}+\frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^3}{d}$$

```
[Out] (I*(a + b*ArcTan[c + d*x])^3)/d + ((c + d*x)*(a + b*ArcTan[c + d*x])^3)/d +
 (3*b*(a + b*ArcTan[c + d*x])^2*Log[2/(1 + I*(c + d*x))])/d + ((3*I)*b^2*(a
 + b*ArcTan[c + d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d + (3*b^3*PolyL
og[3, 1 - 2/(1 + I*(c + d*x))])/(2*d)
```

Rubi [A] time = 0.214405, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5039, 4846, 4920, 4854, 4884, 4994, 6610}

$$\frac{3ib^2\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{d}+\frac{3b^3\text{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d}+\frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^3}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (I*(a + b*ArcTan[c + d*x])^3)/d + ((c + d*x)*(a + b*ArcTan[c + d*x])^3)/d +
 (3*b*(a + b*ArcTan[c + d*x])^2*Log[2/(1 + I*(c + d*x))])/d + ((3*I)*b^2*(a
 + b*ArcTan[c + d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d + (3*b^3*PolyL
og[3, 1 - 2/(1 + I*(c + d*x))])/(2*d)
```

Rule 5039

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \left(a+b\tan^{-1}(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \left(a+b\tan^{-1}(x)\right)^3\,dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^3}{d} - \frac{(3b)\,\text{Subst}\left(\int \frac{x\left(a+b\tan^{-1}(x)\right)^2}{1+x^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{i\left(a+b\tan^{-1}(c+dx)\right)^3}{d} + \frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^3}{d} + \frac{(3b)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{i-x}\right.}{d}$$

$$= \frac{i\left(a+b\tan^{-1}(c+dx)\right)^3}{d} + \frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^3}{d} + \frac{3b\left(a+b\tan^{-1}(c+dx)\right)^2 \text{l}}{d}$$

$$= \frac{i\left(a+b\tan^{-1}(c+dx)\right)^3}{d} + \frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^3}{d} + \frac{3b\left(a+b\tan^{-1}(c+dx)\right)^2 \text{l}}{d}$$

$$= \frac{i\left(a+b\tan^{-1}(c+dx)\right)^3}{d} + \frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^3}{d} + \frac{3b\left(a+b\tan^{-1}(c+dx)\right)^2 \text{l}}{d}$$

<u>Mathematica</u> [A] time = 0.114604, size = 212, normalized size = 1.48

$$\frac{6ab^2\left(\tan^{-1}(c+dx)\left((c+dx-i)\tan^{-1}(c+dx)+2\log\left(1+e^{2i\tan^{-1}(c+dx)}\right)\right)-i\text{PolyLog}\left(2,-e^{2i\tan^{-1}(c+dx)}\right)\right)+2b^3\left(-3i\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (2*a^3*(c + d*x) + 6*a^2*b*(c + d*x)*ArcTan[c + d*x] - 3*a^2*b*Log[1 + (c +
 d*x)^2] + 6*a*b^2*(ArcTan[c + d*x]*((-I + c + d*x)*ArcTan[c + d*x] + 2*Log
[1 + E^((2*I)*ArcTan[c + d*x])]) - I*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])]
) + 2*b^3*(ArcTan[c + d*x]^2*((-I + c + d*x)*ArcTan[c + d*x] + 3*Log[1 + E^
((2*I)*ArcTan[c + d*x])]) - (3*I)*ArcTan[c + d*x]*PolyLog[2, -E^((2*I)*ArcT
an[c + d*x])] + (3*PolyLog[3, -E^((2*I)*ArcTan[c + d*x])])/2))/(2*d)
```

<u>Maple</u> [B] time = 0.155, size = 359, normalized size = 2.5

$$xa^3 + \frac{a^3c}{d} - \frac{3\,ib^3\arctan(dx+c)}{d}\text{polylog}\left(2, -\frac{(1+i(dx+c))^2}{1+(dx+c)^2}\right) + (\arctan(dx+c))^3\,xb^3 + \frac{(\arctan(dx+c))^3\,b^3c}{d} + 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^3,x)
```

```
[Out] x*a^3+1/d*a^3*c-3*I/d*b^3*arctan(d*x+c)*polylog(2,-(1+I*(d*x+c))^2/(1+(d*x+
c)^2))+arctan(d*x+c)^3*x*b^3+1/d*arctan(d*x+c)^3*b^3*c+3/d*b^3*arctan(d*x+c
)^2*ln((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)-3*I/d*arctan(d*x+c)^2*a*b^2+3/2/d*b
^3*polylog(3,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))-3*I/d*polylog(2,-(1+I*(d*x+c))
^2/(1+(d*x+c)^2))*a*b^2+3*arctan(d*x+c)^2*x*a*b^2+3/d*arctan(d*x+c)^2*a*b^2
*c+6/d*arctan(d*x+c)*ln((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)*a*b^2-I/d*b^3*arct
an(d*x+c)^3+3*arctan(d*x+c)*x*a^2*b+3/d*arctan(d*x+c)*a^2*b*c-3/2/d*a^2*b*l
n(1+(d*x+c)^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 7/8*b^3*c^2*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 1/8*b^3*x*arctan(
d*x + c)^3 + 3*a*b^2*c^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 3/32
*b^3*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 - (3*arctan(d*x +
 c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*c^2 -
7/32*(6*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*a
rctan((d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3*c^2 + 7/8*b^3
*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 28*b^3*d^2*integrate(1/32*x^
2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*d^2*integrate
(1/32*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c
*d*x + c^2 + 1), x) + 96*a*b^2*d^2*integrate(1/32*x^2*arctan(d*x + c)^2/(d^
2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b^3*c*d*integrate(1/32*x*arctan(d*x + c
)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*d^2*integrate(1/32*x^2*arcta
n(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1),
x) + 6*b^3*c*d*integrate(1/32*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2
 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*a*b^2*c*d*integrate(1/32*x*
arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d*integrate(
1/32*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x
+ c^2 + 1), x) + 3*b^3*c^2*integrate(1/32*arctan(d*x + c)*log(d^2*x^2 + 2*c
*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*a*b^2*arctan(d*x +
c)^2*arctan((d^2*x + c*d)/d)/d - 12*b^3*d*integrate(1/32*x*arctan(d*x + c)^
2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*d*integrate(1/32*x*log(d^2*x^2
```

```
+ 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - (3*arctan(d*x +
c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2 - 7/32*
(6*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*arctan
((d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3 + a^3*x + 3*b^3*in
tegrate(1/32*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 +
2*c*d*x + c^2 + 1), x) + 3/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2
 + 1))*a^2*b/d
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3 \arctan\left(dx+c\right)^3 + 3\,ab^2 \arctan\left(dx+c\right)^2 + 3\,a^2b \arctan\left(dx+c\right) + a^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arctan
(d*x + c) + a^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\,\text{atan}\left(c + dx\right)\right)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*atan(c + d*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \arctan\left(dx+c\right) + a\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(d*x + c) + a)^3, x)
```

3.39 $\int \frac{(a+b\tan^{-1}(c+dx))^3}{e+fx}\,dx$

Optimal. Leaf size=372

$$\frac{3b^2\left(a+b\tan^{-1}(c+dx)\right)\text{PolyLog}\left(3,1-\frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{2f}-\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{2f}-$$

```
[Out] -(((a + b*ArcTan[c + d*x])^3*Log[2/(1 - I*(c + d*x))])/f) + ((a + b*ArcTan[
c + d*x])^3*Log[(2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f +
 (((3*I)/2)*b*(a + b*ArcTan[c + d*x])^2*PolyLog[2, 1 - 2/(1 - I*(c + d*x))]
)/f - (((3*I)/2)*b*(a + b*ArcTan[c + d*x])^2*PolyLog[2, 1 - (2*d*(e + f*x))
/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f - (3*b^2*(a + b*ArcTan[c + d*x])
*PolyLog[3, 1 - 2/(1 - I*(c + d*x))])/(2*f) + (3*b^2*(a + b*ArcTan[c + d*x]
)*PolyLog[3, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/(2
*f) - (((3*I)/4)*b^3*PolyLog[4, 1 - 2/(1 - I*(c + d*x))])/f + (((3*I)/4)*b^
3*PolyLog[4, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f
```

Rubi [A] time = 0.20282, antiderivative size = 372, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {5047, 4860}

$$\frac{3b^2\left(a+b\tan^{-1}(c+dx)\right)\text{PolyLog}\left(3,1-\frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{2f}-\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{2f}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^3/(e + f*x),x]
```


```
[Out] -(((a + b*ArcTan[c + d*x])^3*Log[2/(1 - I*(c + d*x))])/f) + ((a + b*ArcTan[
c + d*x])^3*Log[(2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f +
 (((3*I)/2)*b*(a + b*ArcTan[c + d*x])^2*PolyLog[2, 1 - 2/(1 - I*(c + d*x))]
)/f - (((3*I)/2)*b*(a + b*ArcTan[c + d*x])^2*PolyLog[2, 1 - (2*d*(e + f*x))
/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f - (3*b^2*(a + b*ArcTan[c + d*x])
*PolyLog[3, 1 - 2/(1 - I*(c + d*x))])/(2*f) + (3*b^2*(a + b*ArcTan[c + d*x]
)*PolyLog[3, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/(2
*f) - (((3*I)/4)*b^3*PolyLog[4, 1 - 2/(1 - I*(c + d*x))])/f + (((3*I)/4)*b^
3*PolyLog[4, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f
```


Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rule 4860

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^3/((d_) + (e_.)*(x_)), x_Symbol] :>
-Simp[((a + b*ArcTan[c*x])^3*Log[2/(1 - I*c*x)])/e, x] + (Simp[((a + b*ArcT
an[c*x])^3*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x] + Simp[(3*
I*b*(a + b*ArcTan[c*x])^2*PolyLog[2, 1 - 2/(1 - I*c*x)])/(2*e), x] - Simp[(
3*I*b*(a + b*ArcTan[c*x])^2*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1
- I*c*x))])/(2*e), x] - Simp[(3*b^2*(a + b*ArcTan[c*x])*PolyLog[3, 1 - 2/(1
 - I*c*x)])/(2*e), x] + Simp[(3*b^2*(a + b*ArcTan[c*x])*PolyLog[3, 1 - (2*c
*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e), x] - Simp[(3*I*b^3*PolyLog[4
, 1 - 2/(1 - I*c*x)])/(4*e), x] + Simp[(3*I*b^3*PolyLog[4, 1 - (2*c*(d + e*
x))/((c*d + I*e)*(1 - I*c*x))])/(4*e), x]) /; FreeQ[{a, b, c, d, e}, x] &&
NeQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{e+fx}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^3}{\frac{de-cf}{d}+\frac{fx}{d}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^3\log\left(\frac{2}{1-i(c+dx)}\right)}{f} + \frac{\left(a+b\tan^{-1}(c+dx)\right)^3\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f}$$

Mathematica [F] time = 71.5293, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^3/(e + f*x),x]
```

```
[Out] Integrate[(a + b*ArcTan[c + d*x])^3/(e + f*x), x]
```

Maple [C] time = 0.765, size = 4389, normalized size = 11.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^3/(f*x+e),x)
```

```
[Out] 3/2*I*a^2*b*ln(f*(d*x+c)-c*f+d*e)/f*ln((I*f-f*(d*x+c))/(d*e+I*f-c*f))-3/2*I
*a^2*b*ln(f*(d*x+c)-c*f+d*e)/f*ln((I*f+f*(d*x+c))/(I*f+c*f-d*e))+3*I*a*b^2/
(I*f+c*f-d*e)*arctan(d*x+c)^2*ln(1-(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)
^2)/(d*e+I*f-c*f))+3*I*a*b^2/f*arctan(d*x+c)*polylog(2,-(1+I*(d*x+c))^2/(1+
(d*x+c)^2))-1/2*I*b^3/f*arctan(d*x+c)^3*Pi*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(
d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^
2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^3-3/2*I*b^3*c/(I*f+c*f-d
*e)*arctan(d*x+c)^2*polylog(2,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(
d*e+I*f-c*f))+3*a*b^2*c/(I*f+c*f-d*e)*arctan(d*x+c)^2*ln(1-(I*f+c*f-d*e)*(1
+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+I*b^3/f*arctan(d*x+c)^3*Pi*csgn(
I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*
(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1
))^2-3*I*a*b^2/f*Pi*arctan(d*x+c)^2+a^3*ln(f*(d*x+c)-c*f+d*e)/f-3/4*b^3/(I*
f+c*f-d*e)*polylog(4,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c
*f))+6*I*d*a*b^2/f*e*arctan(d*x+c)*polylog(2,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/
(1+(d*x+c)^2)/(d*e+I*f-c*f))/(2*I*f+2*c*f-2*d*e)+3/2*I*a*b^2/f*arctan(d*x+c
)^2*Pi*csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(I*(I*f*(1+I*(d*x+c))^
2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d
*x+c)^2)-I*f+c*f-d*e))*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(
d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/((1+
I*(d*x+c))^2/(1+(d*x+c)^2)+1))+3/2*a*b^2*c/(I*f+c*f-d*e)*polylog(3,(I*f+c*f
-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-3*I*a*b^2*c/(I*f+c*f-d*e
)*arctan(d*x+c)*polylog(2,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+
I*f-c*f))+3*I*a*b^2/f*arctan(d*x+c)^2*Pi*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*
x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)
-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2-b^3/f*arctan(d*x+c)^3*ln
(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1
+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)-3/2*b^3/f*arctan(d*x+c)*polylog(3,
-(1+I*(d*x+c))^2/(1+(d*x+c)^2))+I*b^3/(I*f+c*f-d*e)*arctan(d*x+c)^3*ln(1-(I
*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+b^3*c/(I*f+c*f-d*e
)*arctan(d*x+c)^3*ln(1-(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f
-c*f))+3/2*b^3*c/(I*f+c*f-d*e)*arctan(d*x+c)*polylog(3,(I*f+c*f-d*e)*(1+I*(
d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3*a*b^2*ln(f*(d*x+c)-c*f+d*e)/f*arct
an(d*x+c)^2-3*a*b^2/f*arctan(d*x+c)^2*ln(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+
c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f
-d*e)+3*a*b^2/(I*f+c*f-d*e)*arctan(d*x+c)*polylog(2,(I*f+c*f-d*e)*(1+I*(d*x
```

```
+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3*a^2*b*ln(f*(d*x+c)-c*f+d*e)/f*arctan(
d*x+c)+3/2*I*a*b^2/(I*f+c*f-d*e)*polylog(3,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1
+(d*x+c)^2)/(d*e+I*f-c*f))+3/2*I*b^3/(I*f+c*f-d*e)*arctan(d*x+c)*polylog(3,
(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3/2*I*b^3/f*arct
an(d*x+c)^2*polylog(2,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))+3/2*I*a^2*b/f*dilog((
I*f-f*(d*x+c))/(d*e+I*f-c*f))-3/2*I*a^2*b/f*dilog((I*f+f*(d*x+c))/(I*f+c*f-
d*e))-I*b^3/f*Pi*arctan(d*x+c)^3+3/4*I*b^3*c/(I*f+c*f-d*e)*polylog(4,(I*f+c
*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3/2*b^3/(I*f+c*f-d*e)*
arctan(d*x+c)^2*polylog(2,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+
I*f-c*f))+b^3*ln(f*(d*x+c)-c*f+d*e)/f*arctan(d*x+c)^3-3/2*a*b^2/f*polylog(3
,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))-3/4*I*b^3/f*polylog(4,-(1+I*(d*x+c))^2/(1+
(d*x+c)^2))-3*d*a*b^2/f*e/(I*f+c*f-d*e)*arctan(d*x+c)^2*ln(1-(I*f+c*f-d*e)*
(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-3/2*I*a*b^2/f*arctan(d*x+c)^2*
Pi*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)
^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e))*csgn(I*(I*f*(1+I*(d*x+c
))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1
+(d*x+c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2-3/2*I*a*b^2/f
*arctan(d*x+c)^2*Pi*csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(I*(I*f*(
1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*
x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2+1/2
*I*b^3/f*arctan(d*x+c)^3*Pi*csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(
I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*
(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e))*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+
(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)
^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))+3*I*d*b^3/f*e*arctan(d*
x+c)^2*polylog(2,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))
/(2*I*f+2*c*f-2*d*e)-3/2*I*a*b^2/f*arctan(d*x+c)^2*Pi*csgn(I*(I*f*(1+I*(d*x
+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/
(1+(d*x+c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^3-3*I*d*b^3/f
*e*polylog(4,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))/(4*
I*f+4*c*f-4*d*e)-3/2*d*a*b^2/f*e/(I*f+c*f-d*e)*polylog(3,(I*f+c*f-d*e)*(1+I
*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-d*b^3/f*e/(I*f+c*f-d*e)*arctan(d*x
+c)^3*ln(1-(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-3/2*d
*b^3/f*e/(I*f+c*f-d*e)*arctan(d*x+c)*polylog(3,(I*f+c*f-d*e)*(1+I*(d*x+c))^
2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-1/2*I*b^3/f*arctan(d*x+c)^3*Pi*csgn(I*(I*f*(
1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*
x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e))*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^
2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+
c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2-1/2*I*b^3/f*arctan(d*x+c)^3*P
i*csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+
(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)
^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^3 \log\left(fx+e\right)}{f} + \int \frac{28\, b^3 \arctan\left(dx+c\right)^3 + 3\, b^3 \arctan\left(dx+c\right) \log\left(d^2x^2 + 2\, cdx + c^2 + 1\right)^2 + 96\, ab^2 \arctan\left(dx+c\right)}{32\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(f*x+e),x, algorithm="maxima")
```

```
[Out] a^3*log(f*x + e)/f + integrate(1/32*(28*b^3*arctan(d*x + c)^3 + 3*b^3*arcta
n(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 96*a*b^2*arctan(d*x + c)^2
+ 96*a^2*b*arctan(d*x + c))/(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^3 \arctan\left(dx+c\right)^3 + 3\, ab^2 \arctan\left(dx+c\right)^2 + 3\, a^2 b \arctan\left(dx+c\right) + a^3}{fx+e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arcta
n(d*x + c) + a^3)/(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b \operatorname{atan}\left(c + dx\right)\right)^3}{e + fx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**3/(f*x+e),x)
```

```
[Out] Integral((a + b*atan(c + d*x))**3/(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(dx+c)+a)^3}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(d*x + c) + a)^3/(f*x + e), x)
```

3.40 $\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx$

Optimal. Leaf size=1233

result too large to display

```
[Out] (3*a^2*b*d*(d*e - c*f)*ArcTan[c + d*x])/(f*(f^2 + (d*e - c*f)^2)) + ((3*I)*
a*b^2*d*ArcTan[c + d*x]^2)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (3*a*b^2
*d*(d*e - c*f)*ArcTan[c + d*x]^2)/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2))
 + (I*b^3*d*ArcTan[c + d*x]^3)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (b^3
*d*(d*e - c*f)*ArcTan[c + d*x]^3)/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2))
 - (a + b*ArcTan[c + d*x])^3/(f*(e + f*x)) + (3*a^2*b*d*Log[e + f*x])/(f^2
+ (d*e - c*f)^2) - (6*a*b^2*d*ArcTan[c + d*x]*Log[2/(1 - I*(c + d*x))])/(d^
2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (3*b^3*d*ArcTan[c + d*x]^2*Log[2/(1 -
I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (6*a*b^2*d*ArcTan[c
+ d*x]*Log[(2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/(d^2*e^2
 - 2*c*d*e*f + (1 + c^2)*f^2) + (3*b^3*d*ArcTan[c + d*x]^2*Log[(2*d*(e + f*
x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2
)*f^2) + (6*a*b^2*d*ArcTan[c + d*x]*Log[2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*
c*d*e*f + (1 + c^2)*f^2) + (3*b^3*d*ArcTan[c + d*x]^2*Log[2/(1 + I*(c + d*x
))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (3*a^2*b*d*Log[1 + (c + d*x)^2
])/(2*(f^2 + (d*e - c*f)^2)) + ((3*I)*a*b^2*d*PolyLog[2, 1 - 2/(1 - I*(c +
d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + ((3*I)*b^3*d*ArcTan[c + d*x
]*PolyLog[2, 1 - 2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2
) - ((3*I)*a*b^2*d*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I
*(c + d*x)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - ((3*I)*b^3*d*ArcTan[
c + d*x]*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)
))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + ((3*I)*a*b^2*d*PolyLog[2, 1 -
2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + ((3*I)*b^3*d*
ArcTan[c + d*x]*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f +
 (1 + c^2)*f^2) - (3*b^3*d*PolyLog[3, 1 - 2/(1 - I*(c + d*x))])/(2*(d^2*e^2
 - 2*c*d*e*f + (1 + c^2)*f^2)) + (3*b^3*d*PolyLog[3, 1 - (2*d*(e + f*x))/((
d*e + I*f - c*f)*(1 - I*(c + d*x)))])/(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f
^2)) + (3*b^3*d*PolyLog[3, 1 - 2/(1 + I*(c + d*x))])/(2*(d^2*e^2 - 2*c*d*e*
f + (1 + c^2)*f^2))
```

Rubi [A] time = 2.31078, antiderivative size = 1233, normalized size of antiderivative = 1., number of steps used = 35, number of rules used = 22, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.1, Rules used = {5045, 6741, 5057, 6688, 12, 6725, 706, 31, 635, 203, 260, 4856, 2402, 2315, 2447, 4984, 4884, 4920, 4854, 4858, 4994, 6610}

$$\frac{id\tan^{-1}(c+dx)^3b^3}{d^2e^2-2cdfe+\left(c^2+1\right)f^2}+\frac{d(de-cf)\tan^{-1}(c+dx)^3b^3}{f\left(d^2e^2-2cdfe+\left(c^2+1\right)f^2\right)}-\frac{3d\tan^{-1}(c+dx)^2\log\left(\frac{2}{1-i(c+dx)}\right)b^3}{d^2e^2-2cdfe+\left(c^2+1\right)f^2}+\frac{3d\tan^{-1}(c+dx)^2\log}{d^2e^2-2cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^3/(e + f*x)^2,x]
```

```
[Out] (3*a^2*b*d*(d*e - c*f)*ArcTan[c + d*x])/(f*(f^2 + (d*e - c*f)^2)) + ((3*I)*
a*b^2*d*ArcTan[c + d*x]^2)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (3*a*b^2
*d*(d*e - c*f)*ArcTan[c + d*x]^2)/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2))
 + (I*b^3*d*ArcTan[c + d*x]^3)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (b^3
*d*(d*e - c*f)*ArcTan[c + d*x]^3)/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2))
 - (a + b*ArcTan[c + d*x])^3/(f*(e + f*x)) + (3*a^2*b*d*Log[e + f*x])/(f^2
+ (d*e - c*f)^2) - (6*a*b^2*d*ArcTan[c + d*x]*Log[2/(1 - I*(c + d*x))])/(d^
2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (3*b^3*d*ArcTan[c + d*x]^2*Log[2/(1 -
I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (6*a*b^2*d*ArcTan[c
+ d*x]*Log[(2*d*(e + f*x))/((d*e + (I - c)*f)*(1 - I*(c + d*x)))])/(d^2*e^2
 - 2*c*d*e*f + (1 + c^2)*f^2) + (3*b^3*d*ArcTan[c + d*x]^2*Log[(2*d*(e + f*
x))/((d*e + (I - c)*f)*(1 - I*(c + d*x)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2
)*f^2) + (6*a*b^2*d*ArcTan[c + d*x]*Log[2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*
c*d*e*f + (1 + c^2)*f^2) + (3*b^3*d*ArcTan[c + d*x]^2*Log[2/(1 + I*(c + d*x
))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (3*a^2*b*d*Log[1 + (c + d*x)^2
])/(2*(f^2 + (d*e - c*f)^2)) + ((3*I)*a*b^2*d*PolyLog[2, 1 - 2/(1 - I*(c +
d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + ((3*I)*b^3*d*ArcTan[c + d*x
]*PolyLog[2, 1 - 2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2
) - ((3*I)*a*b^2*d*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + (I - c)*f)*(1 - I
*(c + d*x)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - ((3*I)*b^3*d*ArcTan[
c + d*x]*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + (I - c)*f)*(1 - I*(c + d*x)
))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + ((3*I)*a*b^2*d*PolyLog[2, 1 -
2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + ((3*I)*b^3*d*
ArcTan[c + d*x]*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f +
 (1 + c^2)*f^2) - (3*b^3*d*PolyLog[3, 1 - 2/(1 - I*(c + d*x))])/(2*(d^2*e^2
 - 2*c*d*e*f + (1 + c^2)*f^2)) + (3*b^3*d*PolyLog[3, 1 - (2*d*(e + f*x))/((
d*e + (I - c)*f)*(1 - I*(c + d*x)))])/(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f
^2)) + (3*b^3*d*PolyLog[3, 1 - 2/(1 + I*(c + d*x))])/(2*(d^2*e^2 - 2*c*d*e*
f + (1 + c^2)*f^2))
```

Rule 5045

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[((e + f*x)^(m + 1)*(a + b*ArcTan[c + d*x])^p)/(f*(m +
 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcTan[c
```

```
 + d*x])^(p - 1))/(1 + (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 5057

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst
[Int[((d*e - c*f)/d + (f*x)/d)^m*(C/d^2 + (C*x^2)/d^2)^q*(a + b*ArcTan[x])^
p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &&
 EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4984

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4858

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
-Simp[((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e, x] + (Simp[((a + b*ArcT
an[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x] + Simp[(I*
b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/e, x] - Simp[(I*b*(a +
 b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/
e, x] - Simp[(b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e), x] + Simp[(b^2*Poly
Log[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e), x]) /; FreeQ[
{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx = -\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{(3bd)\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{(e+fx)\left(1+(c+dx)^2\right)}\,dx}{f}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{(3bd)\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{(e+fx)\left(1+c^2+2cdx+d^2x^2\right)}\,dx}{f}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{(3b)\,\mathrm{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)\left(1+x^2\right)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{(3b)\,\mathrm{Subst}\left(\int \frac{d\left(a+b\tan^{-1}(x)\right)^2}{(de-cf+fx)\left(1+x^2\right)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{(3bd)\,\mathrm{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{(de-cf+fx)\left(1+x^2\right)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{(3bd)\,\mathrm{Subst}\left(\int \left(\frac{a^2}{(de-cf+fx)\left(1+x^2\right)} + \frac{2ab\tan^{-1}(x)}{(de-cf+fx)\left(1+x^2\right)} + \frac{b^2\tan^{-}\cdots}{(de-cf+f\cdots}\right.\right.}{f}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{\left(3a^2bd\right)\mathrm{Subst}\left(\int \frac{1}{(de-cf+fx)\left(1+x^2\right)}\,dx, x, c+dx\right)}{f} + \frac{\left(6ab^2d\right)\mathrm{S}\cdots}{\cdots}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{\left(6ab^2d\right)\mathrm{Subst}\left(\int \left(\frac{f^2\tan^{-1}(x)}{\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)(de-cf+fx)} + \frac{(de-cf\cdots}{\left(d^2e^2-2cdef\cdots\right.}\right.\right.}{f}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{3a^2bd\log(e+fx)}{f^2+(de-cf)^2} + \frac{\left(6ab^2d\right)\mathrm{Subst}\left(\int \frac{(de-cf-fx)\tan^{-1}(x)}{1+x^2}\,dx, \cdots\right.}{f\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)\cdots}$$

$$= \frac{3a^2bd(de-cf)\tan^{-1}(c+dx)}{f\left(f^2+(de-cf)^2\right)} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{3a^2bd\log(e+fx)}{f^2+(de-cf)^2} - \frac{6ab^2d\,\mathrm{ta}\cdots}{d^2e^2\cdots}$$

$$= \frac{3a^2bd(de-cf)\tan^{-1}(c+dx)}{f\left(f^2+(de-cf)^2\right)} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{3a^2bd\log(e+fx)}{f^2+(de-cf)^2} - \frac{6ab^2d\,\mathrm{ta}\cdots}{d^2e^2\cdots}$$

$$= \frac{3a^2bd(de-cf)\tan^{-1}(c+dx)}{f\left(f^2+(de-cf)^2\right)} + \frac{3iab^2d\tan^{-1}(c+dx)^2}{d^2e^2-2cdef+\left(1+c^2\right)f^2} + \frac{3ab^2d(de-cf)\tan^{-1}(c+\cdots}{f\left(d^2e^2-2cdef+\left(1+c^2\cdots\right.\right.}$$

$$= \frac{3a^2bd(de-cf)\tan^{-1}(c+dx)}{f\left(f^2+(de-cf)^2\right)} + \frac{3iab^2d\tan^{-1}(c+dx)^2}{d^2e^2-2cdef+\left(1+c^2\right)f^2} + \frac{3ab^2d(de-cf)\tan^{-1}(c+\cdots}{f\left(d^2e^2-2cdef+\left(1+c^2\cdots\right.\right.}$$

$$= \frac{3a^2bd(de-cf)\tan^{-1}(c+dx)}{\cdots} + \frac{3iab^2d\tan^{-1}(c+dx)^2}{\cdots} + \frac{3ab^2d(de-cf)\tan^{-1}(c+\cdots}{\cdots}$$

Mathematica [F] time = 70.2675, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^3/(e + f*x)^2,x]
```

```
[Out] Integrate[(a + b*ArcTan[c + d*x])^3/(e + f*x)^2, x]
```

Maple [C] time = 1.013, size = 4764, normalized size = 3.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^3/(f*x+e)^2,x)
```

```
[Out] -3/4*I*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*Pi*csgn(I*(1+I
*(d*x+c))/(1+(d*x+c)^2)^(1/2))^2*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))+3*I*
d*b^3*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)/(I*f+c*f-d*e)*arctan(d*x+c)^2*ln(1-
(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3*d*b^3*f/(c^2*f
^2-2*c*d*e*f+d^2*e^2+f^2)*c/(I*f+c*f-d*e)*arctan(d*x+c)^2*ln(1-(I*f+c*f-d*e
)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-3/2*I*d*b^3/(c^2*f^2-2*c*d*e
*f+d^2*e^2+f^2)*arctan(d*x+c)^2*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)
)*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^2-3/2*I*d*b^3/(c^2*f^2-2*c*d*
e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*Pi*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^
2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+
c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*csgn(I/((1+I*(d*x+c))^2/(1+(d
*x+c)^2)+1))+3*I*d^2*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*e/(I*f+c*f-d*e)*ar
ctan(d*x+c)*polylog(2,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-
c*f))+3/4*I*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*Pi*csgn(I
*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)
+1)^2)-3/2*I*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*Pi*csgn(
I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*
(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e))*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+
(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)
^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2+3/4*I*d*b^3/(c^2*f^2-
2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*Pi*csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c
)^2)+1)^2)*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/((1+I*(d*x+c))^2/(1+(d*x+c)
```

```
^2)+1)^2)^2+3/2*I*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*Pi*
csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)
-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e))*csgn(I*(I*f*(1+I*(d*x+c))^
2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d
*x+c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(I/((1+I*(d*x+
c))^2/(1+(d*x+c)^2)+1))-3/4*I*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(
d*x+c)^2*Pi*csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)*csgn(I*(1+I*(d*x+c)
)^2/(1+(d*x+c)^2))*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/((1+I*(d*x+c))^2/(1
+(d*x+c)^2)+1)^2)-3*I*d*b^3*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*c/(I*f+c*f-d*
e)*arctan(d*x+c)*polylog(2,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e
+I*f-c*f))+3*d*b^3*arctan(d*x+c)^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(f*(d*
x+c)-c*f+d*e)-3/2*d*b^3*arctan(d*x+c)^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(
1+(d*x+c)^2)-3*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*ln(I*f
*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(
d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)+3*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2
)*arctan(d*x+c)^2*ln((1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))-d*b^3/(d*f*x+d*e)/f
*arctan(d*x+c)^3-3/2*d*a^2*b/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(1+(d*x+c)^2
)+3*d*a^2*b/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(f*(d*x+c)-c*f+d*e)-d*b^3*arc
tan(d*x+c)^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*c+3*d*b^3/(c^2*f^2-2*c*d*e*f+d
^2*e^2+f^2)*arctan(d*x+c)^2*ln(2)-I*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*a
rctan(d*x+c)^3+d^2*b^3/f*arctan(d*x+c)^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*e-
3*d*a^2*b/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)*c-3*d*a*b^2/(c^2*f^
2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*c-3*d*a^2*b/(d*f*x+d*e)/f*arctan(d
*x+c)-3*d*a*b^2/(d*f*x+d*e)/f*arctan(d*x+c)^2-3*I*d*b^3/(c^2*f^2-2*c*d*e*f+
d^2*e^2+f^2)*arctan(d*x+c)^2*Pi-3/2*I*d*a*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^
2)*dilog(1/2*I*(d*x+c-I))+3/2*I*d*a*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*dil
og(-1/2*I*(d*x+c+I))-3*I*d*a*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*dilog((I*f
+f*(d*x+c))/(I*f+c*f-d*e))+3/4*I*d*a*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln
(d*x+c-I)^2+3*I*d*a*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*dilog((I*f-f*(d*x+c
))/(d*e+I*f-c*f))-3/4*I*d*a*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(d*x+c+I)
^2-3/2*d^2*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*e/(I*f+c*f-d*e)*polylog(3,(I
*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-3*d*a*b^2*arctan(d
*x+c)/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(1+(d*x+c)^2)+6*d*a*b^2*arctan(d*x+
c)/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(f*(d*x+c)-c*f+d*e)-d*a^3/(d*f*x+d*e)/
f+3/2*I*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*Pi*csgn(I*(1+
I*(d*x+c))/(1+(d*x+c)^2)^(1/2))*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2+3/4
*I*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*
x+c))^2/(1+(d*x+c)^2))*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/((1+I*(d*x+c))^
2/(1+(d*x+c)^2)+1)^2)^2-3/4*I*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(
d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/((1+I*(d*x+c))^2/(1+(d*x+c
)^2)+1)^2)^3+3/4*I*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*Pi
*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^3-3/4*I*d*b^3/(c^2*f^2-2*c*d*e
*f+d^2*e^2+f^2)*arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))^3-
3*d^2*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*e/(I*f+c*f-d*e)*arctan(d*x+c)^2*l
n(1-(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3*d*b^3*f/(c
```

```
^2*f^2-2*c*d*e*f+d^2*e^2+f^2)/(I*f+c*f-d*e)*arctan(d*x+c)*polylog(2,(I*f+c*
f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3*d^2*a^2*b/f/(c^2*f^2-
2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)*e+3/2*d*b^3*f/(c^2*f^2-2*c*d*e*f+d^2*e
^2+f^2)*c/(I*f+c*f-d*e)*polylog(3,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^
2)/(d*e+I*f-c*f))+3*d^2*a*b^2/f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+
c)^2*e-3/2*I*d*a*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(d*x+c+I)*ln(1/2*I*(
d*x+c-I))+3/2*I*d*a*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(d*x+c-I)*ln(-1/2
*I*(d*x+c+I))+3/2*I*d*a*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(1+(d*x+c)^2)
*ln(d*x+c+I)+3*I*d*a*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(f*(d*x+c)-c*f+d
*e)*ln((I*f-f*(d*x+c))/(d*e+I*f-c*f))-3/2*I*d*a*b^2/(c^2*f^2-2*c*d*e*f+d^2*
e^2+f^2)*ln(1+(d*x+c)^2)*ln(d*x+c-I)+3/2*I*d*b^3*f/(c^2*f^2-2*c*d*e*f+d^2*e
^2+f^2)/(I*f+c*f-d*e)*polylog(3,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)
/(d*e+I*f-c*f))-3*I*d*a*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(f*(d*x+c)-c*
f+d*e)*ln((I*f+f*(d*x+c))/(I*f+c*f-d*e))+3*I*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e
^2+f^2)*arctan(d*x+c)^2*Pi*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1
+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/
((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2-3/2*I*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2
+f^2)*arctan(d*x+c)^2*Pi*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I
*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/((
1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3}{2}\left(d\left(\frac{2\left(d^2e-cdf\right)\arctan\left(\frac{d^2x+cd}{d}\right)}{\left(d^2e^2f-2\,cdef^2+\left(c^2+1\right)f^3\right)d}-\frac{\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^2e^2-2\,cdef+\left(c^2+1\right)f^2}+\frac{2\,\log\left(fx+e\right)}{d^2e^2-2\,cdef+\left(c^2+1\right)f^2}\right)-\frac{2\,\arctan\left(dx+c\right)}{f^2x+ef}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] 3/2*(d*(2*(d^2*e - c*d*f)*arctan((d^2*x + c*d)/d)/((d^2*e^2*f - 2*c*d*e*f^2
 + (c^2 + 1)*f^3)*d) - log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*e^2 - 2*c*d*e*
f + (c^2 + 1)*f^2) + 2*log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 + 1)*f^2))
- 2*arctan(d*x + c)/(f^2*x + e*f))*a^2*b - a^3/(f^2*x + e*f) - 1/32*(4*b^3*
arctan(d*x + c)^3 - 3*b^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^
2 - 32*(f^2*x + e*f)*integrate(1/32*(28*(b^3*d^2*f*x^2 + 2*b^3*c*d*f*x + (b
^3*c^2 + b^3)*f)*arctan(d*x + c)^3 + 12*(8*a*b^2*d^2*f*x^2 + b^3*d*e + (16*
a*b^2*c + b^3)*d*f*x + 8*(a*b^2*c^2 + a*b^2)*f)*arctan(d*x + c)^2 - 12*(b^3
*d^2*f*x^2 + b^3*c*d*e + (b^3*d^2*e + b^3*c*d*f)*x)*arctan(d*x + c)*log(d^2
*x^2 + 2*c*d*x + c^2 + 1) - 3*(b^3*d*f*x + b^3*d*e - (b^3*d^2*f*x^2 + 2*b^3
*c*d*f*x + (b^3*c^2 + b^3)*f)*arctan(d*x + c))*log(d^2*x^2 + 2*c*d*x + c^2
```

```
+ 1)^2)/(d^2*f^3*x^4 + (c^2 + 1)*e^2*f + 2*(d^2*e*f^2 + c*d*f^3)*x^3 + (d^2
*e^2*f + 4*c*d*e*f^2 + (c^2 + 1)*f^3)*x^2 + 2*(c*d*e^2*f + (c^2 + 1)*e*f^2)
*x), x))/(f^2*x + e*f)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \arctan\left(dx+c\right)^3 + 3\,ab^2 \arctan\left(dx+c\right)^2 + 3\,a^2b \arctan\left(dx+c\right) + a^3}{f^2x^2 + 2\,efx + e^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arcta
n(d*x + c) + a^3)/(f^2*x^2 + 2*e*f*x + e^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**3/(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\arctan\left(dx+c\right)+a\right)^3}{\left(fx+e\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(d*x + c) + a)^3/(f*x + e)^2, x)
```

3.41 $\int (e+fx)^m \left(a+b\tan^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=177

$$-\frac{ibd(e+fx)^{m+2}\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{d(e+fx)}{-cf+de+if}\right)}{2f(m+1)(m+2)(de+(-c+i)f)}+\frac{ibd(e+fx)^{m+2}\text{Hypergeometric2F1}\left(1,m+2,m\right.}{2f(m+1)(m+2)(de-(c+i)f)}$$

```
[Out] ((e + f*x)^(1 + m)*(a + b*ArcTan[c + d*x]))/(f*(1 + m)) - ((I/2)*b*d*(e + f
*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e + I*f - c
*f)])/(f*(d*e + (I - c)*f)*(1 + m)*(2 + m)) + ((I/2)*b*d*(e + f*x)^(2 + m)*
Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e - (I + c)*f)])/(f*(d*
e - (I + c)*f)*(1 + m)*(2 + m))
```

Rubi [A] time = 0.246324, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5047, 4862, 712, 68}

$$-\frac{ibd(e+fx)^{m+2}\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{d(e+fx)}{-cf+de+if}\right)}{2f(m+1)(m+2)(de+(-c+i)f)}+\frac{ibd(e+fx)^{m+2}\text{Hypergeometric2F1}\left(1,m+2,m\right.}{2f(m+1)(m+2)(de-(c+i)f)}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^m*(a + b*ArcTan[c + d*x]),x]
```


```
[Out] ((e + f*x)^(1 + m)*(a + b*ArcTan[c + d*x]))/(f*(1 + m)) - ((I/2)*b*d*(e + f
*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e + I*f - c
*f)])/(f*(d*e + (I - c)*f)*(1 + m)*(2 + m)) + ((I/2)*b*d*(e + f*x)^(2 + m)*
Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e - (I + c)*f)])/(f*(d*
e - (I + c)*f)*(1 + m)*(2 + m))
```


Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
```

```
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 712

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int (e+fx)^m \left(a+b\tan^{-1}(c+dx)\right) dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^m \left(a+b\tan^{-1}(x)\right) dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^{1+m}\left(a+b\tan^{-1}(c+dx)\right)}{f(1+m)} - \frac{b\,\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^{1+m}}{1+x^2} dx, x, c+dx\right)}{f(1+m)}$$

$$= \frac{(e+fx)^{1+m}\left(a+b\tan^{-1}(c+dx)\right)}{f(1+m)} - \frac{b\,\text{Subst}\left(\int \left(\frac{i\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^{1+m}}{2(i-x)} + \frac{i\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^{1+m}}{2(i+x)}\right.\right.}{f(1+m)}$$

$$= \frac{(e+fx)^{1+m}\left(a+b\tan^{-1}(c+dx)\right)}{f(1+m)} - \frac{(ib)\,\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^{1+m}}{i-x} dx, x, c+dx\right)}{2f(1+m)} -$$

$$= \frac{(e+fx)^{1+m}\left(a+b\tan^{-1}(c+dx)\right)}{f(1+m)} - \frac{ibd(e+fx)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{d(e+f}{de+if-}\right.}{2f(de+(i-c)f)(1+m)(2+m)}$$

Mathematica [A] time = 0.287765, size = 162, normalized size = 0.92

$$\frac{(e+fx)^{m+1}\left(2\left(a+b\tan^{-1}(c+dx)\right)+\frac{bd(e+fx)\left((de-(c+i)f)\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{d(e+fx)}{de-(c-i)f}\right)+(-de+(c-i)f)\text{Hypergeometric2F1}\left(1,\right.\right.}{(m+2)(-icf+ide+f)(de-(c-i)f)}\right.}{2f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^m*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] ((e + f*x)^(1 + m)*(2*(a + b*ArcTan[c + d*x]) + (b*d*(e + f*x)*((d*e - (I +
 c)*f)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e - (-I + c)*f)]
 + (-(d*e) + (-I + c)*f)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(
d*e - (I + c)*f)]))/((I*d*e + f - I*c*f)*(d*e - (-I + c)*f)*(2 + m))))/(2*f
*(1 + m))
```

Maple [F] time = 1.284, size = 0, normalized size = 0.

$$\int \left(fx+e\right)^{m} \left(a+b\arctan \left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*(a+b*arctan(d*x+c)),x)
```

```
[Out] int((f*x+e)^m*(a+b*arctan(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\arctan \left(dx+c\right)+a\right)\left(fx+e\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(d*x + c) + a)*(f*x + e)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*(a+b*atan(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\arctan(dx+c)+a)(fx+e)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(d*x + c) + a)*(f*x + e)^m, x)
```

3.42 $\int (e+fx)^m \left(a+b\tan^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left((e+fx)^m \left(a+b\tan^{-1}(c+dx)\right)^2,x\right)$$

```
[Out] Unintegrable[(e + f*x)^m*(a + b*ArcTan[c + d*x])^2, x]
```

Rubi [A] time = 0.057976, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (e+fx)^m \left(a+b\tan^{-1}(c+dx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(e + f*x)^m*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] Defer[Subst][Defer[Int][((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcTan[x])^2, x]
, x, c + d*x]/d
```

Rubi steps

$$\int (e+fx)^m \left(a+b\tan^{-1}(c+dx)\right)^2 \, dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^m \left(a+b\tan^{-1}(x)\right)^2 \, dx,x,c+dx\right)}{d}$$

Mathematica [A] time = 4.44536, size = 0, normalized size = 0.

$$\int (e+fx)^m \left(a+b\tan^{-1}(c+dx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e + f*x)^m*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] Integrate[(e + f*x)^m*(a + b*ArcTan[c + d*x])^2, x]
```

Maple [A] time = 1.105, size = 0, normalized size = 0.

$$\int \left(fx+e\right)^{m} \left(a+b\arctan\left(dx+c\right)\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*(a+b*arctan(d*x+c))^2,x)
```

```
[Out] int((f*x+e)^m*(a+b*arctan(d*x+c))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^{2}\arctan\left(dx+c\right)^{2}+2\,ab\arctan\left(dx+c\right)+a^{2}\right)\left(fx+e\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2)*(f*x + e)^m,
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*(a+b*atan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b \arctan (dx + c) + a)^2 (fx + e)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(d*x + c) + a)^2*(f*x + e)^m, x)
```

3.43 $\int (e+fx)^m \left(a+b\tan^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left((e+fx)^m \left(a+b\tan^{-1}(c+dx)\right)^3,x\right)$$

```
[Out] Unintegrable[(e + f*x)^m*(a + b*ArcTan[c + d*x])^3, x]
```

Rubi [A] time = 0.0564118, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (e+fx)^m \left(a+b\tan^{-1}(c+dx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(e + f*x)^m*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] Defer[Subst][Defer[Int][((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcTan[x])^3, x]
, x, c + d*x]/d
```

Rubi steps

$$\int (e+fx)^m \left(a+b\tan^{-1}(c+dx)\right)^3 \, dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^m \left(a+b\tan^{-1}(x)\right)^3 \, dx,x,c+dx\right)}{d}$$

Mathematica [A] time = 0.405131, size = 0, normalized size = 0.

$$\int (e+fx)^m \left(a+b\tan^{-1}(c+dx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e + f*x)^m*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] Integrate[(e + f*x)^m*(a + b*ArcTan[c + d*x])^3, x]
```

Maple [A] time = 1.097, size = 0, normalized size = 0.

$$\int \left(fx+e\right)^{m} \left(a+b\arctan\left(dx+c\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*(a+b*arctan(d*x+c))^3,x)
```

```
[Out] int((f*x+e)^m*(a+b*arctan(d*x+c))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^{3}\arctan\left(dx+c\right)^{3}+3\,ab^{2}\arctan\left(dx+c\right)^{2}+3\,a^{2}b\arctan\left(dx+c\right)+a^{3}\right)\left(fx+e\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arcta
n(d*x + c) + a^3)*(f*x + e)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*(a+b*atan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\arctan(dx+c)+a)^3 (fx+e)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(d*x + c) + a)^3*(f*x + e)^m, x)
```

3.44 $\int x^3 \tan^{-1}(a+bx)\,dx$

Optimal. Leaf size=106

$$\frac{(1-6a^2)\,x}{4b^3}-\frac{a\left(1-a^2\right)\log\left((a+bx)^2+1\right)}{2b^4}-\frac{\left(a^4-6a^2+1\right)\tan^{-1}(a+bx)}{4b^4}-\frac{(a+bx)^3}{12b^4}+\frac{a(a+bx)^2}{2b^4}+\frac{1}{4}x^4\tan^{-1}(a+bx)$$

```
[Out] ((1 - 6*a^2)*x)/(4*b^3) + (a*(a + b*x)^2)/(2*b^4) - (a + b*x)^3/(12*b^4) -
((1 - 6*a^2 + a^4)*ArcTan[a + b*x])/(4*b^4) + (x^4*ArcTan[a + b*x])/4 - (a*
(1 - a^2)*Log[1 + (a + b*x)^2])/(2*b^4)
```

Rubi [A] time = 0.110634, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5047, 4862, 702, 635, 203, 260}

$$\frac{(1-6a^2)\,x}{4b^3}-\frac{a\left(1-a^2\right)\log\left((a+bx)^2+1\right)}{2b^4}-\frac{\left(a^4-6a^2+1\right)\tan^{-1}(a+bx)}{4b^4}-\frac{(a+bx)^3}{12b^4}+\frac{a(a+bx)^2}{2b^4}+\frac{1}{4}x^4\tan^{-1}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTan[a + b*x],x]
```

```
[Out] ((1 - 6*a^2)*x)/(4*b^3) + (a*(a + b*x)^2)/(2*b^4) - (a + b*x)^3/(12*b^4) -
((1 - 6*a^2 + a^4)*ArcTan[a + b*x])/(4*b^4) + (x^4*ArcTan[a + b*x])/4 - (a*
(1 - a^2)*Log[1 + (a + b*x)^2])/(2*b^4)
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int x^3 \tan^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^3 \tan^{-1}(x)\,dx, x, a+bx\right)}{b}$$

$$= \frac{1}{4}x^4\tan^{-1}(a+bx) - \frac{1}{4}\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^4}{1+x^2}\,dx, x, a+bx\right)$$

$$= \frac{1}{4}x^4\tan^{-1}(a+bx) - \frac{1}{4}\text{Subst}\left(\int\left(-\frac{1-6a^2}{b^4}-\frac{4ax}{b^4}+\frac{x^2}{b^4}+\frac{1-6a^2+a^4+4a\left(1-a^2\right)x}{b^4\left(1+x^2\right)}\right)dx, x$$

$$= \frac{\left(1-6a^2\right)x}{4b^3}+\frac{a(a+bx)^2}{2b^4}-\frac{(a+bx)^3}{12b^4}+\frac{1}{4}x^4\tan^{-1}(a+bx)-\frac{\text{Subst}\left(\int \frac{1-6a^2+a^4+4a\left(1-a^2\right)x}{1+x^2}\,dx, x,\right.}{4b^4}$$

$$= \frac{\left(1-6a^2\right)x}{4b^3}+\frac{a(a+bx)^2}{2b^4}-\frac{(a+bx)^3}{12b^4}+\frac{1}{4}x^4\tan^{-1}(a+bx)-\frac{\left(a\left(1-a^2\right)\right)\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x,\right.}{b^4}$$

$$= \frac{\left(1-6a^2\right)x}{4b^3}+\frac{a(a+bx)^2}{2b^4}-\frac{(a+bx)^3}{12b^4}-\frac{\left(1-6a^2+a^4\right)\tan^{-1}(a+bx)}{4b^4}+\frac{1}{4}x^4\tan^{-1}(a+bx)-$$

Mathematica [C] time = 0.0699706, size = 95, normalized size = 0.9

$$\frac{6\left(1-6a^2\right)bx+6b^4x^4\tan^{-1}(a+bx)-2(a+bx)^3+12a(a+bx)^2+3i(a-i)^4\log(-a-bx+i)-3i(a+i)^4\log(a+bx+i)}{24b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTan[a + b*x],x]
```

```
[Out] (6*(1 - 6*a^2)*b*x + 12*a*(a + b*x)^2 - 2*(a + b*x)^3 + 6*b^4*x^4*ArcTan[a
+ b*x] + (3*I)*(-I + a)^4*Log[I - a - b*x] - (3*I)*(I + a)^4*Log[I + a + b*
x])/(24*b^4)
```

Maple [A] time = 0.039, size = 132, normalized size = 1.3

$$\frac{x^4\arctan(bx+a)}{4}-\frac{\arctan(bx+a)a^4}{4b^4}-\frac{x^3}{12b}+\frac{ax^2}{4b^2}-\frac{3a^2x}{4b^3}-\frac{13a^3}{12b^4}+\frac{x}{4b^3}+\frac{a}{4b^4}+\frac{\ln\left(1+(bx+a)^2\right)a^3}{2b^4}-\frac{\ln\left(1+(b\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(b*x+a),x)
```

```
[Out] 1/4*x^4*arctan(b*x+a)-1/4/b^4*arctan(b*x+a)*a^4-1/12/b*x^3+1/4/b^2*x^2*a-3/
4/b^3*x*a^2-13/12/b^4*a^3+1/4/b^3*x+1/4/b^4*a+1/2/b^4*ln(1+(b*x+a)^2)*a^3-1
/2/b^4*ln(1+(b*x+a)^2)*a+3/2/b^4*arctan(b*x+a)*a^2-1/4/b^4*arctan(b*x+a)
```

Maxima [A] time = 1.50389, size = 140, normalized size = 1.32

$$\frac{1}{4}x^4\arctan(bx+a)-\frac{1}{12}b\left(\frac{b^2x^3-3abx^2+3\left(3a^2-1\right)x}{b^4}+\frac{3\left(a^4-6a^2+1\right)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^5}-\frac{6\left(a^3-a\right)\log\left(b^2x^2+2a\right.}{b^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arctan(b*x + a) - 1/12*b*((b^2*x^3 - 3*a*b*x^2 + 3*(3*a^2 - 1)*x)/b
^4 + 3*(a^4 - 6*a^2 + 1)*arctan((b^2*x + a*b)/b)/b^5 - 6*(a^3 - a)*log(b^2*
```

```
x^2 + 2*a*b*x + a^2 + 1)/b^5)
```

Fricas [A] time = 1.68514, size = 203, normalized size = 1.92

$$-\frac{b^3x^3 - 3\,ab^2x^2 + 3\left(3\,a^2 - 1\right)bx - 3\left(b^4x^4 - a^4 + 6\,a^2 - 1\right)\arctan\left(bx + a\right) - 6\left(a^3 - a\right)\log\left(b^2x^2 + 2\,abx + a^2 + 1\right)}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/12*(b^3*x^3 - 3*a*b^2*x^2 + 3*(3*a^2 - 1)*b*x - 3*(b^4*x^4 - a^4 + 6*a^2
 - 1)*arctan(b*x + a) - 6*(a^3 - a)*log(b^2*x^2 + 2*a*b*x + a^2 + 1))/b^4
```

Sympy [A] time = 2.36794, size = 155, normalized size = 1.46

$$\begin{cases} -\frac{a^4\operatorname{atan}(a+bx)}{4b^4} + \frac{a^3\log\left(a^2+2abx+b^2x^2+1\right)}{2b^4} - \frac{3a^2x}{4b^3} + \frac{3a^2\operatorname{atan}(a+bx)}{2b^4} + \frac{ax^2}{4b^2} - \frac{a\log\left(a^2+2abx+b^2x^2+1\right)}{2b^4} + \frac{x^4\operatorname{atan}(a+bx)}{4} - \frac{x^3}{12b} + \frac{x}{4b^3} - \text{ata} \\ \frac{x^4\operatorname{atan}(a)}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(b*x+a),x)
```

```
[Out] Piecewise((-a**4*atan(a + b*x)/(4*b**4) + a**3*log(a**2 + 2*a*b*x + b**2*x*
*2 + 1)/(2*b**4) - 3*a**2*x/(4*b**3) + 3*a**2*atan(a + b*x)/(2*b**4) + a*x*
*2/(4*b**2) - a*log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(2*b**4) + x**4*atan(a
+ b*x)/4 - x**3/(12*b) + x/(4*b**3) - atan(a + b*x)/(4*b**4), Ne(b, 0)), (x
**4*atan(a)/4, True))
```

Giac [A] time = 1.10813, size = 139, normalized size = 1.31

$$\frac{1}{4}\,x^4\arctan\left(bx + a\right) - \frac{1}{12}\,b\left(\frac{3\left(a^4 - 6\,a^2 + 1\right)\arctan\left(bx + a\right)}{b^5} - \frac{6\left(a^3 - a\right)\log\left(b^2x^2 + 2\,abx + a^2 + 1\right)}{b^5} + \frac{b^4x^3 - 3\,ab^3x^2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*x^4*arctan(b*x + a) - 1/12*b*(3*(a^4 - 6*a^2 + 1)*arctan(b*x + a)/b^5 -
 6*(a^3 - a)*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^5 + (b^4*x^3 - 3*a*b^3*x^2
+ 9*a^2*b^2*x - 3*b^2*x)/b^6)
```

3.45 $\int x^2 \tan^{-1}(a+bx)\,dx$

Optimal. Leaf size=79

$$\frac{\left(1-3a^2\right)\log\left((a+bx)^2+1\right)}{6b^3}-\frac{a\left(3-a^2\right)\tan^{-1}(a+bx)}{3b^3}+\frac{ax}{b^2}-\frac{(a+bx)^2}{6b^3}+\frac{1}{3}x^3\tan^{-1}(a+bx)$$

```
[Out] (a*x)/b^2 - (a + b*x)^2/(6*b^3) - (a*(3 - a^2)*ArcTan[a + b*x])/(3*b^3) + (
x^3*ArcTan[a + b*x])/3 + ((1 - 3*a^2)*Log[1 + (a + b*x)^2])/(6*b^3)
```

Rubi [A] time = 0.0917566, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5047, 4862, 702, 635, 203, 260}

$$\frac{\left(1-3a^2\right)\log\left((a+bx)^2+1\right)}{6b^3}-\frac{a\left(3-a^2\right)\tan^{-1}(a+bx)}{3b^3}+\frac{ax}{b^2}-\frac{(a+bx)^2}{6b^3}+\frac{1}{3}x^3\tan^{-1}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[a + b*x],x]
```



```
[Out] (a*x)/b^2 - (a + b*x)^2/(6*b^3) - (a*(3 - a^2)*ArcTan[a + b*x])/(3*b^3) + (
x^3*ArcTan[a + b*x])/3 + ((1 - 3*a^2)*Log[1 + (a + b*x)^2])/(6*b^3)
```


Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
```

```
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \tan^{-1}(a+bx)\,dx &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^2 \tan^{-1}(x)\,dx, x, a+bx\right)}{b} \\
&= \frac{1}{3}x^3\tan^{-1}(a+bx) - \frac{1}{3}\,\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}{1+x^2}\,dx, x, a+bx\right) \\
&= \frac{1}{3}x^3\tan^{-1}(a+bx) - \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{3a}{b^3}+\frac{x}{b^3}+\frac{a\left(3-a^2\right)-\left(1-3a^2\right)x}{b^3\left(1+x^2\right)}\right)dx, x, a+bx\right) \\
&= \frac{ax}{b^2} - \frac{(a+bx)^2}{6b^3} + \frac{1}{3}x^3\tan^{-1}(a+bx) - \frac{\text{Subst}\left(\int \frac{a\left(3-a^2\right)-\left(1-3a^2\right)x}{1+x^2}\,dx, x, a+bx\right)}{3b^3} \\
&= \frac{ax}{b^2} - \frac{(a+bx)^2}{6b^3} + \frac{1}{3}x^3\tan^{-1}(a+bx) + \frac{\left(1-3a^2\right)\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, a+bx\right)}{3b^3} - \frac{\left(a\left(3-a^2\right)\right)\text{Sul}}{} \\
&= \frac{ax}{b^2} - \frac{(a+bx)^2}{6b^3} - \frac{a\left(3-a^2\right)\tan^{-1}(a+bx)}{3b^3} + \frac{1}{3}x^3\tan^{-1}(a+bx) + \frac{\left(1-3a^2\right)\log\left(1+(a+bx)^2\right)}{6b^3}
\end{aligned}
$$

Mathematica [C] time = 0.0483124, size = 114, normalized size = 1.44

$$\frac{\frac{1}{3}b\left(\frac{a+bx}{b}-\frac{a}{b}\right)^3\tan^{-1}(a+bx)-\frac{1}{3}b\left(\frac{(a+bx)^2}{2b^3}-\frac{3ax}{b^2}-\frac{(1-ia)^3\log(a+bx+i)}{2b^3}-\frac{(1+ia)^3\log(-a-bx+i)}{2b^3}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[a + b*x],x]
```

```
[Out] ((b*(-(a/b) + (a + b*x)/b)^3*ArcTan[a + b*x])/3 - (b*((-3*a*x)/b^2 + (a + b
*x)^2/(2*b^3) - ((1 + I*a)^3*Log[I - a - b*x])/(2*b^3) - ((1 - I*a)^3*Log[I
 + a + b*x])/(2*b^3)))/3)/b
```

Maple [A] time = 0.036, size = 95, normalized size = 1.2

$$\frac{x^3\arctan(bx+a)}{3}+\frac{\arctan(bx+a)\,a^3}{3\,b^3}-\frac{x^2}{6\,b}+\frac{2\,ax}{3\,b^2}+\frac{5\,a^2}{6\,b^3}-\frac{\ln\left(1+(bx+a)^2\right)a^2}{2\,b^3}+\frac{\ln\left(1+(bx+a)^2\right)}{6\,b^3}-\frac{\arctan(bx}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(b*x+a),x)
```

```
[Out] 1/3*x^3*arctan(b*x+a)+1/3/b^3*arctan(b*x+a)*a^3-1/6/b*x^2+2/3*a*x/b^2+5/6/b
^3*a^2-1/2/b^3*ln(1+(b*x+a)^2)*a^2+1/6/b^3*ln(1+(b*x+a)^2)-1/b^3*arctan(b*x
+a)*a
```

Maxima [A] time = 1.50323, size = 115, normalized size = 1.46

$$\frac{1}{3}x^3\arctan(bx+a)-\frac{1}{6}b\left(\frac{bx^2-4\,ax}{b^3}-\frac{2\left(a^3-3\,a\right)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^4}+\frac{\left(3\,a^2-1\right)\log\left(b^2x^2+2\,abx+a^2+1\right)}{b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(b*x + a) - 1/6*b*((b*x^2 - 4*a*x)/b^3 - 2*(a^3 - 3*a)*arctan
((b^2*x + a*b)/b)/b^4 + (3*a^2 - 1)*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^4)
```

Fricas [A] time = 1.74905, size = 161, normalized size = 2.04

$$-\frac{b^2x^2 - 4\,abx - 2\left(b^3x^3 + a^3 - 3\,a\right)\arctan\left(bx + a\right) + \left(3\,a^2 - 1\right)\log\left(b^2x^2 + 2\,abx + a^2 + 1\right)}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/6*(b^2*x^2 - 4*a*b*x - 2*(b^3*x^3 + a^3 - 3*a)*arctan(b*x + a) + (3*a^2
- 1)*log(b^2*x^2 + 2*a*b*x + a^2 + 1))/b^3
```

Sympy [A] time = 1.4325, size = 117, normalized size = 1.48

$$\begin{cases} \frac{a^3\operatorname{atan}\left(a+bx\right)}{3b^3} - \frac{a^2\log\left(a^2+2abx+b^2x^2+1\right)}{2b^3} + \frac{2ax}{3b^2} - \frac{a\operatorname{atan}\left(a+bx\right)}{b^3} + \frac{x^3\operatorname{atan}\left(a+bx\right)}{3} - \frac{x^2}{6b} + \frac{\log\left(a^2+2abx+b^2x^2+1\right)}{6b^3} & \text{for}\: b \neq 0 \\ \frac{x^3\operatorname{atan}\left(a\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(b*x+a),x)
```

```
[Out] Piecewise((a**3*atan(a + b*x)/(3*b**3) - a**2*log(a**2 + 2*a*b*x + b**2*x**
2 + 1)/(2*b**3) + 2*a*x/(3*b**2) - a*atan(a + b*x)/b**3 + x**3*atan(a + b*x
)/3 - x**2/(6*b) + log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(6*b**3), Ne(b, 0)),
 (x**3*atan(a)/3, True))
```

Giac [A] time = 1.09272, size = 111, normalized size = 1.41

$$\frac{1}{3}\,x^3\arctan\left(bx + a\right) + \frac{1}{6}\,b\left(\frac{2\left(a^3 - 3\,a\right)\arctan\left(bx + a\right)}{b^4} - \frac{\left(3\,a^2 - 1\right)\log\left(b^2x^2 + 2\,abx + a^2 + 1\right)}{b^4} - \frac{b^2x^2 - 4\,abx}{b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(b*x+a),x, algorithm="giac")
```

```
[Out] 1/3*x^3*arctan(b*x + a) + 1/6*b*(2*(a^3 - 3*a)*arctan(b*x + a)/b^4 - (3*a^2
 - 1)*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^4 - (b^2*x^2 - 4*a*b*x)/b^4)
```

3.46 $\int x\tan^{-1}(a+bx)\,dx$

Optimal. Leaf size=60

$$\frac{(1-a^2)\tan^{-1}(a+bx)}{2b^2}+\frac{a\log\left((a+bx)^2+1\right)}{2b^2}+\frac{1}{2}x^2\tan^{-1}(a+bx)-\frac{x}{2b}$$

```
[Out] -x/(2*b) + ((1 - a^2)*ArcTan[a + b*x])/(2*b^2) + (x^2*ArcTan[a + b*x])/2 +
(a*Log[1 + (a + b*x)^2])/(2*b^2)
```

Rubi [A] time = 0.0546893, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {5047, 4862, 702, 635, 203, 260}

$$\frac{(1-a^2)\tan^{-1}(a+bx)}{2b^2}+\frac{a\log\left((a+bx)^2+1\right)}{2b^2}+\frac{1}{2}x^2\tan^{-1}(a+bx)-\frac{x}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[a + b*x],x]
```

```
[Out] -x/(2*b) + ((1 - a^2)*ArcTan[a + b*x])/(2*b^2) + (x^2*ArcTan[a + b*x])/2 +
(a*Log[1 + (a + b*x)^2])/(2*b^2)
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
```

```
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int x\tan^{-1}(a+bx)\,dx &= \frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)\tan^{-1}(x)\,dx,x,a+bx\right)}{b}\\
&= \frac{1}{2}x^2\tan^{-1}(a+bx)-\frac{1}{2}\text{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{1+x^2}\,dx,x,a+bx\right)\\
&= \frac{1}{2}x^2\tan^{-1}(a+bx)-\frac{1}{2}\text{Subst}\left(\int\left(\frac{1}{b^2}-\frac{1-a^2+2ax}{b^2\left(1+x^2\right)}\right)dx,x,a+bx\right)\\
&= -\frac{x}{2b}+\frac{1}{2}x^2\tan^{-1}(a+bx)+\frac{\text{Subst}\left(\int\frac{1-a^2+2ax}{1+x^2}\,dx,x,a+bx\right)}{2b^2}\\
&= -\frac{x}{2b}+\frac{1}{2}x^2\tan^{-1}(a+bx)+\frac{a\,\text{Subst}\left(\int\frac{x}{1+x^2}\,dx,x,a+bx\right)}{b^2}+\frac{\left(1-a^2\right)\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,a+bx\right)}{2b^2}\\
&= -\frac{x}{2b}+\frac{\left(1-a^2\right)\tan^{-1}(a+bx)}{2b^2}+\frac{1}{2}x^2\tan^{-1}(a+bx)+\frac{a\log\left(1+(a+bx)^2\right)}{2b^2}
\end{aligned}$$

Mathematica [C] time = 0.0307717, size = 90, normalized size = 1.5

$$\frac{-ia^2\log(a+bx+i)+2b^2x^2\tan^{-1}(a+bx)+2a\log(a+bx+i)+i(a-i)^2\log(-a-bx+i)+i\log(a+bx+i)-2bx}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[a + b*x],x]
```

```
[Out] (-2*b*x + 2*b^2*x^2*ArcTan[a + b*x] + I*(-I + a)^2*Log[I - a - b*x] + I*Log
[I + a + b*x] + 2*a*Log[I + a + b*x] - I*a^2*Log[I + a + b*x])/(4*b^2)
```

Maple [A] time = 0.035, size = 66, normalized size = 1.1

$$\frac{x^2 \arctan (bx+a)}{2} - \frac{\arctan (bx+a)\, a^2}{2\,b^2} - \frac{x}{2\,b} - \frac{a}{2\,b^2} + \frac{a \ln \left(1+(bx+a)^2\right)}{2\,b^2} + \frac{\arctan (bx+a)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(b*x+a),x)
```

```
[Out] 1/2*x^2*arctan(b*x+a)-1/2/b^2*arctan(b*x+a)*a^2-1/2*x/b-1/2/b^2*a+1/2*a*ln(
1+(b*x+a)^2)/b^2+1/2/b^2*arctan(b*x+a)
```

Maxima [A] time = 1.51409, size = 92, normalized size = 1.53

$$\frac{1}{2}\, x^2 \arctan (bx+a) - \frac{1}{2}\, b \left(\frac{x}{b^2} + \frac{\left(a^2-1\right) \arctan \left(\frac{b^2x+ab}{b}\right)}{b^3} - \frac{a \log \left(b^2x^2+2\,abx+a^2+1\right)}{b^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(b*x + a) - 1/2*b*(x/b^2 + (a^2 - 1)*arctan((b^2*x + a*b)/b)/
b^3 - a*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^3)
```

Fricas [A] time = 1.69324, size = 123, normalized size = 2.05

$$-\frac{bx - \left(b^2x^2 - a^2 + 1\right) \arctan (bx+a) - a \log \left(b^2x^2 + 2\,abx + a^2 + 1\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x - (b^2*x^2 - a^2 + 1)*arctan(b*x + a) - a*log(b^2*x^2 + 2*a*b*x +
 a^2 + 1))/b^2
```

Sympy [A] time = 0.965333, size = 78, normalized size = 1.3

$$\begin{cases} -\frac{a^2 \operatorname{atan}(a+bx)}{2b^2} + \frac{a \log\left(a^2+2abx+b^2x^2+1\right)}{2b^2} + \frac{x^2 \operatorname{atan}(a+bx)}{2} - \frac{x}{2b} + \frac{\operatorname{atan}(a+bx)}{2b^2} & \text{for } b \neq 0 \\ \frac{x^2 \operatorname{atan}(a)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(b*x+a),x)
```

```
[Out] Piecewise(((-a**2*atan(a + b*x)/(2*b**2) + a*log(a**2 + 2*a*b*x + b**2*x**2
+ 1)/(2*b**2) + x**2*atan(a + b*x)/2 - x/(2*b) + atan(a + b*x)/(2*b**2), Ne
(b, 0)), (x**2*atan(a)/2, True))
```

Giac [A] time = 1.10012, size = 81, normalized size = 1.35

$$\frac{1}{2}\,x^2\arctan(bx+a) - \frac{1}{2}\,b\left(\frac{x}{b^2} + \frac{\left(a^2-1\right)\arctan(bx+a)}{b^3} - \frac{a\log\left(b^2x^2+2\,abx+a^2+1\right)}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*x^2*arctan(b*x + a) - 1/2*b*(x/b^2 + (a^2 - 1)*arctan(b*x + a)/b^3 - a*
log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^3)
```

3.47 $\int \tan^{-1}(a+bx)\,dx$

Optimal. Leaf size=33

$$\frac{(a+bx)\tan^{-1}(a+bx)}{b}-\frac{\log\left((a+bx)^2+1\right)}{2b}$$

```
[Out] ((a + b*x)*ArcTan[a + b*x])/b - Log[1 + (a + b*x)^2]/(2*b)
```

Rubi [A] time = 0.0113888, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5039, 4846, 260}

$$\frac{(a+bx)\tan^{-1}(a+bx)}{b}-\frac{\log\left((a+bx)^2+1\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x],x]
```

```
[Out] ((a + b*x)*ArcTan[a + b*x])/b - Log[1 + (a + b*x)^2]/(2*b)
```

Rule 5039

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \tan^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int \tan^{-1}(x)\,dx, x, a+bx\right)}{b}$$
$$= \frac{(a+bx)\tan^{-1}(a+bx)}{b} - \frac{\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, a+bx\right)}{b}$$
$$= \frac{(a+bx)\tan^{-1}(a+bx)}{b} - \frac{\log\left(1+(a+bx)^2\right)}{2b}$$

Mathematica [A] time = 0.0138611, size = 39, normalized size = 1.18

$$-\frac{\log\left(a^2+2abx+b^2x^2+1\right)-2(a+bx)\tan^{-1}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x],x]
```

```
[Out] -(-2*(a + b*x)*ArcTan[a + b*x] + Log[1 + a^2 + 2*a*b*x + b^2*x^2])/(2*b)
```

Maple [A] time = 0.035, size = 36, normalized size = 1.1

$$x\arctan(bx+a)+\frac{\arctan(bx+a)\,a}{b}-\frac{\ln\left(1+(bx+a)^2\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a),x)
```

```
[Out] x*arctan(b*x+a)+1/b*arctan(b*x+a)*a-1/2*ln(1+(b*x+a)^2)/b
```

Maxima [A] time = 1.00836, size = 42, normalized size = 1.27

$$\frac{2\,(bx+a)\arctan(bx+a)-\log\left((bx+a)^2+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b*x + a)*arctan(b*x + a) - log((b*x + a)^2 + 1))/b
```

Fricas [A] time = 1.74759, size = 97, normalized size = 2.94

$$\frac{2\,(bx+a)\arctan\,(bx+a)-\log\left(b^2x^2+2\,abx+a^2+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(b*x + a)*arctan(b*x + a) - log(b^2*x^2 + 2*a*b*x + a^2 + 1))/b
```

Sympy [A] time = 0.558806, size = 46, normalized size = 1.39

$$\begin{cases} \frac{a\,\mathrm{atan}\,(a+bx)}{b}+x\,\mathrm{atan}\,(a+bx)-\frac{\log\left(a^2+2abx+b^2x^2+1\right)}{2b} & \text{for } b\neq 0 \\ x\,\mathrm{atan}\,(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a),x)
```

```
[Out] Piecewise((a*atan(a + b*x)/b + x*atan(a + b*x) - log(a**2 + 2*a*b*x + b**2*
x**2 + 1)/(2*b), Ne(b, 0)), (x*atan(a), True))
```

Giac [A] time = 1.11763, size = 42, normalized size = 1.27

$$\frac{2\,(bx+a)\arctan\,(bx+a)-\log\left((bx+a)^2+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(2*(b*x + a)*arctan(b*x + a) - log((b*x + a)^2 + 1))/b
```

3.48 $\int \frac{\tan^{-1}(a+bx)}{x}\,dx$

Optimal. Leaf size=120

$$\frac{1}{2}i\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)-\frac{1}{2}i\text{PolyLog}\left(2,1-\frac{2bx}{(-a+i)(1-i(a+bx))}\right)+\log\left(\frac{2}{1-i(a+bx)}\right)\left(-\tan^{-1}(a+bx)\right)+$$

```
[Out] -(ArcTan[a + b*x]*Log[2/(1 - I*(a + b*x))]) + ArcTan[a + b*x]*Log[(2*b*x)/(
(I - a)*(1 - I*(a + b*x)))] + (I/2)*PolyLog[2, 1 - 2/(1 - I*(a + b*x))] - (
I/2)*PolyLog[2, 1 - (2*b*x)/((I - a)*(1 - I*(a + b*x)))]
```

Rubi [A] time = 0.106247, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5047, 4856, 2402, 2315, 2447}

$$\frac{1}{2}i\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)-\frac{1}{2}i\text{PolyLog}\left(2,1-\frac{2bx}{(-a+i)(1-i(a+bx))}\right)+\log\left(\frac{2}{1-i(a+bx)}\right)\left(-\tan^{-1}(a+bx)\right)+$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/x,x]
```

```
[Out] -(ArcTan[a + b*x]*Log[2/(1 - I*(a + b*x))]) + ArcTan[a + b*x]*Log[(2*b*x)/(
(I - a)*(1 - I*(a + b*x)))] + (I/2)*PolyLog[2, 1 - 2/(1 - I*(a + b*x))] - (
I/2)*PolyLog[2, 1 - (2*b*x)/((I - a)*(1 - I*(a + b*x)))]
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\tan^{-1}(x)}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bx\right)}{b}$$

$$= -\tan^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)+\tan^{-1}(a+bx)\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right)+\text{Subst}\left(\int \frac{\log}{1}\right.$$

$$= -\tan^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)+\tan^{-1}(a+bx)\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right)-\frac{1}{2}i\text{Li}_2\left(1-\frac{}{(i-}\right.$$

$$= -\tan^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)+\tan^{-1}(a+bx)\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right)+\frac{1}{2}i\text{Li}_2\left(1-\frac{}{1-}\right.$$

Mathematica [A] time = 0.0081672, size = 171, normalized size = 1.42

$$\frac{1}{2}i\text{PolyLog}\left(2,\frac{i(1-i(a+bx))}{a+i}\right)-\frac{1}{2}i\text{PolyLog}\left(2,-\frac{i(1+i(a+bx))}{a-i}\right)-\frac{1}{2}i\log(1+i(a+bx))\log\left(\frac{i\left(\frac{a+bx}{b}-\frac{a}{b}\right)}{-\frac{1}{b}-\frac{ia}{b}}\right)+\frac{1}{2}i\log($$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a + b*x]/x,x]
```

```
[Out] (-I/2)*Log[1 + I*(a + b*x)]*Log[(I*(-(a/b) + (a + b*x)/b))/(-b^(-1) - (I*a)
/b)] + (I/2)*Log[1 - I*(a + b*x)]*Log[((-I)*(-(a/b) + (a + b*x)/b))/(-b^(-1
) + (I*a)/b)] + (I/2)*PolyLog[2, (I*(1 - I*(a + b*x)))/(I + a)] - (I/2)*Pol
yLog[2, ((-I)*(1 + I*(a + b*x)))/(-I + a)]
```

Maple [A] time = 0.047, size = 103, normalized size = 0.9

$$\ln(bx)\arctan(bx+a)+\frac{i}{2}\ln(bx)\ln\left(\frac{i-a-bx}{i-a}\right)-\frac{i}{2}\ln(bx)\ln\left(\frac{i+a+bx}{i+a}\right)+\frac{i}{2}\mathrm{dilog}\left(\frac{i-a-bx}{i-a}\right)-\frac{i}{2}\mathrm{dilog}\left(\frac{i+a+bx}{i+a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/x,x)
```

```
[Out] ln(b*x)*arctan(b*x+a)+1/2*I*ln(b*x)*ln((I-a-b*x)/(I-a))-1/2*I*ln(b*x)*ln((I
+a+b*x)/(I+a))+1/2*I*dilog((I-a-b*x)/(I-a))-1/2*I*dilog((I+a+b*x)/(I+a))
```

Maxima [A] time = 1.71072, size = 181, normalized size = 1.51

$$-\frac{1}{2}\arctan\left(\frac{bx}{a^2+1},-\frac{abx}{a^2+1}\right)\log\left(b^2x^2+2\,abx+a^2+1\right)+\frac{1}{2}\arctan(bx+a)\log\left(\frac{b^2x^2}{a^2+1}\right)+\arctan(bx+a)\log(x)-\mathrm{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] -1/2*arctan2(b*x/(a^2 + 1), -a*b*x/(a^2 + 1))*log(b^2*x^2 + 2*a*b*x + a^2 +
 1) + 1/2*arctan(b*x + a)*log(b^2*x^2/(a^2 + 1)) + arctan(b*x + a)*log(x) -
 arctan((b^2*x + a*b)/b)*log(x) - 1/2*I*dilog((I*b*x + I*a + 1)/(I*a + 1))
+ 1/2*I*dilog((I*b*x + I*a - 1)/(I*a - 1))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\arctan(bx+a)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(arctan(b*x + a)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(arctan(b*x + a)/x, x)
```

3.49 $\int \frac{\tan^{-1}(a+bx)}{x^2} dx$

Optimal. Leaf size=62

$$\frac{b\log(x)}{a^2+1} - \frac{b\log\left((a+bx)^2+1\right)}{2\left(a^2+1\right)} - \frac{ab\tan^{-1}(a+bx)}{a^2+1} - \frac{\tan^{-1}(a+bx)}{x}$$

```
[Out] -((a*b*ArcTan[a + b*x])/(1 + a^2)) - ArcTan[a + b*x]/x + (b*Log[x])/(1 + a^
2) - (b*Log[1 + (a + b*x)^2])/(2*(1 + a^2))
```

Rubi [A] time = 0.038679, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5045, 371, 706, 31, 635, 203, 260}

$$\frac{b\log(x)}{a^2+1} - \frac{b\log\left((a+bx)^2+1\right)}{2\left(a^2+1\right)} - \frac{ab\tan^{-1}(a+bx)}{a^2+1} - \frac{\tan^{-1}(a+bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/x^2,x]
```

```
[Out] -((a*b*ArcTan[a + b*x])/(1 + a^2)) - ArcTan[a + b*x]/x + (b*Log[x])/(1 + a^
2) - (b*Log[1 + (a + b*x)^2])/(2*(1 + a^2))
```

Rule 5045

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[((e + f*x)^(m + 1)*(a + b*ArcTan[c + d*x])^p)/(f*(m +
 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcTan[c
 + d*x])^(p - 1))/(1 + (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(a+bx)}{x^2}\,dx &= -\frac{\tan^{-1}(a+bx)}{x}+b\int \frac{1}{x\left(1+(a+bx)^2\right)}\,dx\\
&= -\frac{\tan^{-1}(a+bx)}{x}+b\,\text{Subst}\left(\int \frac{1}{(-a+x)\left(1+x^2\right)}\,dx,x,a+bx\right)\\
&= -\frac{\tan^{-1}(a+bx)}{x}+\frac{b\,\text{Subst}\left(\int \frac{1}{-a+x}\,dx,x,a+bx\right)}{1+a^2}+\frac{b\,\text{Subst}\left(\int \frac{-a-x}{1+x^2}\,dx,x,a+bx\right)}{1+a^2}\\
&= -\frac{\tan^{-1}(a+bx)}{x}+\frac{b\log(x)}{1+a^2}-\frac{b\,\text{Subst}\left(\int \frac{x}{1+x^2}\,dx,x,a+bx\right)}{1+a^2}-\frac{(ab)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,a+bx\right)}{1+a^2}\\
&= -\frac{ab\tan^{-1}(a+bx)}{1+a^2}-\frac{\tan^{-1}(a+bx)}{x}+\frac{b\log(x)}{1+a^2}-\frac{b\log\left(1+(a+bx)^2\right)}{2\left(1+a^2\right)}
\end{aligned}$$

Mathematica [C] time = 0.0560829, size = 67, normalized size = 1.08

$$-\frac{\tan^{-1}(a+bx)}{x}+\frac{b(i(a+i)\log(-a-bx+i)+(-1-ia)\log(a+bx+i)+2\log(x))}{2\left(a^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/x^2,x]
```

```
[Out] -(ArcTan[a + b*x]/x) + (b*(2*Log[x] + I*(I + a)*Log[I - a - b*x] + (-1 - I*
a)*Log[I + a + b*x]))/(2*(1 + a^2))
```

Maple [A] time = 0.043, size = 63, normalized size = 1.

$$-\frac{\arctan(bx+a)}{x}-\frac{b\ln\left(1+(bx+a)^2\right)}{2a^2+2}-\frac{ab\arctan(bx+a)}{a^2+1}+\frac{b\ln(bx)}{a^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/x^2,x)
```

```
[Out] -arctan(b*x+a)/x-1/2*b*ln(1+(b*x+a)^2)/(a^2+1)-a*b*arctan(b*x+a)/(a^2+1)+b/
(a^2+1)*ln(b*x)
```

Maxima [A] time = 1.54389, size = 104, normalized size = 1.68

$$-\frac{1}{2}b\left(\frac{2a\arctan\left(\frac{b^2x+ab}{b}\right)}{a^2+1}+\frac{\log\left(b^2x^2+2abx+a^2+1\right)}{a^2+1}-\frac{2\log(x)}{a^2+1}\right)-\frac{\arctan(bx+a)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*b*(2*a*arctan((b^2*x + a*b)/b)/(a^2 + 1) + log(b^2*x^2 + 2*a*b*x + a^2
 + 1)/(a^2 + 1) - 2*log(x)/(a^2 + 1)) - arctan(b*x + a)/x
```

Fricas [A] time = 1.71557, size = 151, normalized size = 2.44

$$-\frac{bx\log\left(b^2x^2+2\,abx+a^2+1\right)-2\,bx\log\left(x\right)+2\left(abx+a^2+1\right)\arctan\left(bx+a\right)}{2\left(a^2+1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b*x*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*b*x*log(x) + 2*(a*b*x + a^2
+ 1)*arctan(b*x + a))/((a^2 + 1)*x)
```

Sympy [B] time = 10.9512, size = 323, normalized size = 5.21

$$\begin{cases} -\frac{ib^2x^2\operatorname{atan}(bx-i)}{2bx^2-4ix}-\frac{4bx\operatorname{atan}(bx-i)}{2bx^2-4ix}-\frac{ibx}{2bx^2-4ix}+\frac{4i\operatorname{atan}(bx-i)}{2bx^2-4ix}-\frac{2}{2bx^2-4ix} & \text{for } a=-i \\ \frac{ib^2x^2\operatorname{atan}(bx+i)}{2bx^2+4ix}-\frac{4bx\operatorname{atan}(bx+i)}{2bx^2+4ix}+\frac{ibx}{2bx^2+4ix}-\frac{4i\operatorname{atan}(bx+i)}{2bx^2+4ix}-\frac{2}{2bx^2+4ix} & \text{for } a=i \\ -\frac{2a^2\operatorname{atan}(a+bx)}{2a^2x+2x}-\frac{2abx\operatorname{atan}(a+bx)}{2a^2x+2x}+\frac{2bx\log(x)}{2a^2x+2x}-\frac{bx\log\left(a^2+2abx+b^2x^2+1\right)}{2a^2x+2x}-\frac{2\operatorname{atan}(a+bx)}{2a^2x+2x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/x**2,x)
```

```
[Out] Piecewise((-I*b**2*x**2*atan(b*x - I)/(2*b*x**2 - 4*I*x) - 4*b*x*atan(b*x -
 I)/(2*b*x**2 - 4*I*x) - I*b*x/(2*b*x**2 - 4*I*x) + 4*I*atan(b*x - I)/(2*b*
x**2 - 4*I*x) - 2/(2*b*x**2 - 4*I*x), Eq(a, -I)), (I*b**2*x**2*atan(b*x + I
)/(2*b*x**2 + 4*I*x) - 4*b*x*atan(b*x + I)/(2*b*x**2 + 4*I*x) + I*b*x/(2*b*
x**2 + 4*I*x) - 4*I*atan(b*x + I)/(2*b*x**2 + 4*I*x) - 2/(2*b*x**2 + 4*I*x)
, Eq(a, I)), (-2*a**2*atan(a + b*x)/(2*a**2*x + 2*x) - 2*a*b*x*atan(a + b*x
)/(2*a**2*x + 2*x) + 2*b*x*log(x)/(2*a**2*x + 2*x) - b*x*log(a**2 + 2*a*b*x
 + b**2*x**2 + 1)/(2*a**2*x + 2*x) - 2*atan(a + b*x)/(2*a**2*x + 2*x), True
))
```

Giac [A] time = 1.11186, size = 95, normalized size = 1.53

$$-\frac{1}{2}\,b\left(\frac{2\,a\arctan\left(bx+a\right)}{a^2+1}+\frac{\log\left(b^2x^2+2\,abx+a^2+1\right)}{a^2+1}-\frac{2\,\log\left(|x|\right)}{a^2+1}\right)-\frac{\arctan\left(bx+a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*b*(2*a*arctan(b*x + a)/(a^2 + 1) + log(b^2*x^2 + 2*a*b*x + a^2 + 1)/(a
^2 + 1) - 2*log(abs(x))/(a^2 + 1)) - arctan(b*x + a)/x
```

3.50 $\int \frac{\tan^{-1}(a+bx)}{x^3} dx$

Optimal. Leaf size=96

$$-\frac{ab^2\log(x)}{\left(a^2+1\right)^2}+\frac{ab^2\log\left((a+bx)^2+1\right)}{2\left(a^2+1\right)^2}-\frac{\left(1-a^2\right)b^2\tan^{-1}(a+bx)}{2\left(a^2+1\right)^2}-\frac{b}{2\left(a^2+1\right)x}-\frac{\tan^{-1}(a+bx)}{2x^2}$$

```
[Out] -b/(2*(1 + a^2)*x) - ((1 - a^2)*b^2*ArcTan[a + b*x])/(2*(1 + a^2)^2) - ArcT
an[a + b*x]/(2*x^2) - (a*b^2*Log[x])/(1 + a^2)^2 + (a*b^2*Log[1 + (a + b*x)
^2])/(2*(1 + a^2)^2)
```

Rubi [A] time = 0.0833072, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5045, 371, 710, 801, 635, 203, 260}

$$-\frac{ab^2\log(x)}{\left(a^2+1\right)^2}+\frac{ab^2\log\left((a+bx)^2+1\right)}{2\left(a^2+1\right)^2}-\frac{\left(1-a^2\right)b^2\tan^{-1}(a+bx)}{2\left(a^2+1\right)^2}-\frac{b}{2\left(a^2+1\right)x}-\frac{\tan^{-1}(a+bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/x^3,x]
```


```
[Out] -b/(2*(1 + a^2)*x) - ((1 - a^2)*b^2*ArcTan[a + b*x])/(2*(1 + a^2)^2) - ArcT
an[a + b*x]/(2*x^2) - (a*b^2*Log[x])/(1 + a^2)^2 + (a*b^2*Log[1 + (a + b*x)
^2])/(2*(1 + a^2)^2)
```


Rule 5045

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[((e + f*x)^(m + 1)*(a + b*ArcTan[c + d*x])^p)/(f*(m +
 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcTan[c
 + d*x])^(p - 1))/(1 + (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
```

```
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{x^3}\,dx = -\frac{\tan^{-1}(a+bx)}{2x^2} + \frac{1}{2}b\int \frac{1}{x^2\left(1+(a+bx)^2\right)}\,dx$$

$$= -\frac{\tan^{-1}(a+bx)}{2x^2} + \frac{1}{2}b^2\,\mathrm{Subst}\left(\int \frac{1}{(-a+x)^2\left(1+x^2\right)}\,dx, x, a+bx\right)$$

$$= -\frac{b}{2\left(1+a^2\right)x} - \frac{\tan^{-1}(a+bx)}{2x^2} + \frac{b^2\,\mathrm{Subst}\left(\int \frac{-a-x}{(-a+x)\left(1+x^2\right)}\,dx, x, a+bx\right)}{2\left(1+a^2\right)}$$

$$= -\frac{b}{2\left(1+a^2\right)x} - \frac{\tan^{-1}(a+bx)}{2x^2} + \frac{b^2\,\mathrm{Subst}\left(\int \left(\frac{2a}{\left(1+a^2\right)(a-x)} + \frac{-1+a^2+2ax}{\left(1+a^2\right)\left(1+x^2\right)}\right)dx, x, a+bx\right)}{2\left(1+a^2\right)}$$

$$= -\frac{b}{2\left(1+a^2\right)x} - \frac{\tan^{-1}(a+bx)}{2x^2} - \frac{ab^2\log(x)}{\left(1+a^2\right)^2} + \frac{b^2\,\mathrm{Subst}\left(\int \frac{-1+a^2+2ax}{1+x^2}\,dx, x, a+bx\right)}{2\left(1+a^2\right)^2}$$

$$= -\frac{b}{2\left(1+a^2\right)x} - \frac{\tan^{-1}(a+bx)}{2x^2} - \frac{ab^2\log(x)}{\left(1+a^2\right)^2} + \frac{\left(ab^2\right)\mathrm{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, a+bx\right)}{\left(1+a^2\right)^2} - \frac{\left(\left(1-a^2\right)b^2\right)}{}$$

$$= -\frac{b}{2\left(1+a^2\right)x} - \frac{\left(1-a^2\right)b^2\tan^{-1}(a+bx)}{2\left(1+a^2\right)^2} - \frac{\tan^{-1}(a+bx)}{2x^2} - \frac{ab^2\log(x)}{\left(1+a^2\right)^2} + \frac{ab^2\log\left(1+(a+bx)^2\right.}{2\left(1+a^2\right)^2}$$

Mathematica [C] time = 0.0958245, size = 92, normalized size = 0.96

$$\frac{-2\tan^{-1}(a+bx) + \frac{bx\left(-i(a+i)^2bx\log(-a-bx+i)-4abx\log(x)+(a-i)((1+ia)bx\log(a+bx+i)-2(a+i))\right)}{\left(a^2+1\right)^2}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/x^3,x]
```

```
[Out] (-2*ArcTan[a + b*x] + (b*x*(-4*a*b*x*Log[x] - I*(I + a)^2*b*x*Log[I - a - b
*x] + (-I + a)*(-2*(I + a) + (1 + I*a)*b*x*Log[I + a + b*x])))/(1 + a^2)^2)
/(4*x^2)
```

Maple [A] time = 0.043, size = 105, normalized size = 1.1

$$-\frac{\arctan\left(bx+a\right)}{2\,x^{2}}+\frac{b^{2}\arctan\left(bx+a\right)a^{2}}{2\,\left(a^{2}+1\right)^{2}}+\frac{ab^{2}\ln\left(1+\left(bx+a\right)^{2}\right)}{2\,\left(a^{2}+1\right)^{2}}-\frac{b^{2}\arctan\left(bx+a\right)}{2\,\left(a^{2}+1\right)^{2}}-\frac{b}{\left(2\,a^{2}+2\right)x}-\frac{ab^{2}\ln\left(bx\right)}{\left(a^{2}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/x^3,x)
```

```
[Out] -1/2*arctan(b*x+a)/x^2+1/2*b^2/(a^2+1)^2*arctan(b*x+a)*a^2+1/2*a*b^2*ln(1+(
b*x+a)^2)/(a^2+1)^2-1/2*b^2/(a^2+1)^2*arctan(b*x+a)-1/2*b/(a^2+1)/x-b^2/(a^
2+1)^2*a*ln(b*x)
```

Maxima [A] time = 1.52569, size = 151, normalized size = 1.57

$$\frac{1}{2}\left(\frac{\left(a^{2}-1\right)b\arctan\left(\frac{b^{2}x+ab}{b}\right)}{a^{4}+2\,a^{2}+1}+\frac{ab\log\left(b^{2}x^{2}+2\,abx+a^{2}+1\right)}{a^{4}+2\,a^{2}+1}-\frac{2\,ab\log\left(x\right)}{a^{4}+2\,a^{2}+1}-\frac{1}{\left(a^{2}+1\right)x}\right)b-\frac{\arctan\left(bx+a\right)}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2 - 1)*b*arctan((b^2*x + a*b)/b)/(a^4 + 2*a^2 + 1) + a*b*log(b^2*x^
2 + 2*a*b*x + a^2 + 1)/(a^4 + 2*a^2 + 1) - 2*a*b*log(x)/(a^4 + 2*a^2 + 1) -
 1/((a^2 + 1)*x))*b - 1/2*arctan(b*x + a)/x^2
```

Fricas [A] time = 1.89237, size = 225, normalized size = 2.34

$$\frac{ab^{2}x^{2}\log\left(b^{2}x^{2}+2\,abx+a^{2}+1\right)-2\,ab^{2}x^{2}\log\left(x\right)-\left(a^{2}+1\right)bx+\left(\left(a^{2}-1\right)b^{2}x^{2}-a^{4}-2\,a^{2}-1\right)\arctan\left(bx+a\right)}{2\left(a^{4}+2\,a^{2}+1\right)x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a*b^2*x^2*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*a*b^2*x^2*log(x) - (a^2
 + 1)*b*x + ((a^2 - 1)*b^2*x^2 - a^4 - 2*a^2 - 1)*arctan(b*x + a))/((a^4 +
```

```
2*a^2 + 1)*x^2)
```

Sympy [B] time = 16.5722, size = 644, normalized size = 6.71

$$\begin{cases} -\frac{2b^3x^3\operatorname{atan}(bx-i)}{16bx^3-32ix^2}+\frac{ib^3x^3}{16bx^3-32ix^2}+\frac{4ib^2x^2\operatorname{atan}(bx-i)}{16bx^3-32ix^2}-\frac{8bx\operatorname{atan}(bx-i)}{16bx^3-32ix^2}+\frac{2ibx}{16bx^3-32ix^2}+\frac{16i\operatorname{atan}(bx-i)}{16bx^3-32ix^2}-\frac{4}{16bx^3-32ix^2} \\ -\frac{2b^3x^3\operatorname{atan}(bx+i)}{16bx^3+32ix^2}-\frac{ib^3x^3}{16bx^3+32ix^2}-\frac{4ib^2x^2\operatorname{atan}(bx+i)}{16bx^3+32ix^2}-\frac{8bx\operatorname{atan}(bx+i)}{16bx^3+32ix^2}-\frac{2ibx}{16bx^3+32ix^2}-\frac{16i\operatorname{atan}(bx+i)}{16bx^3+32ix^2}-\frac{4}{16bx^3+32ix^2} \\ -\frac{a^4\operatorname{atan}(a+bx)}{2a^4x^2+4a^2x^2+2x^2}+\frac{a^2b^2x^2\operatorname{atan}(a+bx)}{2a^4x^2+4a^2x^2+2x^2}-\frac{a^2bx}{2a^4x^2+4a^2x^2+2x^2}-\frac{2a^2\operatorname{atan}(a+bx)}{2a^4x^2+4a^2x^2+2x^2}-\frac{2ab^2x^2\log(x)}{2a^4x^2+4a^2x^2+2x^2}+\frac{ab^2x^2\log\left(a^2+2abx+b^2x^2+1\right)}{2a^4x^2+4a^2x^2+2x^2}-\frac{b^2x^2\operatorname{a}}{2a^4x^2+} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/x**3,x)
```

```
[Out] Piecewise((-2*b**3*x**3*atan(b*x - I)/(16*b*x**3 - 32*I*x**2) + I*b**3*x**3
/(16*b*x**3 - 32*I*x**2) + 4*I*b**2*x**2*atan(b*x - I)/(16*b*x**3 - 32*I*x*
*2) - 8*b*x*atan(b*x - I)/(16*b*x**3 - 32*I*x**2) + 2*I*b*x/(16*b*x**3 - 32
*I*x**2) + 16*I*atan(b*x - I)/(16*b*x**3 - 32*I*x**2) - 4/(16*b*x**3 - 32*I
*x**2), Eq(a, -I)), (-2*b**3*x**3*atan(b*x + I)/(16*b*x**3 + 32*I*x**2) - I
*b**3*x**3/(16*b*x**3 + 32*I*x**2) - 4*I*b**2*x**2*atan(b*x + I)/(16*b*x**3
 + 32*I*x**2) - 8*b*x*atan(b*x + I)/(16*b*x**3 + 32*I*x**2) - 2*I*b*x/(16*b
*x**3 + 32*I*x**2) - 16*I*atan(b*x + I)/(16*b*x**3 + 32*I*x**2) - 4/(16*b*x
**3 + 32*I*x**2), Eq(a, I)), (-a**4*atan(a + b*x)/(2*a**4*x**2 + 4*a**2*x**
2 + 2*x**2) + a**2*b**2*x**2*atan(a + b*x)/(2*a**4*x**2 + 4*a**2*x**2 + 2*x
**2) - a**2*b*x/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2) - 2*a**2*atan(a + b*x)
/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2) - 2*a*b**2*x**2*log(x)/(2*a**4*x**2 +
 4*a**2*x**2 + 2*x**2) + a*b**2*x**2*log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(2
*a**4*x**2 + 4*a**2*x**2 + 2*x**2) - b**2*x**2*atan(a + b*x)/(2*a**4*x**2 +
 4*a**2*x**2 + 2*x**2) - b*x/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2) - atan(a
+ b*x)/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2), True))
```

Giac [A] time = 1.08494, size = 155, normalized size = 1.61

$$\frac{1}{2}\left(\frac{ab\log\left(b^2x^2+2\,abx+a^2+1\right)}{a^4+2\,a^2+1}-\frac{2\,ab\log\left(|x|\right)}{a^4+2\,a^2+1}+\frac{\left(a^2b^2-b^2\right)\arctan\left(bx+a\right)}{\left(a^4+2\,a^2+1\right)b}-\frac{1}{\left(a^2+1\right)x}\right)b-\frac{\arctan\left(bx+a\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*(a*b*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/(a^4 + 2*a^2 + 1) - 2*a*b*log(abs
(x))/(a^4 + 2*a^2 + 1) + (a^2*b^2 - b^2)*arctan(b*x + a)/((a^4 + 2*a^2 + 1)
*b) - 1/((a^2 + 1)*x))*b - 1/2*arctan(b*x + a)/x^2
```

3.51 $\int \frac{\tan^{-1}(a+bx)}{x^4} dx$

Optimal. Leaf size=129

$$\frac{2ab^2}{3\left(a^2+1\right)^2 x}-\frac{\left(1-3a^2\right) b^3 \log(x)}{3\left(a^2+1\right)^3}+\frac{\left(1-3a^2\right) b^3 \log\left((a+bx)^2+1\right)}{6\left(a^2+1\right)^3}+\frac{a\left(3-a^2\right) b^3 \tan^{-1}(a+bx)}{3\left(a^2+1\right)^3}-\frac{b}{6\left(a^2+1\right) x^2}-\frac{\text{ta}}{}$$

```
[Out] -b/(6*(1 + a^2)*x^2) + (2*a*b^2)/(3*(1 + a^2)^2*x) + (a*(3 - a^2)*b^3*ArcTa
n[a + b*x])/(3*(1 + a^2)^3) - ArcTan[a + b*x]/(3*x^3) - ((1 - 3*a^2)*b^3*Lo
g[x])/(3*(1 + a^2)^3) + ((1 - 3*a^2)*b^3*Log[1 + (a + b*x)^2])/(6*(1 + a^2)
^3)
```

Rubi [A] time = 0.114371, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5045, 371, 710, 801, 635, 203, 260}

$$\frac{2ab^2}{3\left(a^2+1\right)^2 x}-\frac{\left(1-3a^2\right) b^3 \log(x)}{3\left(a^2+1\right)^3}+\frac{\left(1-3a^2\right) b^3 \log\left((a+bx)^2+1\right)}{6\left(a^2+1\right)^3}+\frac{a\left(3-a^2\right) b^3 \tan^{-1}(a+bx)}{3\left(a^2+1\right)^3}-\frac{b}{6\left(a^2+1\right) x^2}-\frac{\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/x^4,x]
```


```
[Out] -b/(6*(1 + a^2)*x^2) + (2*a*b^2)/(3*(1 + a^2)^2*x) + (a*(3 - a^2)*b^3*ArcTa
n[a + b*x])/(3*(1 + a^2)^3) - ArcTan[a + b*x]/(3*x^3) - ((1 - 3*a^2)*b^3*Lo
g[x])/(3*(1 + a^2)^3) + ((1 - 3*a^2)*b^3*Log[1 + (a + b*x)^2])/(6*(1 + a^2)
^3)
```


Rule 5045

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[((e + f*x)^(m + 1)*(a + b*ArcTan[c + d*x])^p)/(f*(m +
 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcTan[c
 + d*x])^(p - 1))/(1 + (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
```

```
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{x^4}\,dx = -\frac{\tan^{-1}(a+bx)}{3x^3} + \frac{1}{3}b\int \frac{1}{x^3\left(1+(a+bx)^2\right)}\,dx$$

$$= -\frac{\tan^{-1}(a+bx)}{3x^3} + \frac{1}{3}b^3\,\mathrm{Subst}\left(\int \frac{1}{(-a+x)^3\left(1+x^2\right)}\,dx, x, a+bx\right)$$

$$= -\frac{b}{6\left(1+a^2\right)x^2} - \frac{\tan^{-1}(a+bx)}{3x^3} + \frac{b^3\,\mathrm{Subst}\left(\int \frac{-a-x}{(-a+x)^2\left(1+x^2\right)}\,dx, x, a+bx\right)}{3\left(1+a^2\right)}$$

$$= -\frac{b}{6\left(1+a^2\right)x^2} - \frac{\tan^{-1}(a+bx)}{3x^3} + \frac{b^3\,\mathrm{Subst}\left(\int\left(-\frac{2a}{\left(1+a^2\right)(a-x)^2} + \frac{1-3a^2}{\left(1+a^2\right)^2(a-x)} + \frac{a\left(3-a^2\right)+\left(1-3a^2\right)x}{\left(1+a^2\right)^2\left(1+x^2\right)}\right) dx}{3\left(1+a^2\right)}$$

$$= -\frac{b}{6\left(1+a^2\right)x^2} + \frac{2ab^2}{3\left(1+a^2\right)^2 x} - \frac{\tan^{-1}(a+bx)}{3x^3} - \frac{\left(1-3a^2\right)b^3\log(x)}{3\left(1+a^2\right)^3} + \frac{b^3\,\mathrm{Subst}\left(\int \frac{a\left(3-a^2\right)+\left(1-3\right.}{1+x^2}\right.}{3\left(1+a^2\right.}$$

$$= -\frac{b}{6\left(1+a^2\right)x^2} + \frac{2ab^2}{3\left(1+a^2\right)^2 x} - \frac{\tan^{-1}(a+bx)}{3x^3} - \frac{\left(1-3a^2\right)b^3\log(x)}{3\left(1+a^2\right)^3} + \frac{\left(\left(1-3a^2\right)b^3\right)\mathrm{Subst}\left(\int\right.}{3\left(1+a^2\right.}$$

$$= -\frac{b}{6\left(1+a^2\right)x^2} + \frac{2ab^2}{3\left(1+a^2\right)^2 x} + \frac{a\left(3-a^2\right)b^3\tan^{-1}(a+bx)}{3\left(1+a^2\right)^3} - \frac{\tan^{-1}(a+bx)}{3x^3} - \frac{\left(1-3a^2\right)b^3\log(}{3\left(1+a^2\right)^3}$$

Mathematica [C] time = 0.13592, size = 128, normalized size = 0.99

$$\frac{2\left(3a^2-1\right)b^3x^3\log(x) - (a-i)bx\left((a+i)\left(a^2-4abx+1\right) + i(a-i)^2b^2x^2\log(a+bx+i)\right) - 2\left(a^2+1\right)^3\tan^{-1}(a+bx) +}{6\left(a^2+1\right)^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/x^4,x]
```

```
[Out] (-2*(1 + a^2)^3*ArcTan[a + b*x] + 2*(-1 + 3*a^2)*b^3*x^3*Log[x] + I*(I + a)
^3*b^3*x^3*Log[I - a - b*x] - (-I + a)*b*x*((I + a)*(1 + a^2 - 4*a*b*x) + I
*(-I + a)^2*b^2*x^2*Log[I + a + b*x]))/(6*(1 + a^2)^3*x^3)
```

Maple [A] time = 0.043, size = 162, normalized size = 1.3

$$-\frac{\arctan(bx+a)}{3x^3}-\frac{b^3\ln\left(1+(bx+a)^2\right)a^2}{2\left(a^2+1\right)^3}+\frac{b^3\ln\left(1+(bx+a)^2\right)}{6\left(a^2+1\right)^3}-\frac{b^3\arctan(bx+a)a^3}{3\left(a^2+1\right)^3}+\frac{b^3\arctan(bx+a)a}{\left(a^2+1\right)^3}-\frac{}{(6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/x^4,x)
```

```
[Out] -1/3*arctan(b*x+a)/x^3-1/2*b^3/(a^2+1)^3*ln(1+(b*x+a)^2)*a^2+1/6*b^3/(a^2+1
)^3*ln(1+(b*x+a)^2)-1/3*b^3/(a^2+1)^3*arctan(b*x+a)*a^3+b^3/(a^2+1)^3*arcta
n(b*x+a)*a-1/6*b/(a^2+1)/x^2+b^3/(a^2+1)^3*ln(b*x)*a^2-1/3*b^3/(a^2+1)^3*ln
(b*x)+2/3*a*b^2/(a^2+1)^2/x
```

Maxima [A] time = 1.54328, size = 223, normalized size = 1.73

$$-\frac{1}{6}\left(\frac{2\left(a^3-3a\right)b^2\arctan\left(\frac{b^2x+ab}{b}\right)}{a^6+3a^4+3a^2+1}+\frac{\left(3a^2-1\right)b^2\log\left(b^2x^2+2abx+a^2+1\right)}{a^6+3a^4+3a^2+1}-\frac{2\left(3a^2-1\right)b^2\log(x)}{a^6+3a^4+3a^2+1}-\frac{4abx-a^2-1}{\left(a^4+2a^2+1\right)x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(a^3 - 3*a)*b^2*arctan((b^2*x + a*b)/b)/(a^6 + 3*a^4 + 3*a^2 + 1) +
 (3*a^2 - 1)*b^2*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/(a^6 + 3*a^4 + 3*a^2 + 1)
 - 2*(3*a^2 - 1)*b^2*log(x)/(a^6 + 3*a^4 + 3*a^2 + 1) - (4*a*b*x - a^2 - 1)
/((a^4 + 2*a^2 + 1)*x^2))*b - 1/3*arctan(b*x + a)/x^3
```

Fricas [A] time = 1.65264, size = 321, normalized size = 2.49

$$-\frac{\left(3a^2-1\right)b^3x^3\log\left(b^2x^2+2abx+a^2+1\right)-2\left(3a^2-1\right)b^3x^3\log(x)-4\left(a^3+a\right)b^2x^2+\left(a^4+2a^2+1\right)bx+2\left(\left(a^3-3a\right)\right.}{6\left(a^6+3a^4+3a^2+1\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*((3*a^2 - 1)*b^3*x^3*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*(3*a^2 - 1)*
b^3*x^3*log(x) - 4*(a^3 + a)*b^2*x^2 + (a^4 + 2*a^2 + 1)*b*x + 2*((a^3 - 3*
a)*b^3*x^3 + a^6 + 3*a^4 + 3*a^2 + 1)*arctan(b*x + a))/((a^6 + 3*a^4 + 3*a^
2 + 1)*x^3)
```

Sympy [B] time = 26.9557, size = 1127, normalized size = 8.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/x**4,x)
```

```
[Out] Piecewise((3*I*b**4*x**4*atan(b*x - I)/(72*b*x**4 - 144*I*x**3) + 6*b**3*x*
*3*atan(b*x - I)/(72*b*x**4 - 144*I*x**3) + 3*I*b**3*x**3/(72*b*x**4 - 144*
I*x**3) + 3*b**2*x**2/(72*b*x**4 - 144*I*x**3) - 24*b*x*atan(b*x - I)/(72*b
*x**4 - 144*I*x**3) + 2*I*b*x/(72*b*x**4 - 144*I*x**3) + 48*I*atan(b*x - I)
/(72*b*x**4 - 144*I*x**3) - 8/(72*b*x**4 - 144*I*x**3), Eq(a, -I)), (-3*I*b
**4*x**4*atan(b*x + I)/(72*b*x**4 + 144*I*x**3) + 6*b**3*x**3*atan(b*x + I)
/(72*b*x**4 + 144*I*x**3) - 3*I*b**3*x**3/(72*b*x**4 + 144*I*x**3) + 3*b**2
*x**2/(72*b*x**4 + 144*I*x**3) - 24*b*x*atan(b*x + I)/(72*b*x**4 + 144*I*x*
*3) - 2*I*b*x/(72*b*x**4 + 144*I*x**3) - 48*I*atan(b*x + I)/(72*b*x**4 + 14
4*I*x**3) - 8/(72*b*x**4 + 144*I*x**3), Eq(a, I)), (-2*a**6*atan(a + b*x)/(
6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - a**4*b*x/(6*a**6*x**3
 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 6*a**4*atan(a + b*x)/(6*a**6*x**
3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 2*a**3*b**3*x**3*atan(a + b*x)/
(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) + 4*a**3*b**2*x**2/(6*
a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) + 6*a**2*b**3*x**3*log(x)
/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 3*a**2*b**3*x**3*lo
g(a**2 + 2*a*b*x + b**2*x**2 + 1)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**
3 + 6*x**3) - 2*a**2*b**3*x**3/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 +
 6*x**3) - 2*a**2*b*x/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3)
- 6*a**2*atan(a + b*x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3)
 + 6*a*b**3*x**3*atan(a + b*x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 +
 6*x**3) + 4*a*b**2*x**2/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**
3) - 2*b**3*x**3*log(x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3
) + b**3*x**3*log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(6*a**6*x**3 + 18*a**4*x*
*3 + 18*a**2*x**3 + 6*x**3) - 2*b**3*x**3/(6*a**6*x**3 + 18*a**4*x**3 + 18*
a**2*x**3 + 6*x**3) - b*x/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x*
*3) - 2*atan(a + b*x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3),
 True))
```

Giac [A] time = 1.1008, size = 239, normalized size = 1.85

$$-\frac{1}{6}b\left(\frac{(3a^2b^2-b^2)\log(b^2x^2+2abx+a^2+1)}{a^6+3a^4+3a^2+1}-\frac{2(3a^2b^2-b^2)\log(|x|)}{a^6+3a^4+3a^2+1}+\frac{2(a^3b^3-3ab^3)\arctan(bx+a)}{(a^6+3a^4+3a^2+1)b}+\frac{a^4+2a^2-\cdots}{(\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] -1/6*b*((3*a^2*b^2 - b^2)*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/(a^6 + 3*a^4 + 3
*a^2 + 1) - 2*(3*a^2*b^2 - b^2)*log(abs(x))/(a^6 + 3*a^4 + 3*a^2 + 1) + 2*(
a^3*b^3 - 3*a*b^3)*arctan(b*x + a)/((a^6 + 3*a^4 + 3*a^2 + 1)*b) + (a^4 + 2
*a^2 - 4*(a^3*b + a*b)*x + 1)/((a^2 + 1)^3*x^2)) - 1/3*arctan(b*x + a)/x^3
```

3.52 $\int \frac{\tan^{-1}(a+bx)}{c+dx^3}\,dx$

Optimal. Leaf size=863

$$-\frac{i\log(ia+ibx+1)\log\left(\frac{b\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\sqrt[3]{d}(i-a)+b\sqrt[3]{c}}\right)}{6c^{2/3}\sqrt[3]{d}}+\frac{i\log(-ia-ibx+1)\log\left(\frac{b\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{b\sqrt[3]{c}-(a+i)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}}+\frac{\sqrt[6]{-1}\log(ia+ibx+1)\log\left(\frac{b\left(\sqrt[3]{c}-\sqrt[3]{-1}\sqrt[3]{d}\right.}{b\sqrt[3]{c}-\sqrt[3]{-1}(i-a)}\right.}{6c^{2/3}\sqrt[3]{d}}$$

```
[Out] ((-I/6)*Log[1 + I*a + I*b*x]*Log[(b*(c^(1/3) + d^(1/3)*x))/(b*c^(1/3) + (I
- a)*d^(1/3))])/(c^(2/3)*d^(1/3)) + ((I/6)*Log[1 - I*a - I*b*x]*Log[(b*(c^(
1/3) + d^(1/3)*x))/(b*c^(1/3) - (I + a)*d^(1/3))])/(c^(2/3)*d^(1/3)) + ((-1
)^(1/6)*Log[1 + I*a + I*b*x]*Log[(b*(c^(1/3) - (-1)^(1/3)*d^(1/3)*x))/(b*c^
(1/3) - (-1)^(1/3)*(I - a)*d^(1/3))])/(6*c^(2/3)*d^(1/3)) - ((-1)^(1/6)*Log
[1 - I*a - I*b*x]*Log[(b*(c^(1/3) - (-1)^(1/3)*d^(1/3)*x))/(b*c^(1/3) + (-1
)^(1/3)*(I + a)*d^(1/3))])/(6*c^(2/3)*d^(1/3)) + ((-1)^(5/6)*Log[1 + I*a +
I*b*x]*Log[(b*(c^(1/3) + (-1)^(2/3)*d^(1/3)*x))/(b*c^(1/3) + (-1)^(2/3)*(I
- a)*d^(1/3))])/(6*c^(2/3)*d^(1/3)) - ((-1)^(5/6)*Log[1 - I*a - I*b*x]*Log[
(b*(c^(1/3) + (-1)^(2/3)*d^(1/3)*x))/(b*c^(1/3) + (-1)^(1/6)*(1 - I*a)*d^(1
/3))])/(6*c^(2/3)*d^(1/3)) - ((I/6)*PolyLog[2, (d^(1/3)*(I - a - b*x))/(b*c
^(1/3) + (I - a)*d^(1/3))])/(c^(2/3)*d^(1/3)) + ((-1)^(5/6)*PolyLog[2, -(((
-1)^(1/6)*d^(1/3)*(I - a - b*x))/(I*b*c^(1/3) - (-1)^(1/6)*(I - a)*d^(1/3))
)])/(6*c^(2/3)*d^(1/3)) + ((-1)^(1/6)*PolyLog[2, -(((-1)^(1/3)*d^(1/3)*(I -
 a - b*x))/(b*c^(1/3) - (-1)^(1/3)*(I - a)*d^(1/3)))])/(6*c^(2/3)*d^(1/3))
 + ((I/6)*PolyLog[2, -((d^(1/3)*(I + a + b*x))/(b*c^(1/3) - (I + a)*d^(1/3))
)])/(c^(2/3)*d^(1/3)) - ((-1)^(1/6)*PolyLog[2, ((-1)^(1/3)*d^(1/3)*(I + a +
 b*x))/(b*c^(1/3) + (-1)^(1/3)*(I + a)*d^(1/3))])/(6*c^(2/3)*d^(1/3)) - ((-
1)^(5/6)*PolyLog[2, -(((-1)^(2/3)*d^(1/3)*(I + a + b*x))/(b*c^(1/3) - (-1)^
(2/3)*(I + a)*d^(1/3)))])/(6*c^(2/3)*d^(1/3))
```

Rubi [A] time = 1.20694, antiderivative size = 863, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5051, 2409, 2394, 2393, 2391}

$$-\frac{i\log(ia+ibx+1)\log\left(\frac{b\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\sqrt[3]{d}(i-a)+b\sqrt[3]{c}}\right)}{6c^{2/3}\sqrt[3]{d}}+\frac{i\log(-ia-ibx+1)\log\left(\frac{b\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{b\sqrt[3]{c}-(a+i)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}}+\frac{\sqrt[6]{-1}\log(ia+ibx+1)\log\left(\frac{b\left(\sqrt[3]{c}-\sqrt[3]{-1}\sqrt[3]{d}\right.}{b\sqrt[3]{c}-\sqrt[3]{-1}(i-a)}\right.}{6c^{2/3}\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/(c + d*x^3),x]
```

```
[Out] ((-I/6)*Log[1 + I*a + I*b*x]*Log[(b*(c^(1/3) + d^(1/3)*x))/(b*c^(1/3) + (I
- a)*d^(1/3))]])/(c^(2/3)*d^(1/3)) + ((I/6)*Log[1 - I*a - I*b*x]*Log[(b*(c^(
1/3) + d^(1/3)*x))/(b*c^(1/3) - (I + a)*d^(1/3))]])/(c^(2/3)*d^(1/3)) + ((-1
)^(1/6)*Log[1 + I*a + I*b*x]*Log[(b*(c^(1/3) - (-1)^(1/3)*d^(1/3)*x))/(b*c^
(1/3) - (-1)^(1/3)*(I - a)*d^(1/3))]])/(6*c^(2/3)*d^(1/3)) - ((-1)^(1/6)*Log
[1 - I*a - I*b*x]*Log[(b*(c^(1/3) - (-1)^(1/3)*d^(1/3)*x))/(b*c^(1/3) + (-1
)^(1/3)*(I + a)*d^(1/3))]])/(6*c^(2/3)*d^(1/3)) + ((-1)^(5/6)*Log[1 + I*a +
I*b*x]*Log[(b*(c^(1/3) + (-1)^(2/3)*d^(1/3)*x))/(b*c^(1/3) + (-1)^(2/3)*(I
- a)*d^(1/3))]])/(6*c^(2/3)*d^(1/3)) - ((-1)^(5/6)*Log[1 - I*a - I*b*x]*Log[
(b*(c^(1/3) + (-1)^(2/3)*d^(1/3)*x))/(b*c^(1/3) + (-1)^(1/6)*(1 - I*a)*d^(1
/3))]])/(6*c^(2/3)*d^(1/3)) - ((I/6)*PolyLog[2, (d^(1/3)*(I - a - b*x))/(b*c
^(1/3) + (I - a)*d^(1/3))]])/(c^(2/3)*d^(1/3)) + ((-1)^(5/6)*PolyLog[2, -(((
-1)^(1/6)*d^(1/3)*(I - a - b*x))/(I*b*c^(1/3) - (-1)^(1/6)*(I - a)*d^(1/3))
)]])/(6*c^(2/3)*d^(1/3)) + ((-1)^(1/6)*PolyLog[2, -(((-1)^(1/3)*d^(1/3)*(I -
 a - b*x))/(b*c^(1/3) - (-1)^(1/3)*(I - a)*d^(1/3)))]])/(6*c^(2/3)*d^(1/3))
+ ((I/6)*PolyLog[2, -((d^(1/3)*(I + a + b*x))/(b*c^(1/3) - (I + a)*d^(1/3))
)]])/(c^(2/3)*d^(1/3)) - ((-1)^(1/6)*PolyLog[2, ((-1)^(1/3)*d^(1/3)*(I + a +
 b*x))/(b*c^(1/3) + (-1)^(1/3)*(I + a)*d^(1/3))]])/(6*c^(2/3)*d^(1/3)) - ((-
1)^(5/6)*PolyLog[2, -(((-1)^(2/3)*d^(1/3)*(I + a + b*x))/(b*c^(1/3) - (-1)^
(2/3)*(I + a)*d^(1/3)))]])/(6*c^(2/3)*d^(1/3))
```

Rule 5051

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{c+dx^3}\,dx = \frac{1}{2}i\int \frac{\log(1-ia-ibx)}{c+dx^3}\,dx - \frac{1}{2}i\int \frac{\log(1+ia+ibx)}{c+dx^3}\,dx$$

$$= \frac{1}{2}i\int \left(-\frac{\log(1-ia-ibx)}{3c^{2/3}\left(-\sqrt[3]{c}-\sqrt[3]{d}x\right)} - \frac{\log(1-ia-ibx)}{3c^{2/3}\left(-\sqrt[3]{c}+\sqrt[3]{-1}\sqrt[3]{d}x\right)} - \frac{\log(1-ia-ibx)}{3c^{2/3}\left(-\sqrt[3]{c}-(-1)^{2/3}\sqrt[3]{d}x\right)}\right)dx - \frac{1}{2}i\int \Bigg(\ldots$$

$$= -\frac{i\int \frac{\log(1-ia-ibx)}{-\sqrt[3]{c}-\sqrt[3]{d}x}\,dx}{6c^{2/3}} - \frac{i\int \frac{\log(1-ia-ibx)}{-\sqrt[3]{c}+\sqrt[3]{-1}\sqrt[3]{d}x}\,dx}{6c^{2/3}} - \frac{i\int \frac{\log(1-ia-ibx)}{-\sqrt[3]{c}-(-1)^{2/3}\sqrt[3]{d}x}\,dx}{6c^{2/3}} + \frac{i\int \frac{\log(1+ia+ibx)}{-\sqrt[3]{c}-\sqrt[3]{d}x}\,dx}{6c^{2/3}} + \frac{i\int \frac{\log(1\ldots}{-\sqrt[3]{c}+\ldots}}{6\ldots}$$

$$= -\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{b\sqrt[3]{c}+(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{b\sqrt[3]{c}-(i+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{\sqrt[6]{-1}\log(1+ia+\ldots}{\ldots}$$

$$= -\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{b\sqrt[3]{c}+(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{b\sqrt[3]{c}-(i+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{\sqrt[6]{-1}\log(1+ia+\ldots}{\ldots}$$

$$= -\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{b\sqrt[3]{c}+(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{b\sqrt[3]{c}-(i+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{\sqrt[6]{-1}\log(1+ia+\ldots}{\ldots}$$

Mathematica [A] time = 0.804645, size = 701, normalized size = 0.81

$$\frac{-i\text{PolyLog}\left(2,\frac{\sqrt[3]{d}(a+bx-i)}{-b\sqrt[3]{c}+(a-i)\sqrt[3]{d}}\right) + (-1)^{5/6}\text{PolyLog}\left(2,\frac{\sqrt[6]{-1}\sqrt[3]{d}(a+bx-i)}{\sqrt[6]{-1}(a-i)\sqrt[3]{d}+ib\sqrt[3]{c}}\right) + \sqrt[6]{-1}\text{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{d}(a+bx-i)}{b\sqrt[3]{c}+\sqrt[3]{-1}(a-i)\sqrt[3]{d}}\right) + i\text{PolyLog}\Big(2\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/(c + d*x^3),x]
```

```
[Out] ((-I)*Log[1 + I*a + I*b*x]*Log[(b*(c^(1/3) + d^(1/3)*x))/(b*c^(1/3) - (-I +
 a)*d^(1/3))] + I*Log[(-I)*(I + a + b*x)]*Log[(b*(c^(1/3) + d^(1/3)*x))/(b*
c^(1/3) - (I + a)*d^(1/3))] + (-1)^(1/6)*Log[1 + I*a + I*b*x]*Log[(b*(c^(1/
3) - (-1)^(1/3)*d^(1/3)*x))/(b*c^(1/3) + (-1)^(1/3)*(-I + a)*d^(1/3))] - (-
1)^(1/6)*Log[(-I)*(I + a + b*x)]*Log[(b*(c^(1/3) - (-1)^(1/3)*d^(1/3)*x))/(
b*c^(1/3) + (-1)^(1/3)*(I + a)*d^(1/3))] - (-1)^(5/6)*Log[(-I)*(I + a + b*x
)]*Log[(b*(c^(1/3) + (-1)^(2/3)*d^(1/3)*x))/(b*c^(1/3) + (-1)^(1/6)*(1 - I*
a)*d^(1/3))] + (-1)^(5/6)*Log[1 + I*a + I*b*x]*Log[(b*(c^(1/3) + (-1)^(2/3)
*d^(1/3)*x))/(b*c^(1/3) - (-1)^(2/3)*(-I + a)*d^(1/3))] - I*PolyLog[2, (d^(
1/3)*(-I + a + b*x))/(-(b*c^(1/3)) + (-I + a)*d^(1/3))] + (-1)^(5/6)*PolyLo
g[2, ((-1)^(1/6)*d^(1/3)*(-I + a + b*x))/(I*b*c^(1/3) + (-1)^(1/6)*(-I + a)
*d^(1/3))] + (-1)^(1/6)*PolyLog[2, ((-1)^(1/3)*d^(1/3)*(-I + a + b*x))/(b*c
^(1/3) + (-1)^(1/3)*(-I + a)*d^(1/3))] + I*PolyLog[2, (d^(1/3)*(I + a + b*x
))/(-(b*c^(1/3)) + (I + a)*d^(1/3))] - (-1)^(1/6)*PolyLog[2, ((-1)^(1/3)*d^
(1/3)*(I + a + b*x))/(b*c^(1/3) + (-1)^(1/3)*(I + a)*d^(1/3))] - (-1)^(5/6)
*PolyLog[2, ((-1)^(2/3)*d^(1/3)*(I + a + b*x))/(-(b*c^(1/3)) + (-1)^(2/3)*(
I + a)*d^(1/3))])/(6*c^(2/3)*d^(1/3))
```

Maple [C] time = 0.636, size = 631, normalized size = 0.7

$$\frac{2b^2}{3}\sum_{\_R1=\mathrm{RootOf}\left(\left(3ia^2d+a^3d-cb^3-id-3ad\right)\_Z^6+\left(3ia^2d+3a^3d-3cb^3+3id+3ad\right)\_Z^4+\left(-3ia^2d+3a^3d-3cb^3-3id+3ad\right)\_Z^2-3ia^2d+a^3d-cb^3+id-3ad\right)} \overline{a^3 d\_}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(d*x^3+c),x)
```

```
[Out] 2/3*b^2*sum(1/(a^3*d*_R1^4+3*I*a^2*d*_R1^4-b^3*c*_R1^4-3*a*d*_R1^4-I*d*_R1^
4+2*a^3*d*_R1^2+2*I*a^2*d*_R1^2-2*b^3*c*_R1^2+2*_R1^2*a*d+2*I*d*_R1^2+a^3*d
-I*a^2*d-c*b^3+a*d-I*d)*(I*arctan(b*x+a)*ln((_R1-(1+I*(b*x+a))/(1+(b*x+a)^2
)^(1/2))/_R1)+dilog((_R1-(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))/_R1)),_R1=RootO
f((3*I*a^2*d+a^3*d-c*b^3-I*d-3*a*d)*_Z^6+(3*I*a^2*d+3*a^3*d-3*c*b^3+3*I*d+3
*a*d)*_Z^4+(-3*I*a^2*d+3*a^3*d-3*c*b^3-3*I*d+3*a*d)*_Z^2-3*I*a^2*d+a^3*d-c*
b^3+I*d-3*a*d))+2/3*b^2*sum(_R1^2/(a^3*d*_R1^4+3*I*a^2*d*_R1^4-b^3*c*_R1^4-
3*a*d*_R1^4-I*d*_R1^4+2*a^3*d*_R1^2+2*I*a^2*d*_R1^2-2*b^3*c*_R1^2+2*_R1^2*a
*d+2*I*d*_R1^2+a^3*d-I*a^2*d-c*b^3+a*d-I*d)*(I*arctan(b*x+a)*ln((_R1-(1+I*(
b*x+a))/(1+(b*x+a)^2)^(1/2))/_R1)+dilog((_R1-(1+I*(b*x+a))/(1+(b*x+a)^2)^(1
/2))/_R1)),_R1=RootOf((3*I*a^2*d+a^3*d-c*b^3-I*d-3*a*d)*_Z^6+(3*I*a^2*d+3*a
^3*d-3*c*b^3+3*I*d+3*a*d)*_Z^4+(-3*I*a^2*d+3*a^3*d-3*c*b^3-3*I*d+3*a*d)*_Z^
2-3*I*a^2*d+a^3*d-c*b^3+I*d-3*a*d))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan\left(bx+a\right)}{dx^3+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] integral(arctan(b*x + a)/(d*x^3 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left(bx+a\right)}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(arctan(b*x + a)/(d*x^3 + c), x)
```

3.53 $\int \frac{\tan^{-1}(a+bx)}{c+dx^2}\,dx$

Optimal. Leaf size=543

$$-\frac{i\text{PolyLog}\left(2,-\frac{\sqrt{d}(-a-bx+i)}{b\sqrt{-c}-(-a+i)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{i\text{PolyLog}\left(2,\frac{\sqrt{d}(-a-bx+i)}{b\sqrt{-c}+(-a+i)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}-\frac{i\text{PolyLog}\left(2,-\frac{\sqrt{d}(a+bx+i)}{b\sqrt{-c}-(a+i)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{i\text{PolyLog}\left(2,\frac{\sqrt{d}(a+bx\cdots}{b\sqrt{-c}+(a+i\cdots}\right.}{4\sqrt{-c}\sqrt{d}}$$

```
[Out] ((-I/4)*Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] - (
I - a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d]) + ((I/4)*Log[1 - I*a - I*b*x]*Log[(b*(
Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] + (I + a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d])
+ ((I/4)*Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] +
(I - a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d]) - ((I/4)*Log[1 - I*a - I*b*x]*Log[(b*
(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] - (I + a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d])
 - ((I/4)*PolyLog[2, -((Sqrt[d]*(I - a - b*x))/(b*Sqrt[-c] - (I - a)*Sqrt[d
]))])/(Sqrt[-c]*Sqrt[d]) + ((I/4)*PolyLog[2, (Sqrt[d]*(I - a - b*x))/(b*Sqr
t[-c] + (I - a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d]) - ((I/4)*PolyLog[2, -((Sqrt[d
]*(I + a + b*x))/(b*Sqrt[-c] - (I + a)*Sqrt[d]))])/(Sqrt[-c]*Sqrt[d]) + ((I
/4)*PolyLog[2, (Sqrt[d]*(I + a + b*x))/(b*Sqrt[-c] + (I + a)*Sqrt[d])])/(Sq
rt[-c]*Sqrt[d])
```

Rubi [A] time = 0.607336, antiderivative size = 543, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5051, 2409, 2394, 2393, 2391}

$$-\frac{i\text{PolyLog}\left(2,-\frac{\sqrt{d}(-a-bx+i)}{b\sqrt{-c}-(-a+i)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{i\text{PolyLog}\left(2,\frac{\sqrt{d}(-a-bx+i)}{b\sqrt{-c}+(-a+i)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}-\frac{i\text{PolyLog}\left(2,-\frac{\sqrt{d}(a+bx+i)}{b\sqrt{-c}-(a+i)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{i\text{PolyLog}\left(2,\frac{\sqrt{d}(a+bx\cdots}{b\sqrt{-c}+(a+i\cdots}\right.}{4\sqrt{-c}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/(c + d*x^2),x]
```

```
[Out] ((-I/4)*Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] - (
I - a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d]) + ((I/4)*Log[1 - I*a - I*b*x]*Log[(b*(
Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] + (I + a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d])
+ ((I/4)*Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] +
(I - a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d]) - ((I/4)*Log[1 - I*a - I*b*x]*Log[(b*
(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] - (I + a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d])
 - ((I/4)*PolyLog[2, -((Sqrt[d]*(I - a - b*x))/(b*Sqrt[-c] - (I - a)*Sqrt[d
]))])/(Sqrt[-c]*Sqrt[d]) + ((I/4)*PolyLog[2, (Sqrt[d]*(I - a - b*x))/(b*Sqr
```

```
t[-c] + (I - a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d]) - ((I/4)*PolyLog[2, -((Sqrt[d
]*(I + a + b*x))/(b*Sqrt[-c] - (I + a)*Sqrt[d]))])/(Sqrt[-c]*Sqrt[d]) + ((I
/4)*PolyLog[2, (Sqrt[d]*(I + a + b*x))/(b*Sqrt[-c] + (I + a)*Sqrt[d])])/(Sq
rt[-c]*Sqrt[d])
```

Rule 5051

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{c+dx^2}\,dx = \frac{1}{2}i\int \frac{\log(1-ia-ibx)}{c+dx^2}\,dx - \frac{1}{2}i\int \frac{\log(1+ia+ibx)}{c+dx^2}\,dx$$

$$= \frac{1}{2}i\int \left(\frac{\sqrt{-c}\log(1-ia-ibx)}{2c\left(\sqrt{-c}-\sqrt{d}x\right)} + \frac{\sqrt{-c}\log(1-ia-ibx)}{2c\left(\sqrt{-c}+\sqrt{d}x\right)}\right)dx - \frac{1}{2}i\int \left(\frac{\sqrt{-c}\log(1+ia+ibx)}{2c\left(\sqrt{-c}-\sqrt{d}x\right)} + \frac{\sqrt{-c}\ldots}{2\ldots}\right.$$

$$= -\frac{i\int \frac{\log(1-ia-ibx)}{\sqrt{-c}-\sqrt{d}x}\,dx}{4\sqrt{-c}} - \frac{i\int \frac{\log(1-ia-ibx)}{\sqrt{-c}+\sqrt{d}x}\,dx}{4\sqrt{-c}} + \frac{i\int \frac{\log(1+ia+ibx)}{\sqrt{-c}-\sqrt{d}x}\,dx}{4\sqrt{-c}} + \frac{i\int \frac{\log(1+ia+ibx)}{\sqrt{-c}+\sqrt{d}x}\,dx}{4\sqrt{-c}}$$

$$= -\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}x\right)}{b\sqrt{-c}-(i-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}x\right)}{b\sqrt{-c}+(i+a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{i\log(1+ia+ib\ldots}{4\ldots}$$

$$= -\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}x\right)}{b\sqrt{-c}-(i-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}x\right)}{b\sqrt{-c}+(i+a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{i\log(1+ia+ib\ldots}{4\ldots}$$

$$= -\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}x\right)}{b\sqrt{-c}-(i-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}x\right)}{b\sqrt{-c}+(i+a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{i\log(1+ia+ib\ldots}{4\ldots}$$

Mathematica [A] time = 0.329175, size = 409, normalized size = 0.75

$$-\frac{i\left(-\text{PolyLog}\left(2,\frac{\sqrt{d}(a+bx-i)}{-b\sqrt{-c}+(a-i)\sqrt{d}}\right)+\text{PolyLog}\left(2,\frac{\sqrt{d}(a+bx-i)}{b\sqrt{-c}+(a-i)\sqrt{d}}\right)+\text{PolyLog}\left(2,\frac{\sqrt{d}(a+bx+i)}{-b\sqrt{-c}+(a+i)\sqrt{d}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{d}(a+bx+i)}{b\sqrt{-c}+(a+i)\sqrt{d}}\right)\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/(c + d*x^2),x]
```

```
[Out] ((-I/4)*(Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] +
(-I + a)*Sqrt[d])] - Log[(-I)*(I + a + b*x)]*Log[(b*(Sqrt[-c] - Sqrt[d]*x))
/(b*Sqrt[-c] + (I + a)*Sqrt[d])] - Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[-c] +
Sqrt[d]*x))/(b*Sqrt[-c] - (-I + a)*Sqrt[d])] + Log[(-I)*(I + a + b*x)]*Log[
(b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] - (I + a)*Sqrt[d])] - PolyLog[2, (Sq
rt[d]*(-I + a + b*x))/(-(b*Sqrt[-c]) + (-I + a)*Sqrt[d])] + PolyLog[2, (Sqr
t[d]*(-I + a + b*x))/(b*Sqrt[-c] + (-I + a)*Sqrt[d])] + PolyLog[2, (Sqrt[d]
*(I + a + b*x))/(-(b*Sqrt[-c]) + (I + a)*Sqrt[d])] - PolyLog[2, (Sqrt[d]*(I
 + a + b*x))/(b*Sqrt[-c] + (I + a)*Sqrt[d])]))/(Sqrt[-c]*Sqrt[d])
```

Maple [B] time = 0.713, size = 2192, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(d*x^2+c),x)
```

```
[Out] 1/2*I/b*(b^2*c*d)^(1/2)/c/(a^2*d+c*b^2+2*(b^2*c*d)^(1/2)+d)*ln(1-(2*I*a*d+a
^2*d+c*b^2-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-c*b^2-2*(b^2*c*d)^(1/2)
-d))*arctan(b*x+a)-1/2*I/b*(b^2*c*d)^(1/2)/c/(a^2*d+c*b^2-2*(b^2*c*d)^(1/2)
+d)*ln(1-(2*I*a*d+a^2*d+c*b^2-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-c*b^
2+2*(b^2*c*d)^(1/2)-d))*arctan(b*x+a)+I*b/(a^2*d+c*b^2-2*(b^2*c*d)^(1/2)+d)
*ln(1-(2*I*a*d+a^2*d+c*b^2-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-c*b^2+2
*(b^2*c*d)^(1/2)-d))*arctan(b*x+a)+I*b/(a^2*d+c*b^2+2*(b^2*c*d)^(1/2)+d)*ln
(1-(2*I*a*d+a^2*d+c*b^2-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-c*b^2-2*(b
^2*c*d)^(1/2)-d))*arctan(b*x+a)-1/2*b/d*(b^2*c*d)^(1/2)/(a^2*d+c*b^2-2*(b^2
*c*d)^(1/2)+d)*arctan(b*x+a)^2+b/(a^2*d+c*b^2-2*(b^2*c*d)^(1/2)+d)*arctan(b
*x+a)^2-1/2/b*(b^2*c*d)^(1/2)/c/(a^2*d+c*b^2-2*(b^2*c*d)^(1/2)+d)*arctan(b*
x+a)^2-1/2/b*(b^2*c*d)^(1/2)/c/(a^2*d+c*b^2-2*(b^2*c*d)^(1/2)+d)*arctan(b*x
+a)^2*a^2-1/4*b/d*(b^2*c*d)^(1/2)/(a^2*d+c*b^2-2*(b^2*c*d)^(1/2)+d)*polylog
(2,(2*I*a*d+a^2*d+c*b^2-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-c*b^2+2*(b
^2*c*d)^(1/2)-d))+1/2*b/(a^2*d+c*b^2-2*(b^2*c*d)^(1/2)+d)*polylog(2,(2*I*a*
d+a^2*d+c*b^2-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-c*b^2+2*(b^2*c*d)^(1
/2)-d))-1/4/b*(b^2*c*d)^(1/2)/c/(a^2*d+c*b^2-2*(b^2*c*d)^(1/2)+d)*polylog(2
,(2*I*a*d+a^2*d+c*b^2-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-c*b^2+2*(b^2
*c*d)^(1/2)-d))-1/4/b*(b^2*c*d)^(1/2)/c/(a^2*d+c*b^2-2*(b^2*c*d)^(1/2)+d)*p
olylog(2,(2*I*a*d+a^2*d+c*b^2-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-c*b^
2+2*(b^2*c*d)^(1/2)-d))*a^2+1/2*I/b*(b^2*c*d)^(1/2)/c/(a^2*d+c*b^2+2*(b^2*c
*d)^(1/2)+d)*ln(1-(2*I*a*d+a^2*d+c*b^2-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a
^2*d-c*b^2-2*(b^2*c*d)^(1/2)-d))*arctan(b*x+a)*a^2+1/2*I*b/d*(b^2*c*d)^(1/2
)/(a^2*d+c*b^2+2*(b^2*c*d)^(1/2)+d)*ln(1-(2*I*a*d+a^2*d+c*b^2-d)*(1+I*(b*x+
a))^2/(1+(b*x+a)^2)/(-a^2*d-c*b^2-2*(b^2*c*d)^(1/2)-d))*arctan(b*x+a)-1/2*I
/b*(b^2*c*d)^(1/2)/c/(a^2*d+c*b^2-2*(b^2*c*d)^(1/2)+d)*ln(1-(2*I*a*d+a^2*d+
c*b^2-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-c*b^2+2*(b^2*c*d)^(1/2)-d))*
arctan(b*x+a)*a^2-1/2*I*b/d*(b^2*c*d)^(1/2)/(a^2*d+c*b^2-2*(b^2*c*d)^(1/2)+
d)*ln(1-(2*I*a*d+a^2*d+c*b^2-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-c*b^2
+2*(b^2*c*d)^(1/2)-d))*arctan(b*x+a)+1/2*b/d*(b^2*c*d)^(1/2)/(a^2*d+c*b^2+2
*(b^2*c*d)^(1/2)+d)*arctan(b*x+a)^2+b/(a^2*d+c*b^2+2*(b^2*c*d)^(1/2)+d)*arc
tan(b*x+a)^2+1/2/b*(b^2*c*d)^(1/2)/c/(a^2*d+c*b^2+2*(b^2*c*d)^(1/2)+d)*arct
an(b*x+a)^2+1/2/b*(b^2*c*d)^(1/2)/c/(a^2*d+c*b^2+2*(b^2*c*d)^(1/2)+d)*arcta
n(b*x+a)^2*a^2+1/4*b/d*(b^2*c*d)^(1/2)/(a^2*d+c*b^2+2*(b^2*c*d)^(1/2)+d)*po
lylog(2,(2*I*a*d+a^2*d+c*b^2-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-c*b^2
```

```
-2*(b^2*c*d)^(1/2)-d))+1/2*b/(a^2*d+c*b^2+2*(b^2*c*d)^(1/2)+d)*polylog(2,(2
*I*a*d+a^2*d+c*b^2-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-c*b^2-2*(b^2*c*
d)^(1/2)-d))+1/4/b*(b^2*c*d)^(1/2)/c/(a^2*d+c*b^2+2*(b^2*c*d)^(1/2)+d)*poly
log(2,(2*I*a*d+a^2*d+c*b^2-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-c*b^2-2
*(b^2*c*d)^(1/2)-d))+1/4/b*(b^2*c*d)^(1/2)/c/(a^2*d+c*b^2+2*(b^2*c*d)^(1/2)
+d)*polylog(2,(2*I*a*d+a^2*d+c*b^2-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d
-c*b^2-2*(b^2*c*d)^(1/2)-d))*a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan\left(bx+a\right)}{dx^2+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arctan(b*x + a)/(d*x^2 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan (bx + a)}{dx^2 + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(arctan(b*x + a)/(d*x^2 + c), x)
```

3.54 $\int \frac{\tan^{-1}(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=152

$$-\frac{i\mathrm{PolyLog}\left(2,1-\frac{2b(c+dx)}{(1-i(a+bx))(-ad+bc+id)}\right)}{2d}+\frac{i\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)}{2d}+\frac{\tan^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(1-i(a+bx))(-ad+bc+id)}\right)}{d}-\frac{\log\left(\right.}{}$$

```
[Out] -((ArcTan[a + b*x]*Log[2/(1 - I*(a + b*x))])/d) + (ArcTan[a + b*x]*Log[(2*b
*(c + d*x))/((b*c + I*d - a*d)*(1 - I*(a + b*x)))])/d + ((I/2)*PolyLog[2, 1
 - 2/(1 - I*(a + b*x))])/d - ((I/2)*PolyLog[2, 1 - (2*b*(c + d*x))/((b*c +
I*d - a*d)*(1 - I*(a + b*x)))])/d
```

Rubi [A] time = 0.138446, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5047, 4856, 2402, 2315, 2447}

$$-\frac{i\mathrm{PolyLog}\left(2,1-\frac{2b(c+dx)}{(1-i(a+bx))(-ad+bc+id)}\right)}{2d}+\frac{i\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)}{2d}+\frac{\tan^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(1-i(a+bx))(-ad+bc+id)}\right)}{d}-\frac{\log\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/(c + d*x),x]
```

```
[Out] -((ArcTan[a + b*x]*Log[2/(1 - I*(a + b*x))])/d) + (ArcTan[a + b*x]*Log[(2*b
*(c + d*x))/((b*c + I*d - a*d)*(1 - I*(a + b*x)))])/d + ((I/2)*PolyLog[2, 1
 - 2/(1 - I*(a + b*x))])/d - ((I/2)*PolyLog[2, 1 - (2*b*(c + d*x))/((b*c +
I*d - a*d)*(1 - I*(a + b*x)))])/d
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
```

```
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{c+dx}\,dx = \frac{\text{Subst}\left(\int \frac{\tan^{-1}(x)}{\frac{bc-ad}{b}+\frac{dx}{b}}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\tan^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{d} + \frac{\tan^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{d} + \frac{\text{Subst}\left(\int \frac{\log\left(\frac{2}{1-ix}\right)}{1+x^2}\,dx, x\right.}{d}$$

$$= -\frac{\tan^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{d} + \frac{\tan^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{d} - \frac{i\text{Li}_2\left(1-\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+\right.}}{2d}$$

$$= -\frac{\tan^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{d} + \frac{\tan^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{d} + \frac{i\text{Li}_2\left(1-\frac{2}{1-i(a+bx)}\right)}{2d} - \frac{i\text{I}}{}$$

Mathematica [A] time = 0.0195464, size = 231, normalized size = 1.52

$$\frac{i\text{PolyLog}\left(2,-\frac{id(1-i(a+bx))}{-ad+bc-id}\right)}{2d}-\frac{i\text{PolyLog}\left(2,\frac{id(1+i(a+bx))}{-ad+bc+id}\right)}{2d}+\frac{i\log(1-i(a+bx))\log\left(-\frac{i\left(\frac{bc-ad}{b}+\frac{d(a+bx)}{b}\right)}{-\frac{d}{b}-\frac{i(bc-ad)}{b}}\right)}{2d}-\frac{i\log(1+i(a+bx$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a + b*x]/(c + d*x),x]
```

```
[Out] ((I/2)*Log[1 - I*(a + b*x)]*Log[((-I)*((b*c - a*d)/b + (d*(a + b*x))/b))/(-
(d/b) - (I*(b*c - a*d))/b)])/d - ((I/2)*Log[1 + I*(a + b*x)]*Log[(I*((b*c -
 a*d)/b + (d*(a + b*x))/b))/(-(d/b) + (I*(b*c - a*d))/b)])/d + ((I/2)*PolyL
og[2, ((-I)*d*(1 - I*(a + b*x)))/(b*c - I*d - a*d)])/d - ((I/2)*PolyLog[2,
(I*d*(1 + I*(a + b*x)))/(b*c + I*d - a*d)])/d
```

Maple [A] time = 0.054, size = 198, normalized size = 1.3

$$\frac{\ln(d(bx+a)-ad+bc)\arctan(bx+a)}{d}+\frac{\frac{i}{2}\ln(d(bx+a)-ad+bc)}{d}\ln\left(\frac{id-d(bx+a)}{bc+id-ad}\right)-\frac{\frac{i}{2}\ln(d(bx+a)-ad+bc)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(d*x+c),x)
```

```
[Out] ln(d*(b*x+a)-a*d+b*c)/d*arctan(b*x+a)+1/2*I*ln(d*(b*x+a)-a*d+b*c)/d*ln((I*d
-d*(b*x+a))/(b*c+I*d-a*d))-1/2*I*ln(d*(b*x+a)-a*d+b*c)/d*ln((I*d+d*(b*x+a))
/(I*d+a*d-b*c))+1/2*I/d*dilog((I*d-d*(b*x+a))/(b*c+I*d-a*d))-1/2*I/d*dilog(
(I*d+d*(b*x+a))/(I*d+a*d-b*c))
```

Maxima [B] time = 1.97429, size = 383, normalized size = 2.52

$$\frac{\arctan(bx+a)\log(dx+c)}{d}-\frac{\arctan\left(\frac{b^2x+ab}{b}\right)\log(dx+c)}{d}-\frac{\arctan\left(\frac{bd^2x+bcd}{b^2c^2-2\,abcd+(a^2+1)d^2},\frac{b^2c^2-abcd+(b^2cd-abd^2)x}{b^2c^2-2\,abcd+(a^2+1)d^2}\right)\log\left(b^2x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] arctan(b*x + a)*log(d*x + c)/d - arctan((b^2*x + a*b)/b)*log(d*x + c)/d - 1
/2*(arctan2((b*d^2*x + b*c*d)/(b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2), (b^2*c
^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x)/(b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)
)*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - arctan(b*x + a)*log((b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2)/(b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)) + I*dilog((I*b*d
*x + (I*a + 1)*d)/(-I*b*c + (I*a + 1)*d)) - I*dilog((I*b*d*x + (I*a - 1)*d)
/(-I*b*c + (I*a - 1)*d)))/d
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan\left(bx+a\right)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(arctan(b*x + a)/(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(atan(a + b*x)/(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left(bx+a\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(arctan(b*x + a)/(d*x + c), x)
```

3.55 $\int \frac{\tan^{-1}(a+bx)}{c+\frac{d}{x}}\,dx$

Optimal. Leaf size=244

$$\frac{id\text{PolyLog}\left(2,\frac{c(-a-bx+i)}{-ac+bd+ic}\right)}{2c^2}-\frac{id\text{PolyLog}\left(2,\frac{c(a+bx+i)}{-bd+(a+i)c}\right)}{2c^2}+\frac{id\log(ia+ibx+1)\log\left(\frac{b(cx+d)}{bd+(-a+i)c}\right)}{2c^2}-\frac{id\log(-ia-ibx+1)\log\left(\cdots\right.}{2c^2}$$

```
[Out] -((1 + I*a + I*b*x)*Log[1 + I*a + I*b*x])/(2*b*c) - ((1 - I*a - I*b*x)*Log[
(-I)*(I + a + b*x)])/(2*b*c) - ((I/2)*d*Log[1 - I*a - I*b*x]*Log[-((b*(d +
c*x))/((I + a)*c - b*d))])/c^2 + ((I/2)*d*Log[1 + I*a + I*b*x]*Log[(b*(d +
c*x))/((I - a)*c + b*d)])/c^2 + ((I/2)*d*PolyLog[2, (c*(I - a - b*x))/(I*c
- a*c + b*d)])/c^2 - ((I/2)*d*PolyLog[2, (c*(I + a + b*x))/((I + a)*c - b*d
)])/c^2
```

Rubi [A] time = 0.239131, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5051, 2409, 2389, 2295, 2394, 2393, 2391}

$$\frac{id\text{PolyLog}\left(2,\frac{c(-a-bx+i)}{-ac+bd+ic}\right)}{2c^2}-\frac{id\text{PolyLog}\left(2,\frac{c(a+bx+i)}{-bd+(a+i)c}\right)}{2c^2}+\frac{id\log(ia+ibx+1)\log\left(\frac{b(cx+d)}{bd+(-a+i)c}\right)}{2c^2}-\frac{id\log(-ia-ibx+1)\log\left(\cdots\right.}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/(c + d/x),x]
```

```
[Out] -((1 + I*a + I*b*x)*Log[1 + I*a + I*b*x])/(2*b*c) - ((1 - I*a - I*b*x)*Log[
(-I)*(I + a + b*x)])/(2*b*c) - ((I/2)*d*Log[1 - I*a - I*b*x]*Log[-((b*(d +
c*x))/((I + a)*c - b*d))])/c^2 + ((I/2)*d*Log[1 + I*a + I*b*x]*Log[(b*(d +
c*x))/((I - a)*c + b*d)])/c^2 + ((I/2)*d*PolyLog[2, (c*(I - a - b*x))/(I*c
- a*c + b*d)])/c^2 - ((I/2)*d*PolyLog[2, (c*(I + a + b*x))/((I + a)*c - b*d
)])/c^2
```

Rule 5051

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{c+\frac{d}{x}}\,dx = \frac{1}{2}i\int \frac{\log(1-ia-ibx)}{c+\frac{d}{x}}\,dx - \frac{1}{2}i\int \frac{\log(1+ia+ibx)}{c+\frac{d}{x}}\,dx$$

$$= \frac{1}{2}i\int \left(\frac{\log(1-ia-ibx)}{c} - \frac{d\log(1-ia-ibx)}{c(d+cx)}\right)dx - \frac{1}{2}i\int \left(\frac{\log(1+ia+ibx)}{c} - \frac{d\log(1+ia+ibx)}{c(d+cx)}\right.$$

$$= \frac{i\int \log(1-ia-ibx)\,dx}{2c} - \frac{i\int \log(1+ia+ibx)\,dx}{2c} - \frac{(id)\int \frac{\log(1-ia-ibx)}{d+cx}\,dx}{2c} + \frac{(id)\int \frac{\log(1+ia+ibx)}{d+cx}\,dx}{2c}$$

$$= -\frac{id\log(1-ia-ibx)\log\left(-\frac{b(d+cx)}{(i+a)c-bd}\right)}{2c^2} + \frac{id\log(1+ia+ibx)\log\left(\frac{b(d+cx)}{(i-a)c+bd}\right)}{2c^2} - \frac{\text{Subst}(\int \log(x)\,dx, x,}{2bc}$$

$$= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} - \frac{id\log(1-ia-ibx)\log\left(-\frac{b}{(i}\right.}{2c^2}$$

$$= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} - \frac{id\log(1-ia-ibx)\log\left(-\frac{b}{(i}\right.}{2c^2}$$

Mathematica [B] time = 11.0746, size = 771, normalized size = 3.16

$$\frac{ibd(bd-ac)\text{PolyLog}\left(2,\exp\left(2i\left(\tan^{-1}(a+bx)-\tan^{-1}\left(a-\frac{bd}{c}\right)\right)\right)\right)+ibd(ac-bd)\text{PolyLog}\left(2,-e^{2i\tan^{-1}(a+bx)}\right)+bcd\sqrt{a}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a + b*x]/(c + d/x),x]
```

```
[Out] (-2*a^2*c^2*ArcTan[a + b*x] + 2*a*b*c*d*ArcTan[a + b*x] + I*a*b*c*d*Pi*ArcT
an[a + b*x] - I*b^2*d^2*Pi*ArcTan[a + b*x] - 2*a*b*c^2*x*ArcTan[a + b*x] +
2*b^2*c*d*x*ArcTan[a + b*x] + (2*I)*a*b*c*d*ArcTan[a - (b*d)/c]*ArcTan[a +
b*x] - (2*I)*b^2*d^2*ArcTan[a - (b*d)/c]*ArcTan[a + b*x] - b*c*d*ArcTan[a +
 b*x]^2 + I*a*b*c*d*ArcTan[a + b*x]^2 - I*b^2*d^2*ArcTan[a + b*x]^2 + (b*c*
d*Sqrt[1 + a^2 - (2*a*b*d)/c + (b^2*d^2)/c^2]*ArcTan[a + b*x]^2)/E^(I*ArcTa
n[a - (b*d)/c]) + a*b*c*d*Pi*Log[1 + E^((-2*I)*ArcTan[a + b*x])] - b^2*d^2*
Pi*Log[1 + E^((-2*I)*ArcTan[a + b*x])] - 2*a*b*c*d*ArcTan[a + b*x]*Log[1 +
E^((2*I)*ArcTan[a + b*x])] + 2*b^2*d^2*ArcTan[a + b*x]*Log[1 + E^((2*I)*Arc
Tan[a + b*x])] - 2*a*b*c*d*ArcTan[a - (b*d)/c]*Log[1 - E^((2*I)*(-ArcTan[a
- (b*d)/c] + ArcTan[a + b*x]))] + 2*b^2*d^2*ArcTan[a - (b*d)/c]*Log[1 - E^(
(2*I)*(-ArcTan[a - (b*d)/c] + ArcTan[a + b*x]))] + 2*a*b*c*d*ArcTan[a + b*x
]*Log[1 - E^((2*I)*(-ArcTan[a - (b*d)/c] + ArcTan[a + b*x]))] - 2*b^2*d^2*A
rcTan[a + b*x]*Log[1 - E^((2*I)*(-ArcTan[a - (b*d)/c] + ArcTan[a + b*x]))]
```

```
- 2*a*c^2*Log[1/Sqrt[1 + (a + b*x)^2]] + 2*b*c*d*Log[1/Sqrt[1 + (a + b*x)^2
]] - a*b*c*d*Pi*Log[1/Sqrt[1 + (a + b*x)^2]] + b^2*d^2*Pi*Log[1/Sqrt[1 + (a
 + b*x)^2]] + 2*a*b*c*d*ArcTan[a - (b*d)/c]*Log[-Sin[ArcTan[a - (b*d)/c] -
ArcTan[a + b*x]]] - 2*b^2*d^2*ArcTan[a - (b*d)/c]*Log[-Sin[ArcTan[a - (b*d)
/c] - ArcTan[a + b*x]]] + I*b*d*(a*c - b*d)*PolyLog[2, -E^((2*I)*ArcTan[a +
 b*x])] + I*b*d*(-(a*c) + b*d)*PolyLog[2, E^((2*I)*(-ArcTan[a - (b*d)/c] +
ArcTan[a + b*x]))])/(b*c^2*(-2*a*c + 2*b*d))
```

Maple [A] time = 0.06, size = 317, normalized size = 1.3

$$\frac{x\arctan(bx+a)}{c}+\frac{\arctan(bx+a)\,a}{bc}-\frac{\arctan(bx+a)\,d\ln(c(bx+a)-ac+bd)}{c^2}-\frac{\ln\left(a^2c^2-2\,abcd+b^2d^2+2\,(c(b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(c+1/x*d),x)
```

```
[Out] arctan(b*x+a)/c*x+1/b*arctan(b*x+a)/c*a-arctan(b*x+a)*d/c^2*ln(c*(b*x+a)-a*
c+b*d)-1/2/b/c*ln(a^2*c^2-2*a*b*c*d+b^2*d^2+2*(c*(b*x+a)-a*c+b*d)*a*c-2*(c*
(b*x+a)-a*c+b*d)*b*d+(c*(b*x+a)-a*c+b*d)^2+c^2)-1/2*I/c^2*d*ln(c*(b*x+a)-a*
c+b*d)*ln((I*c-c*(b*x+a))/(I*c-a*c+b*d))+1/2*I/c^2*d*ln(c*(b*x+a)-a*c+b*d)*
ln((I*c+c*(b*x+a))/(I*c+a*c-b*d))-1/2*I/c^2*d*dilog((I*c-c*(b*x+a))/(I*c-a*
c+b*d))+1/2*I/c^2*d*dilog((I*c+c*(b*x+a))/(I*c+a*c-b*d))
```

Maxima [A] time = 2.01758, size = 383, normalized size = 1.57

$$-\frac{bd\arctan(bx+a)\log\left(-\frac{b^2c^2x^2+2\,b^2cdx+b^2d^2}{2\,abcd-b^2d^2-(a^2+1)c^2}\right)+i\,bd\mathrm{Li}_2\left(-\frac{i\,bcx+(i\,a-1)c}{(-i\,a+1)c+i\,bd}\right)-i\,bd\mathrm{Li}_2\left(-\frac{i\,bcx+(i\,a+1)c}{(-i\,a-1)c+i\,bd}\right)-2\,(bcx+ac)\arctan(b}{2\,bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x),x, algorithm="maxima")
```

```
[Out] -1/2*(b*d*arctan(b*x + a)*log(-(b^2*c^2*x^2 + 2*b^2*c*d*x + b^2*d^2)/(2*a*b
*c*d - b^2*d^2 - (a^2 + 1)*c^2)) + I*b*d*dilog(-(I*b*c*x + (I*a - 1)*c)/((-
I*a + 1)*c + I*b*d)) - I*b*d*dilog(-(I*b*c*x + (I*a + 1)*c)/((-I*a - 1)*c +
 I*b*d)) - 2*(b*c*x + a*c)*arctan(b*x + a) - (b*d*arctan2(-(b*c^2*x + b*c*d
)/(2*a*b*c*d - b^2*d^2 - (a^2 + 1)*c^2), (a*b*c*d - b^2*d^2 + (a*b*c^2 - b^
```

```
2*c*d)*x)/(2*a*b*c*d - b^2*d^2 - (a^2 + 1)*c^2)) - c)*log(b^2*x^2 + 2*a*b*x
 + a^2 + 1))/(b*c^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x \arctan(bx + a)}{cx + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x),x, algorithm="fricas")
```

```
[Out] integral(x*arctan(b*x + a)/(c*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(c+d/x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(bx + a)}{c + \frac{d}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x),x, algorithm="giac")
```

```
[Out] integrate(arctan(b*x + a)/(c + d/x), x)
```

3.56 $\int \frac{\tan^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx$

Optimal. Leaf size=668

$$\frac{i\sqrt{d}\text{PolyLog}\left(2,\frac{\sqrt{-c}(-a-bx+i)}{a\left(-\sqrt{-c}\right)-b\sqrt{d}+i\sqrt{-c}}\right)}{4(-c)^{3/2}}-\frac{i\sqrt{d}\text{PolyLog}\left(2,\frac{\sqrt{-c}(ia+ibx+1)}{(1+ia)\sqrt{-c}-ib\sqrt{d}}\right)}{4(-c)^{3/2}}+\frac{i\sqrt{d}\text{PolyLog}\left(2,\frac{\sqrt{-c}(a+bx+i)}{a\sqrt{-c}-b\sqrt{d}+i\sqrt{-c}}\right)}{4(-c)^{3/2}}-\frac{i\sqrt{d}\text{PolyLo}}{}$$

```
[Out] -((1 + I*a + I*b*x)*Log[1 + I*a + I*b*x])/(2*b*c) - ((1 - I*a - I*b*x)*Log[
(-I)*(I + a + b*x)])/(2*b*c) + ((I/4)*Sqrt[d]*Log[1 + I*a + I*b*x]*Log[-((b
*(Sqrt[d] - Sqrt[-c]*x))/(I*Sqrt[-c] - a*Sqrt[-c] - b*Sqrt[d]))])/(-c)^(3/2
) - ((I/4)*Sqrt[d]*Log[1 - I*a - I*b*x]*Log[(b*(Sqrt[d] - Sqrt[-c]*x))/(I*S
qrt[-c] + a*Sqrt[-c] + b*Sqrt[d])])/(-c)^(3/2) + ((I/4)*Sqrt[d]*Log[1 - I*a
 - I*b*x]*Log[-((b*(Sqrt[d] + Sqrt[-c]*x))/((I + a)*Sqrt[-c] - b*Sqrt[d]))]
)/(-c)^(3/2) - ((I/4)*Sqrt[d]*Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[d] + Sqrt[-
c]*x))/(I*Sqrt[-c] - a*Sqrt[-c] + b*Sqrt[d])])/(-c)^(3/2) + ((I/4)*Sqrt[d]*
PolyLog[2, (Sqrt[-c]*(I - a - b*x))/(I*Sqrt[-c] - a*Sqrt[-c] - b*Sqrt[d])])
/(-c)^(3/2) - ((I/4)*Sqrt[d]*PolyLog[2, (Sqrt[-c]*(1 + I*a + I*b*x))/((1 +
I*a)*Sqrt[-c] - I*b*Sqrt[d])])/(-c)^(3/2) + ((I/4)*Sqrt[d]*PolyLog[2, (Sqrt
[-c]*(I + a + b*x))/(I*Sqrt[-c] + a*Sqrt[-c] - b*Sqrt[d])])/(-c)^(3/2) - ((
I/4)*Sqrt[d]*PolyLog[2, (Sqrt[-c]*(I + a + b*x))/(I*Sqrt[-c] + a*Sqrt[-c] +
 b*Sqrt[d])])/(-c)^(3/2)
```

Rubi [A] time = 0.854872, antiderivative size = 668, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5051, 2409, 2389, 2295, 2394, 2393, 2391}

$$\frac{i\sqrt{d}\text{PolyLog}\left(2,\frac{\sqrt{-c}(-a-bx+i)}{a\left(-\sqrt{-c}\right)-b\sqrt{d}+i\sqrt{-c}}\right)}{4(-c)^{3/2}}-\frac{i\sqrt{d}\text{PolyLog}\left(2,\frac{\sqrt{-c}(ia+ibx+1)}{(1+ia)\sqrt{-c}-ib\sqrt{d}}\right)}{4(-c)^{3/2}}+\frac{i\sqrt{d}\text{PolyLog}\left(2,\frac{\sqrt{-c}(a+bx+i)}{a\sqrt{-c}-b\sqrt{d}+i\sqrt{-c}}\right)}{4(-c)^{3/2}}-\frac{i\sqrt{d}\text{PolyLo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/(c + d/x^2),x]
```

```
[Out] -((1 + I*a + I*b*x)*Log[1 + I*a + I*b*x])/(2*b*c) - ((1 - I*a - I*b*x)*Log[
(-I)*(I + a + b*x)])/(2*b*c) + ((I/4)*Sqrt[d]*Log[1 + I*a + I*b*x]*Log[-((b
*(Sqrt[d] - Sqrt[-c]*x))/(I*Sqrt[-c] - a*Sqrt[-c] - b*Sqrt[d]))])/(-c)^(3/2
) - ((I/4)*Sqrt[d]*Log[1 - I*a - I*b*x]*Log[(b*(Sqrt[d] - Sqrt[-c]*x))/(I*S
qrt[-c] + a*Sqrt[-c] + b*Sqrt[d])])/(-c)^(3/2) + ((I/4)*Sqrt[d]*Log[1 - I*a
```

```
 - I*b*x]*Log[-((b*(Sqrt[d] + Sqrt[-c]*x))/((I + a)*Sqrt[-c] - b*Sqrt[d]))]
)/(-c)^(3/2) - ((I/4)*Sqrt[d]*Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[d] + Sqrt[-
c]*x))/(I*Sqrt[-c] - a*Sqrt[-c] + b*Sqrt[d])])/(-c)^(3/2) + ((I/4)*Sqrt[d]*
PolyLog[2, (Sqrt[-c]*(I - a - b*x))/(I*Sqrt[-c] - a*Sqrt[-c] - b*Sqrt[d])])
/(-c)^(3/2) - ((I/4)*Sqrt[d]*PolyLog[2, (Sqrt[-c]*(1 + I*a + I*b*x))/((1 +
I*a)*Sqrt[-c] - I*b*Sqrt[d])])/(-c)^(3/2) + ((I/4)*Sqrt[d]*PolyLog[2, (Sqrt
[-c]*(I + a + b*x))/(I*Sqrt[-c] + a*Sqrt[-c] - b*Sqrt[d])])/(-c)^(3/2) - ((
I/4)*Sqrt[d]*PolyLog[2, (Sqrt[-c]*(I + a + b*x))/(I*Sqrt[-c] + a*Sqrt[-c] +
 b*Sqrt[d])])/(-c)^(3/2)
```

<u>Rule 5051</u>

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

<u>Rule 2409</u>

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

<u>Rule 2389</u>

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

<u>Rule 2295</u>

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

<u>Rule 2394</u>

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

<u>Rule 2393</u>

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx = \frac{1}{2}i\int \frac{\log(1-ia-ibx)}{c+\frac{d}{x^2}}\,dx - \frac{1}{2}i\int \frac{\log(1+ia+ibx)}{c+\frac{d}{x^2}}\,dx$$

$$= \frac{1}{2}i\int \left(\frac{\log(1-ia-ibx)}{c} - \frac{d\log(1-ia-ibx)}{c\left(d+cx^2\right)}\right)dx - \frac{1}{2}i\int \left(\frac{\log(1+ia+ibx)}{c} - \frac{d\log(1+ia+ibx}{c\left(d+cx^2\right)}\right.$$

$$= \frac{i\int \log(1-ia-ibx)\,dx}{2c} - \frac{i\int \log(1+ia+ibx)\,dx}{2c} - \frac{(id)\int \frac{\log(1-ia-ibx)}{d+cx^2}\,dx}{2c} + \frac{(id)\int \frac{\log(1+ia+ibx)}{d+cx^2}\,dx}{2c}$$

$$= -\frac{\text{Subst}(\int \log(x)\,dx, x, 1-ia-ibx)}{2bc} - \frac{\text{Subst}(\int \log(x)\,dx, x, 1+ia+ibx)}{2bc} - \frac{(id)\int \left(\frac{\log(1-ia-ibx)}{2\sqrt{d}\left(\sqrt{d}-\sqrt{-c}x\right)}\right.}{2}$$

$$= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} - \frac{\left(i\sqrt{d}\right)\int \frac{\log(1-ia-ibx)}{\sqrt{d}-\sqrt{-c}x}\,dx}{4c} -$$

$$= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} + \frac{i\sqrt{d}\log(1+ia+ibx)\log}{4(-c)^{3/2}}$$

$$= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} + \frac{i\sqrt{d}\log(1+ia+ibx)\log}{4(-c)^{3/2}}$$

$$= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} + \frac{i\sqrt{d}\log(1+ia+ibx)\log}{4(-c)^{3/2}}$$

Mathematica [B] time = 21.489, size = 1536, normalized size = 2.3

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a + b*x]/(c + d/x^2),x]

[Out] ((a + b*x)*ArcTan[a + b*x] + Log[1/Sqrt[1 + (a + b*x)^2]])/(b*c) - (Sqrt[d]
*(-2*Sqrt[c]*ArcTan[((-I + a)*Sqrt[c])/(b*Sqrt[d])]*ArcTan[(Sqrt[c]*x)/Sqrt
[d]] - 2*a^2*Sqrt[c]*ArcTan[((-I + a)*Sqrt[c])/(b*Sqrt[d])]*ArcTan[(Sqrt[c]
*x)/Sqrt[d]] + 2*Sqrt[c]*ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])]*ArcTan[(Sqrt
[c]*x)/Sqrt[d]] + 2*a^2*Sqrt[c]*ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])]*ArcTa
n[(Sqrt[c]*x)/Sqrt[d]] - 2*b*Sqrt[d]*ArcTan[(Sqrt[c]*x)/Sqrt[d]]^2 + (b*Sqr
t[d]*Sqrt[((-I + a)^2*c + b^2*d)/(b^2*d)]*ArcTan[(Sqrt[c]*x)/Sqrt[d]]^2)/E^
(I*ArcTan[((-I + a)*Sqrt[c])/(b*Sqrt[d])]) - (I*a*b*Sqrt[d]*Sqrt[((-I + a)^
2*c + b^2*d)/(b^2*d)]*ArcTan[(Sqrt[c]*x)/Sqrt[d]]^2)/E^(I*ArcTan[((-I + a)*
Sqrt[c])/(b*Sqrt[d])]) + (b*Sqrt[d]*Sqrt[((I + a)^2*c + b^2*d)/(b^2*d)]*Arc
Tan[(Sqrt[c]*x)/Sqrt[d]]^2)/E^(I*ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])]) + (
I*a*b*Sqrt[d]*Sqrt[((I + a)^2*c + b^2*d)/(b^2*d)]*ArcTan[(Sqrt[c]*x)/Sqrt[d
]]^2)/E^(I*ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])]) + 4*(1 + a^2)*Sqrt[c]*Arc
Tan[(Sqrt[c]*x)/Sqrt[d]]*ArcTan[a + b*x] + (2*I)*Sqrt[c]*ArcTan[((-I + a)*S
qrt[c])/(b*Sqrt[d])]*Log[1 - E^((-2*I)*(ArcTan[((-I + a)*Sqrt[c])/(b*Sqrt[d
])] + ArcTan[(Sqrt[c]*x)/Sqrt[d]]))] + (2*I)*a^2*Sqrt[c]*ArcTan[((-I + a)*S
qrt[c])/(b*Sqrt[d])]*Log[1 - E^((-2*I)*(ArcTan[((-I + a)*Sqrt[c])/(b*Sqrt[d
])] + ArcTan[(Sqrt[c]*x)/Sqrt[d]]))] + (2*I)*Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqr
t[d]]*Log[1 - E^((-2*I)*(ArcTan[((-I + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan[(S
qrt[c]*x)/Sqrt[d]]))] + (2*I)*a^2*Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[d]]*Log[1
 - E^((-2*I)*(ArcTan[((-I + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan[(Sqrt[c]*x)/S
qrt[d]]))] - (2*I)*Sqrt[c]*ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])]*Log[1 - E^
((-2*I)*(ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan[(Sqrt[c]*x)/Sqrt[d]
]))] - (2*I)*a^2*Sqrt[c]*ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])]*Log[1 - E^((
-2*I)*(ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan[(Sqrt[c]*x)/Sqrt[d]])
)] - (2*I)*Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[d]]*Log[1 - E^((-2*I)*(ArcTan[((
I + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan[(Sqrt[c]*x)/Sqrt[d]]))] - (2*I)*a^2*S
qrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[d]]*Log[1 - E^((-2*I)*(ArcTan[((I + a)*Sqrt[
c])/(b*Sqrt[d])] + ArcTan[(Sqrt[c]*x)/Sqrt[d]]))] - (2*I)*Sqrt[c]*ArcTan[((
-I + a)*Sqrt[c])/(b*Sqrt[d])]*Log[-Sin[ArcTan[((-I + a)*Sqrt[c])/(b*Sqrt[d]
)] + ArcTan[(Sqrt[c]*x)/Sqrt[d]]]] - (2*I)*a^2*Sqrt[c]*ArcTan[((-I + a)*Sqr
t[c])/(b*Sqrt[d])]*Log[-Sin[ArcTan[((-I + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan
[(Sqrt[c]*x)/Sqrt[d]]]] + (2*I)*Sqrt[c]*ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d]
)]*Log[-Sin[ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan[(Sqrt[c]*x)/Sqrt
[d]]]] + (2*I)*a^2*Sqrt[c]*ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])]*Log[-Sin[A
rcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan[(Sqrt[c]*x)/Sqrt[d]]]] - (1 +
 a^2)*Sqrt[c]*PolyLog[2, E^((-2*I)*(ArcTan[((-I + a)*Sqrt[c])/(b*Sqrt[d])]
+ ArcTan[(Sqrt[c]*x)/Sqrt[d]]))] + (1 + a^2)*Sqrt[c]*PolyLog[2, E^((-2*I)*(
ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan[(Sqrt[c]*x)/Sqrt[d]]))]))/(4
*(1 + a^2)*c^2)
```

Maple [C] time = 2.177, size = 53434, normalized size = 80.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(c+d/x^2),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2 \arctan(bx+a)}{cx^2+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x^2),x, algorithm="fricas")
```

```
[Out] integral(x^2*arctan(b*x + a)/(c*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(c+d/x**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(bx+a)}{c+\frac{d}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x^2),x, algorithm="giac")
```

```
[Out] integrate(arctan(b*x + a)/(c + d/x^2), x)
```

3.57 $\int \frac{\tan^{-1}(a+bx)}{c+\frac{d}{x^3}}\,dx$

Optimal. Leaf size=933

result too large to display

```
[Out] -((1 + I*a + I*b*x)*Log[1 + I*a + I*b*x])/(2*b*c) - ((1 - I*a - I*b*x)*Log[
(-I)*(I + a + b*x)])/(2*b*c) - ((I/6)*d^(1/3)*Log[1 - I*a - I*b*x]*Log[-((b
*(d^(1/3) + c^(1/3)*x))/((I + a)*c^(1/3) - b*d^(1/3)))])/c^(4/3) + ((I/6)*d
^(1/3)*Log[1 + I*a + I*b*x]*Log[(b*(d^(1/3) + c^(1/3)*x))/((I - a)*c^(1/3)
+ b*d^(1/3))])/c^(4/3) - ((-1)^(1/6)*d^(1/3)*Log[1 + I*a + I*b*x]*Log[-((b*
(d^(1/3) - (-1)^(1/3)*c^(1/3)*x))/((-1)^(1/3)*(I - a)*c^(1/3) - b*d^(1/3)))
])/(6*c^(4/3)) + ((-1)^(1/6)*d^(1/3)*Log[1 - I*a - I*b*x]*Log[(b*(d^(1/3) -
 (-1)^(1/3)*c^(1/3)*x))/((-1)^(1/3)*(I + a)*c^(1/3) + b*d^(1/3))])/(6*c^(4/
3)) - ((-1)^(5/6)*d^(1/3)*Log[1 + I*a + I*b*x]*Log[(b*(d^(1/3) + (-1)^(2/3)
*c^(1/3)*x))/((-1)^(2/3)*(I - a)*c^(1/3) + b*d^(1/3))])/(6*c^(4/3)) + ((-1)
^(5/6)*d^(1/3)*Log[1 - I*a - I*b*x]*Log[(b*(d^(1/3) + (-1)^(2/3)*c^(1/3)*x)
)/((-1)^(1/6)*(1 - I*a)*c^(1/3) + b*d^(1/3))])/(6*c^(4/3)) - ((-1)^(1/6)*d^
(1/3)*PolyLog[2, ((-1)^(1/3)*c^(1/3)*(I - a - b*x))/((-1)^(1/3)*(I - a)*c^(
1/3) - b*d^(1/3))])/(6*c^(4/3)) - ((-1)^(5/6)*d^(1/3)*PolyLog[2, ((-1)^(1/6
)*c^(1/3)*(I - a - b*x))/((-1)^(1/6)*(I - a)*c^(1/3) - I*b*d^(1/3))])/(6*c^
(4/3)) + ((I/6)*d^(1/3)*PolyLog[2, (c^(1/3)*(I - a - b*x))/((I - a)*c^(1/3)
 + b*d^(1/3))])/c^(4/3) - ((I/6)*d^(1/3)*PolyLog[2, (c^(1/3)*(I + a + b*x))
/((I + a)*c^(1/3) - b*d^(1/3))])/c^(4/3) + ((-1)^(5/6)*d^(1/3)*PolyLog[2, (
(-1)^(2/3)*c^(1/3)*(I + a + b*x))/((-1)^(2/3)*(I + a)*c^(1/3) - b*d^(1/3))]
)/(6*c^(4/3)) + ((-1)^(1/6)*d^(1/3)*PolyLog[2, ((-1)^(1/3)*c^(1/3)*(I + a +
 b*x))/((-1)^(1/3)*(I + a)*c^(1/3) + b*d^(1/3))])/(6*c^(4/3))
```

Rubi [A] time = 1.36743, antiderivative size = 933, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5051, 2409, 2389, 2295, 2394, 2393, 2391}

$$-\frac{(ia+ibx+1)\log(ia+ibx+1)}{2bc}+\frac{i\sqrt[3]{d}\log\left(\frac{b\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)}{\sqrt[3]{c}(i-a)+b\sqrt[3]{d}}\right)\log(ia+ibx+1)}{6c^{4/3}}-\frac{\sqrt[6]{-1}\sqrt[3]{d}\log\left(-\frac{b\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{c}x\right)}{\sqrt[3]{-1}(i-a)\sqrt[3]{c}-b\sqrt[3]{d}}\right)\log(ia+ibx}{6c^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/(c + d/x^3),x]
```

```
[Out] -((1 + I*a + I*b*x)*Log[1 + I*a + I*b*x])/(2*b*c) - ((1 - I*a - I*b*x)*Log[
(-I)*(I + a + b*x)])/(2*b*c) - ((I/6)*d^(1/3)*Log[1 - I*a - I*b*x]*Log[-((b
*(d^(1/3) + c^(1/3)*x))/((I + a)*c^(1/3) - b*d^(1/3)))])/c^(4/3) + ((I/6)*d
^(1/3)*Log[1 + I*a + I*b*x]*Log[(b*(d^(1/3) + c^(1/3)*x))/((I - a)*c^(1/3)
+ b*d^(1/3))])/c^(4/3) - ((-1)^(1/6)*d^(1/3)*Log[1 + I*a + I*b*x]*Log[-((b*
(d^(1/3) - (-1)^(1/3)*c^(1/3)*x))/((-1)^(1/3)*(I - a)*c^(1/3) - b*d^(1/3)))
])/(6*c^(4/3)) + ((-1)^(1/6)*d^(1/3)*Log[1 - I*a - I*b*x]*Log[(b*(d^(1/3) -
 (-1)^(1/3)*c^(1/3)*x))/((-1)^(1/3)*(I + a)*c^(1/3) + b*d^(1/3))])/(6*c^(4/
3)) - ((-1)^(5/6)*d^(1/3)*Log[1 + I*a + I*b*x]*Log[(b*(d^(1/3) + (-1)^(2/3)
*c^(1/3)*x))/((-1)^(2/3)*(I - a)*c^(1/3) + b*d^(1/3))])/(6*c^(4/3)) + ((-1)
^(5/6)*d^(1/3)*Log[1 - I*a - I*b*x]*Log[(b*(d^(1/3) + (-1)^(2/3)*c^(1/3)*x)
)/((-1)^(1/6)*(1 - I*a)*c^(1/3) + b*d^(1/3))])/(6*c^(4/3)) - ((-1)^(1/6)*d^
(1/3)*PolyLog[2, ((-1)^(1/3)*c^(1/3)*(I - a - b*x))/((-1)^(1/3)*(I - a)*c^(
1/3) - b*d^(1/3))])/(6*c^(4/3)) - ((-1)^(5/6)*d^(1/3)*PolyLog[2, ((-1)^(1/6
)*c^(1/3)*(I - a - b*x))/((-1)^(1/6)*(I - a)*c^(1/3) - I*b*d^(1/3))])/(6*c^
(4/3)) + ((I/6)*d^(1/3)*PolyLog[2, (c^(1/3)*(I - a - b*x))/((I - a)*c^(1/3)
 + b*d^(1/3))])/c^(4/3) - ((I/6)*d^(1/3)*PolyLog[2, (c^(1/3)*(I + a + b*x))
/((I + a)*c^(1/3) - b*d^(1/3))])/c^(4/3) + ((-1)^(5/6)*d^(1/3)*PolyLog[2, (
(-1)^(2/3)*c^(1/3)*(I + a + b*x))/((-1)^(2/3)*(I + a)*c^(1/3) - b*d^(1/3))]
)/(6*c^(4/3)) + ((-1)^(1/6)*d^(1/3)*PolyLog[2, ((-1)^(1/3)*c^(1/3)*(I + a +
 b*x))/((-1)^(1/3)*(I + a)*c^(1/3) + b*d^(1/3))])/(6*c^(4/3))
```

Rule 5051

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{c+\frac{d}{x^3}}\,dx = \frac{1}{2}i\int \frac{\log(1-ia-ibx)}{c+\frac{d}{x^3}}\,dx - \frac{1}{2}i\int \frac{\log(1+ia+ibx)}{c+\frac{d}{x^3}}\,dx$$

$$= \frac{1}{2}i\int\left(\frac{\log(1-ia-ibx)}{c} - \frac{d\log(1-ia-ibx)}{c\left(d+cx^3\right)}\right)dx - \frac{1}{2}i\int\left(\frac{\log(1+ia+ibx)}{c} - \frac{d\log(1+ia+ibx)}{c\left(d+cx^3\right)}\right.$$

$$= \frac{i\int\log(1-ia-ibx)\,dx}{2c} - \frac{i\int\log(1+ia+ibx)\,dx}{2c} - \frac{(id)\int\frac{\log(1-ia-ibx)}{d+cx^3}\,dx}{2c} + \frac{(id)\int\frac{\log(1+ia+ibx)}{d+cx^3}\,dx}{2c}$$

$$= -\frac{\text{Subst}(\int\log(x)\,dx, x, 1-ia-ibx)}{2bc} - \frac{\text{Subst}(\int\log(x)\,dx, x, 1+ia+ibx)}{2bc} - \frac{(id)\int\left(-\frac{\log(1-ia-ibx)}{3d^{2/3}\left(-\sqrt[3]{d}-\sqrt[3]{c}x\right.}\right.}{}$$

$$= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} + \frac{\left(i\sqrt[3]{d}\right)\int\frac{\log(1-ia-ibx)}{-\sqrt[3]{d}-\sqrt[3]{c}x}\,dx}{6c} + $$

$$= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} - \frac{i\sqrt[3]{d}\log(1-ia-ibx)\log\left(-\right.}{6c^{4/3}}$$

$$= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} - \frac{i\sqrt[3]{d}\log(1-ia-ibx)\log\left(-\right.}{6c^{4/3}}$$

$$= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} - \frac{i\sqrt[3]{d}\log(1-ia-ibx)\log\left(-\right.}{6c^{4/3}}$$

Mathematica [C] time = 7.07409, size = 933, normalized size = 1.

$$\frac{6\left((a+bx)\tan^{-1}(a+bx)+\log\left(\frac{1}{\sqrt{(a+bx)^2+1}}\right)\right) - b^3d\,\text{RootSum}\left[c\#1^3a^3+3c\#1^2a^3+ca^3+3c\#1a^3+3ic\#1^3a^2+3ic\#1^2a^2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a + b*x]/(c + d/x^3),x]
```

```
[Out] (6*((a + b*x)*ArcTan[a + b*x] + Log[1/Sqrt[1 + (a + b*x)^2]]) - b^3*d*RootS
um[I*c - 3*a*c - (3*I)*a^2*c + a^3*c - b^3*d - (3*I)*c*#1 + 3*a*c*#1 - (3*I
)*a^2*c*#1 + 3*a^3*c*#1 - 3*b^3*d*#1 + (3*I)*c*#1^2 + 3*a*c*#1^2 + (3*I)*a^
```

```
2*c*#1^2 + 3*a^3*c*#1^2 - 3*b^3*d*#1^2 - I*c*#1^3 - 3*a*c*#1^3 + (3*I)*a^2*
c*#1^3 + a^3*c*#1^3 - b^3*d*#1^3 & , (-(Pi*ArcTan[a + b*x]) - 2*ArcTan[a +
b*x]^2 + (2*I)*ArcTan[a + b*x]*ArcTanh[(-1 + #1)/(1 + #1)] + I*Pi*Log[1 + E
^((-2*I)*ArcTan[a + b*x])] + (2*I)*ArcTan[a + b*x]*Log[1 - E^((2*I)*ArcTan[
a + b*x] - 2*ArcTanh[(-1 + #1)/(1 + #1)])] - 2*ArcTanh[(-1 + #1)/(1 + #1)]*
Log[1 - E^((2*I)*ArcTan[a + b*x] - 2*ArcTanh[(-1 + #1)/(1 + #1)])] - I*Pi*L
og[1/Sqrt[1 + (a + b*x)^2]] + 2*ArcTanh[(-1 + #1)/(1 + #1)]*Log[Sin[ArcTan[
a + b*x] + I*ArcTanh[(-1 + #1)/(1 + #1)]]] + PolyLog[2, E^((2*I)*ArcTan[a +
 b*x] - 2*ArcTanh[(-1 + #1)/(1 + #1)])] - 2*ArcTan[a + b*x]^2*#1 + Pi*ArcTa
n[a + b*x]*#1^2 - (2*I)*ArcTan[a + b*x]*ArcTanh[(-1 + #1)/(1 + #1)]*#1^2 -
I*Pi*Log[1 + E^((-2*I)*ArcTan[a + b*x])]*#1^2 - (2*I)*ArcTan[a + b*x]*Log[1
 - E^((2*I)*ArcTan[a + b*x] - 2*ArcTanh[(-1 + #1)/(1 + #1)])]*#1^2 + 2*ArcT
anh[(-1 + #1)/(1 + #1)]*Log[1 - E^((2*I)*ArcTan[a + b*x] - 2*ArcTanh[(-1 +
#1)/(1 + #1)])]*#1^2 + I*Pi*Log[1/Sqrt[1 + (a + b*x)^2]]*#1^2 - 2*ArcTanh[(
-1 + #1)/(1 + #1)]*Log[Sin[ArcTan[a + b*x] + I*ArcTanh[(-1 + #1)/(1 + #1)]]
]*#1^2 - PolyLog[2, E^((2*I)*ArcTan[a + b*x] - 2*ArcTanh[(-1 + #1)/(1 + #1)
])]*#1^2 + 2*E^ArcTanh[(1 - #1)/(1 + #1)]*ArcTan[a + b*x]^2*Sqrt[#1/(1 + #1
)^2] + 4*E^ArcTanh[(1 - #1)/(1 + #1)]*ArcTan[a + b*x]^2*#1*Sqrt[#1/(1 + #1)
^2] + 2*E^ArcTanh[(1 - #1)/(1 + #1)]*ArcTan[a + b*x]^2*#1^2*Sqrt[#1/(1 + #1
)^2])/(-(a*c) - (2*I)*a^2*c + a^3*c - b^3*d + 2*a*c*#1 + 2*a^3*c*#1 - 2*b^3
*d*#1 - a*c*#1^2 + (2*I)*a^2*c*#1^2 + a^3*c*#1^2 - b^3*d*#1^2) & ])/(6*b*c)
```

Maple [C] time = 0.658, size = 682, normalized size = 0.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(c+d/x^3),x)
```

```
[Out] arctan(b*x+a)/c*x+1/b*arctan(b*x+a)/c*a-1/2/b/c*ln(1+(b*x+a)^2)-2/3*b^2/c*d
*sum(1/(a^3*c*_R1^4+3*I*_R1^4*a^2*c-b^3*d*_R1^4-3*a*c*_R1^4-I*c*_R1^4+2*a^3
*c*_R1^2+2*I*_R1^2*a^2*c-2*b^3*d*_R1^2+2*_R1^2*a*c+2*I*c*_R1^2+a^3*c-I*a^2*
c-b^3*d+a*c-I*c)*(I*arctan(b*x+a)*ln((_R1-(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)
)/_R1)+dilog((_R1-(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))/_R1)),_R1=RootOf((3*I*
a^2*c+a^3*c-b^3*d-I*c-3*a*c)*_Z^6+(3*I*a^2*c+3*a^3*c-3*b^3*d+3*I*c+3*a*c)*_
Z^4+(-3*I*a^2*c+3*a^3*c-3*b^3*d-3*I*c+3*a*c)*_Z^2-3*I*a^2*c+a^3*c-b^3*d+I*c
-3*a*c))-2/3*b^2/c*d*sum(_R1^2/(a^3*c*_R1^4+3*I*_R1^4*a^2*c-b^3*d*_R1^4-3*a
*c*_R1^4-I*c*_R1^4+2*a^3*c*_R1^2+2*I*_R1^2*a^2*c-2*b^3*d*_R1^2+2*_R1^2*a*c+
2*I*c*_R1^2+a^3*c-I*a^2*c-b^3*d+a*c-I*c)*(I*arctan(b*x+a)*ln((_R1-(1+I*(b*x
+a))/(1+(b*x+a)^2)^(1/2))/_R1)+dilog((_R1-(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)
)/_R1)),_R1=RootOf((3*I*a^2*c+a^3*c-b^3*d-I*c-3*a*c)*_Z^6+(3*I*a^2*c+3*a^3*
```

```
c-3*b^3*d+3*I*c+3*a*c)*_Z^4+(-3*I*a^2*c+3*a^3*c-3*b^3*d-3*I*c+3*a*c)*_Z^2-3
*I*a^2*c+a^3*c-b^3*d+I*c-3*a*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3 \arctan(bx+a)}{cx^3+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x^3),x, algorithm="fricas")
```

```
[Out] integral(x^3*arctan(b*x + a)/(c*x^3 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(c+d/x**3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(bx+a)}{c+\frac{d}{x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x^3),x, algorithm="giac")
```

```
[Out] integrate(arctan(b*x + a)/(c + d/x^3), x)
```

3.58 $\int \frac{\tan^{-1}(a+bx)}{c+d\sqrt{x}}\,dx$

Optimal. Leaf size=673

$$\frac{ic\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{-a-id}}\right)}{d^2}+\frac{ic\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{-a-id}}\right)}{d^2}-\frac{ic\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{-a+id}}\right)}{d^2}-\frac{ic\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{-a+id}}\right)}{d^2}+ -$$

```
[Out] ((2*I)*Sqrt[I + a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]])/(Sqrt[b]*d) - ((2
*I)*Sqrt[I - a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]])/(Sqrt[b]*d) + (I*c*
Log[(d*(Sqrt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-I - a]*d)]*Log[
c + d*Sqrt[x]])/d^2 - (I*c*Log[(d*(Sqrt[I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]
*c + Sqrt[I - a]*d)]*Log[c + d*Sqrt[x]])/d^2 + (I*c*Log[-((d*(Sqrt[-I - a]
+ Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c - Sqrt[-I - a]*d))]*Log[c + d*Sqrt[x]])/d^2
- (I*c*Log[-((d*(Sqrt[I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c - Sqrt[I - a]*d
))]*Log[c + d*Sqrt[x]])/d^2 + (I*Sqrt[x]*Log[1 - I*a - I*b*x])/d - (I*c*Log
[c + d*Sqrt[x]]*Log[1 - I*a - I*b*x])/d^2 - (I*Sqrt[x]*Log[1 + I*a + I*b*x]
)/d + (I*c*Log[c + d*Sqrt[x]]*Log[1 + I*a + I*b*x])/d^2 + (I*c*PolyLog[2, (
Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[-I - a]*d)])/d^2 + (I*c*PolyLog[
2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-I - a]*d)])/d^2 - (I*c*Poly
Log[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[I - a]*d)])/d^2 - (I*c*P
olyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[I - a]*d)])/d^2
```

Rubi [A] time = 0.898424, antiderivative size = 673, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {5051, 2408, 2466, 2448, 321, 205, 2462, 260, 2416, 2394, 2393, 2391, 208}

$$\frac{ic\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{-a-id}}\right)}{d^2}+\frac{ic\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{-a-id}}\right)}{d^2}-\frac{ic\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{-a+id}}\right)}{d^2}-\frac{ic\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{-a+id}}\right)}{d^2}+ -$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/(c + d*Sqrt[x]),x]
```

```
[Out] ((2*I)*Sqrt[I + a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]])/(Sqrt[b]*d) - ((2
*I)*Sqrt[I - a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]])/(Sqrt[b]*d) + (I*c*
Log[(d*(Sqrt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-I - a]*d)]*Log[
c + d*Sqrt[x]])/d^2 - (I*c*Log[(d*(Sqrt[I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]
*c + Sqrt[I - a]*d)]*Log[c + d*Sqrt[x]])/d^2 + (I*c*Log[-((d*(Sqrt[-I - a]
```

```
+ Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c - Sqrt[-I - a]*d))]*Log[c + d*Sqrt[x]])/d^2
- (I*c*Log[-((d*(Sqrt[I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c - Sqrt[I - a]*d
))]*Log[c + d*Sqrt[x]])/d^2 + (I*Sqrt[x]*Log[1 - I*a - I*b*x])/d - (I*c*Log
[c + d*Sqrt[x]]*Log[1 - I*a - I*b*x])/d^2 - (I*Sqrt[x]*Log[1 + I*a + I*b*x]
)/d + (I*c*Log[c + d*Sqrt[x]]*Log[1 + I*a + I*b*x])/d^2 + (I*c*PolyLog[2, (
Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[-I - a]*d)])/d^2 + (I*c*PolyLog[
2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-I - a]*d)])/d^2 - (I*c*Poly
Log[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[I - a]*d)])/d^2 - (I*c*P
olyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[I - a]*d)])/d^2
```

Rule 5051

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rule 2408

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_)^(r_))^(q_.), x_Symbol] :> With[{k = Denominator[r]}, Dist[k, Subst[
Int[x^(k - 1)*(f + g*x^(k*r))^q*(a + b*Log[c*(d + e*x^k)^n])^p, x], x, x^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && FractionQ[r] && IG
tQ[p, 0]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{c+d\sqrt{x}}\,dx = \frac{1}{2}i\int \frac{\log(1-ia-ibx)}{c+d\sqrt{x}}\,dx - \frac{1}{2}i\int \frac{\log(1+ia+ibx)}{c+d\sqrt{x}}\,dx$$

$$= i\,\text{Subst}\left(\int \frac{x\log\left(1-ia-ibx^2\right)}{c+dx}\,dx, x, \sqrt{x}\right) - i\,\text{Subst}\left(\int \frac{x\log\left(1+ia+ibx^2\right)}{c+dx}\,dx, x, \sqrt{x}\right)$$

$$= i\,\text{Subst}\left(\int\left(\frac{\log\left(1-ia-ibx^2\right)}{d} - \frac{c\log\left(1-ia-ibx^2\right)}{d(c+dx)}\right)dx, x, \sqrt{x}\right) - i\,\text{Subst}\left(\int\left(\frac{\log\left(1+ia+i\right.}{d}\right.\right.$$

$$= \frac{i\,\text{Subst}\left(\int \log\left(1-ia-ibx^2\right)dx, x, \sqrt{x}\right)}{d} - \frac{i\,\text{Subst}\left(\int \log\left(1+ia+ibx^2\right)dx, x, \sqrt{x}\right)}{d} - \frac{(ic)\,\text{Subs}}{}$$

$$= \frac{i\sqrt{x}\log(1-ia-ibx)}{d} - \frac{ic\log\left(c+d\sqrt{x}\right)\log(1-ia-ibx)}{d^2} - \frac{i\sqrt{x}\log(1+ia+ibx)}{d} + \frac{ic\log\left(c+d\right.}{}$$

$$= \frac{i\sqrt{x}\log(1-ia-ibx)}{d} - \frac{ic\log\left(c+d\sqrt{x}\right)\log(1-ia-ibx)}{d^2} - \frac{i\sqrt{x}\log(1+ia+ibx)}{d} + \frac{ic\log\left(c+d\right.}{}$$

$$= \frac{2i\sqrt{i+a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}d} - \frac{2i\sqrt{i-a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}d} + \frac{i\sqrt{x}\log(1-ia-ibx)}{d} - \frac{ic\log\left(c+d\sqrt{x}\right)\log}{d^2}$$

$$= \frac{2i\sqrt{i+a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}d} - \frac{2i\sqrt{i-a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}d} + \frac{ic\log\left(\frac{d\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2} - \frac{ic\,\text{lo}}{}$$

$$= \frac{2i\sqrt{i+a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}d} - \frac{2i\sqrt{i-a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}d} + \frac{ic\log\left(\frac{d\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2} - \frac{ic\,\text{lo}}{}$$

$$= \frac{2i\sqrt{i+a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}d} - \frac{2i\sqrt{i-a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}d} + \frac{ic\log\left(\frac{d\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2} - \frac{ic\,\text{lo}}{}$$

Mathematica [A] time = 0.517766, size = 604, normalized size = 0.9

$$i\left(c\text{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}c-\sqrt{-a-id}}\right)+c\text{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-a-id}}\right)-c\text{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}c-\sqrt{-a+id}}\right)-c\text{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-a+id}}\right)+c\log\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/(c + d*Sqrt[x]),x]
```

```
[Out] (I*((2*Sqrt[I + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]])/Sqrt[b] - (2*Sq
rt[I - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]])/Sqrt[b] + c*Log[(d*(Sqr
t[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-I - a]*d)]*Log[c + d*Sqrt[
x]] - c*Log[(d*(Sqrt[I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[I - a]*d)
]*Log[c + d*Sqrt[x]] + c*Log[(d*(Sqrt[-I - a] + Sqrt[b]*Sqrt[x]))/(-(Sqrt[b
]*c) + Sqrt[-I - a]*d)]*Log[c + d*Sqrt[x]] - c*Log[(d*(Sqrt[I - a] + Sqrt[b
]*Sqrt[x]))/(-(Sqrt[b]*c) + Sqrt[I - a]*d)]*Log[c + d*Sqrt[x]] - d*Sqrt[x]*
Log[1 + I*a + I*b*x] + c*Log[c + d*Sqrt[x]]*Log[1 + I*a + I*b*x] + d*Sqrt[x
]*Log[(-I)*(I + a + b*x)] - c*Log[c + d*Sqrt[x]]*Log[(-I)*(I + a + b*x)] +
c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[-I - a]*d)] + c*Po
lyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-I - a]*d)] - c*PolyLo
g[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[I - a]*d)] - c*PolyLog[2,
(Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[I - a]*d)]))/d^2
```

Maple [C] time = 0.338, size = 344, normalized size = 0.5

$$2\,\frac{\arctan\left(bx+a\right)\sqrt{x}}{d}-2\,\frac{\arctan\left(bx+a\right)c\ln\left(c+d\sqrt{x}\right)}{d^{2}}+c\sum_{\_R1=\text{RootOf}\left(b^{2}\_Z^{4}-4\,cb^{2}\_Z^{3}+\left(2\,abd^{2}+6\,b^{2}c^{2}\right)\_Z^{2}+\left(-4\,bcad^{2}-4\,c^{3}b^{2}\right)\_Z+a^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(c+d*x^(1/2)),x)
```

```
[Out] 2*arctan(b*x+a)/d*x^(1/2)-2*arctan(b*x+a)*c/d^2*ln(c+d*x^(1/2))+c*sum(1/(_R
1^2*b-2*_R1*b*c+a*d^2+b*c^2)*(ln(c+d*x^(1/2))*ln((-d*x^(1/2)+_R1-c)/_R1)+di
log((-d*x^(1/2)+_R1-c)/_R1)),_R1=RootOf(b^2*_Z^4-4*c*b^2*_Z^3+(2*a*b*d^2+6*
b^2*c^2)*_Z^2+(-4*a*b*c*d^2-4*b^2*c^3)*_Z+a^2*d^4+2*a*b*c^2*d^2+b^2*c^4+d^4
))-sum((_R^2-2*_R*c+c^2)/(_R^3*b-3*_R^2*b*c+_R*a*d^2+3*_R*b*c^2-a*c*d^2-b*c
^3)*ln(d*x^(1/2)-_R+c),_R=RootOf(b^2*_Z^4-4*c*b^2*_Z^3+(2*a*b*d^2+6*b^2*c^2
```

```
)*_Z^2+(-4*a*b*c*d^2-4*b^2*c^3)*_Z+a^2*d^4+2*a*b*c^2*d^2+b^2*c^4+d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(bx+a)}{d\sqrt{x}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d*x^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(arctan(b*x + a)/(d*sqrt(x) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d\sqrt{x}\arctan(bx+a) - c\arctan(bx+a)}{d^2x - c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d*x^(1/2)),x, algorithm="fricas")
```

```
[Out] integral((d*sqrt(x)*arctan(b*x + a) - c*arctan(b*x + a))/(d^2*x - c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(c+d*x**(1/2)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(bx+a)}{d\sqrt{x}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d*x^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(arctan(b*x + a)/(d*sqrt(x) + c), x)
```

3.59 $\int \frac{\tan^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx$

Optimal. Leaf size=770

$$-\frac{id^2\text{PolyLog}\left(2,-\frac{\sqrt{b}\left(c\sqrt{x}+d\right)}{-\sqrt{b}d+\sqrt{-a-ic}}\right)}{c^3}+\frac{id^2\text{PolyLog}\left(2,-\frac{\sqrt{b}\left(c\sqrt{x}+d\right)}{-\sqrt{b}d+\sqrt{-a+ic}}\right)}{c^3}-\frac{id^2\text{PolyLog}\left(2,\frac{\sqrt{b}\left(c\sqrt{x}+d\right)}{\sqrt{b}d+\sqrt{-a-ic}}\right)}{c^3}+\frac{id^2\text{PolyLog}\left(2,\frac{\sqrt{b}}{\sqrt{bc}}\right.}{c^3}$$

```
[Out] ((-2*I)*Sqrt[I + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]])/(Sqrt[b]*c^2)
+ ((2*I)*Sqrt[I - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]])/(Sqrt[b]*c^2
) - (I*d^2*Log[(c*(Sqrt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[
b]*d)]*Log[d + c*Sqrt[x]])/c^3 + (I*d^2*Log[(c*(Sqrt[I - a] - Sqrt[b]*Sqrt[
x]))/(Sqrt[I - a]*c + Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 - (I*d^2*Log[(c*(
Sqrt[-I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c - Sqrt[b]*d)]*Log[d + c*Sq
rt[x]])/c^3 + (I*d^2*Log[(c*(Sqrt[I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[I - a]*c
 - Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 - (I*d*Sqrt[x]*Log[1 - I*a - I*b*x])
/c^2 + (I*d^2*Log[d + c*Sqrt[x]]*Log[1 - I*a - I*b*x])/c^3 + (I*d*Sqrt[x]*L
og[1 + I*a + I*b*x])/c^2 - ((1 + I*a + I*b*x)*Log[1 + I*a + I*b*x])/(2*b*c)
 - (I*d^2*Log[d + c*Sqrt[x]]*Log[1 + I*a + I*b*x])/c^3 - ((1 - I*a - I*b*x)
*Log[(-I)*(I + a + b*x)])/(2*b*c) - (I*d^2*PolyLog[2, -((Sqrt[b]*(d + c*Sqr
t[x]))/(Sqrt[-I - a]*c - Sqrt[b]*d))])/c^3 + (I*d^2*PolyLog[2, -((Sqrt[b]*(
d + c*Sqrt[x]))/(Sqrt[I - a]*c - Sqrt[b]*d))])/c^3 - (I*d^2*PolyLog[2, (Sqr
t[b]*(d + c*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[b]*d)])/c^3 + (I*d^2*PolyLog[2
, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[I - a]*c + Sqrt[b]*d)])/c^3
```

Rubi [A] time = 0.982378, antiderivative size = 770, normalized size of antiderivative = 1., number of steps used = 37, number of rules used = 16, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {5051, 2408, 2476, 2448, 321, 205, 2454, 2389, 2295, 2462, 260, 2416, 2394, 2393, 2391, 208}

$$-\frac{id^2\text{PolyLog}\left(2,-\frac{\sqrt{b}\left(c\sqrt{x}+d\right)}{-\sqrt{b}d+\sqrt{-a-ic}}\right)}{c^3}+\frac{id^2\text{PolyLog}\left(2,-\frac{\sqrt{b}\left(c\sqrt{x}+d\right)}{-\sqrt{b}d+\sqrt{-a+ic}}\right)}{c^3}-\frac{id^2\text{PolyLog}\left(2,\frac{\sqrt{b}\left(c\sqrt{x}+d\right)}{\sqrt{b}d+\sqrt{-a-ic}}\right)}{c^3}+\frac{id^2\text{PolyLog}\left(2,\frac{\sqrt{b}}{\sqrt{bc}}\right.}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/(c + d/Sqrt[x]),x]
```

```
[Out] ((-2*I)*Sqrt[I + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]])/(Sqrt[b]*c^2)
+ ((2*I)*Sqrt[I - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]])/(Sqrt[b]*c^2
```

```
) - (I*d^2*Log[(c*(Sqrt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[
b]*d)]*Log[d + c*Sqrt[x]])/c^3 + (I*d^2*Log[(c*(Sqrt[I - a] - Sqrt[b]*Sqrt[
x]))/(Sqrt[I - a]*c + Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 - (I*d^2*Log[(c*(
Sqrt[-I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c - Sqrt[b]*d)]*Log[d + c*Sq
rt[x]])/c^3 + (I*d^2*Log[(c*(Sqrt[I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[I - a]*c
 - Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 - (I*d*Sqrt[x]*Log[1 - I*a - I*b*x])
/c^2 + (I*d^2*Log[d + c*Sqrt[x]]*Log[1 - I*a - I*b*x])/c^3 + (I*d*Sqrt[x]*L
og[1 + I*a + I*b*x])/c^2 - ((1 + I*a + I*b*x)*Log[1 + I*a + I*b*x])/(2*b*c)
 - (I*d^2*Log[d + c*Sqrt[x]]*Log[1 + I*a + I*b*x])/c^3 - ((1 - I*a - I*b*x)
*Log[(-I)*(I + a + b*x)])/(2*b*c) - (I*d^2*PolyLog[2, -((Sqrt[b]*(d + c*Sqr
t[x]))/(Sqrt[-I - a]*c - Sqrt[b]*d))])/c^3 + (I*d^2*PolyLog[2, -((Sqrt[b]*(
d + c*Sqrt[x]))/(Sqrt[I - a]*c - Sqrt[b]*d))])/c^3 - (I*d^2*PolyLog[2, (Sqr
t[b]*(d + c*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[b]*d)])/c^3 + (I*d^2*PolyLog[2
, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[I - a]*c + Sqrt[b]*d)])/c^3
```

Rule 5051

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rule 2408

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_)^(r_))^(q_.), x_Symbol] :> With[{k = Denominator[r]}, Dist[k, Subst[
Int[x^(k - 1)*(f + g*x^(k*r))^q*(a + b*Log[c*(d + e*x^k)^n])^p, x], x, x^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && FractionQ[r] && IG
tQ[p, 0]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \frac{1}{2}i\int \frac{\log(1-ia-ibx)}{c+\frac{d}{\sqrt{x}}}\,dx - \frac{1}{2}i\int \frac{\log(1+ia+ibx)}{c+\frac{d}{\sqrt{x}}}\,dx$$

$$= i\,\text{Subst}\left(\int \frac{x\log\left(1-ia-ibx^2\right)}{c+\frac{d}{x}}\,dx, x, \sqrt{x}\right) - i\,\text{Subst}\left(\int \frac{x\log\left(1+ia+ibx^2\right)}{c+\frac{d}{x}}\,dx, x, \sqrt{x}\right)$$

$$= i\,\text{Subst}\left(\int \left(-\frac{d\log\left(1-ia-ibx^2\right)}{c^2} + \frac{x\log\left(1-ia-ibx^2\right)}{c} + \frac{d^2\log\left(1-ia-ibx^2\right)}{c^2(d+cx)}\right)dx, x, \sqrt{x}\right) -$$

$$= \frac{i\,\text{Subst}\left(\int x\log\left(1-ia-ibx^2\right)dx, x, \sqrt{x}\right)}{c} - \frac{i\,\text{Subst}\left(\int x\log\left(1+ia+ibx^2\right)dx, x, \sqrt{x}\right)}{c} - \frac{(id)\,\text{S}}{}$$

$$= -\frac{id\sqrt{x}\log(1-ia-ibx)}{c^2} + \frac{id^2\log\left(d+c\sqrt{x}\right)\log(1-ia-ibx)}{c^3} + \frac{id\sqrt{x}\log(1+ia+ibx)}{c^2} - \frac{id^2\log}{}$$

$$= -\frac{id\sqrt{x}\log(1-ia-ibx)}{c^2} + \frac{id^2\log\left(d+c\sqrt{x}\right)\log(1-ia-ibx)}{c^3} + \frac{id\sqrt{x}\log(1+ia+ibx)}{c^2} - \frac{id^2\log}{}$$

$$= -\frac{2i\sqrt{i+a}\,d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}c^2} + \frac{2i\sqrt{i-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}c^2} - \frac{id\sqrt{x}\log(1-ia-ibx)}{c^2} + \frac{id^2\log\left(d+c\right.}{}$$

$$= -\frac{2i\sqrt{i+a}\,d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}c^2} + \frac{2i\sqrt{i-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}c^2} - \frac{id^2\log\left(\frac{c\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} +$$

$$= -\frac{2i\sqrt{i+a}\,d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}c^2} + \frac{2i\sqrt{i-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}c^2} - \frac{id^2\log\left(\frac{c\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} +$$

$$= -\frac{2i\sqrt{i+a}\,d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}c^2} + \frac{2i\sqrt{i-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}c^2} - \frac{id^2\log\left(\frac{c\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} +$$

<u>Mathematica</u> [A] time = 0.718889, size = 666, normalized size = 0.86

$$\frac{i\left(-2d^2\left(\text{PolyLog}\left(2, \frac{\sqrt{b}\left(c\sqrt{x}+d\right)}{\sqrt{b}d-\sqrt{-a-i}c}\right) + \text{PolyLog}\left(2, \frac{\sqrt{b}\left(c\sqrt{x}+d\right)}{\sqrt{b}d+\sqrt{-a-i}c}\right) + \log\left(c\sqrt{x}+d\right)\left(\log\left(\frac{c\left(-\sqrt{b}\sqrt{x}+\sqrt{-a-i}\right)}{\sqrt{b}d+\sqrt{-a-i}c}\right) + \log\left(\frac{c\left(\sqrt{b}\sqrt{x}+\sqrt{-a-}\right.}{-\sqrt{b}d+\sqrt{-a-i}}\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/(c + d/Sqrt[x]),x]
```

```
[Out] ((I/2)*(4*c*d*(Sqrt[x] - (Sqrt[I + a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]]
)/Sqrt[b]) - 4*c*d*(Sqrt[x] - (Sqrt[I - a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I
 - a]])/Sqrt[b]) + 2*c*d*Sqrt[x]*Log[1 + I*a + I*b*x] - (c^2*(-I + a + b*x)
*Log[1 + I*a + I*b*x])/b - 2*d^2*Log[d + c*Sqrt[x]]*Log[1 + I*a + I*b*x] -
2*c*d*Sqrt[x]*Log[(-I)*(I + a + b*x)] + (c^2*(I + a + b*x)*Log[(-I)*(I + a
+ b*x)])/b + 2*d^2*Log[d + c*Sqrt[x]]*Log[(-I)*(I + a + b*x)] - 2*d^2*((Log
[(c*(Sqrt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[b]*d)] + Log[(
c*(Sqrt[-I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c - Sqrt[b]*d)])*Log[d +
c*Sqrt[x]] + PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(-(Sqrt[-I - a]*c) + Sqrt
[b]*d)] + PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[b]*d)
]) + 2*d^2*((Log[(c*(Sqrt[I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[I - a]*c + Sqrt[
b]*d)] + Log[(c*(Sqrt[I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[I - a]*c - Sqrt[b]*d
)])*Log[d + c*Sqrt[x]] + PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(-(Sqrt[I - a
]*c) + Sqrt[b]*d)] + PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[I - a]*c +
Sqrt[b]*d)])))/c^3
```

Maple [C] time = 0.318, size = 377, normalized size = 0.5

$$\frac{x\arctan(bx+a)}{c}-2\,\frac{\arctan(bx+a)\,d\sqrt{x}}{c^2}+2\,\frac{\arctan(bx+a)\,d^2\ln\left(d+c\sqrt{x}\right)}{c^3}-\frac{d^2}{c}{}_{\_R1=\mathrm{RootOf}\left(b^2\_Z^4-4\,b^2d\_Z^3+\left(2\,c^2ab+6\,b^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(c+d/x^(1/2)),x)
```

```
[Out] arctan(b*x+a)/c*x-2*arctan(b*x+a)/c^2*d*x^(1/2)+2*arctan(b*x+a)/c^3*d^2*ln(
d+c*x^(1/2))-1/c*d^2*sum(1/(_R1^2*b-2*_R1*b*d+a*c^2+b*d^2)*(ln(d+c*x^(1/2))
*ln((-c*x^(1/2)+_R1-d)/_R1)+dilog((-c*x^(1/2)+_R1-d)/_R1)),_R1=RootOf(b^2*_
Z^4-4*b^2*d*_Z^3+(2*a*b*c^2+6*b^2*d^2)*_Z^2+(-4*a*b*c^2*d-4*b^2*d^3)*_Z+a^2
*c^4+2*a*b*c^2*d^2+b^2*d^4+c^4))-1/2/c*sum((_R^3-5*_R^2*d+7*_R*d^2-3*d^3)/(
_R^3*b-3*_R^2*b*d+_R*a*c^2+3*_R*b*d^2-a*c^2*d-b*d^3)*ln(c*x^(1/2)-_R+d),_R=
RootOf(b^2*_Z^4-4*b^2*d*_Z^3+(2*a*b*c^2+6*b^2*d^2)*_Z^2+(-4*a*b*c^2*d-4*b^2
*d^3)*_Z+a^2*c^4+2*a*b*c^2*d^2+b^2*d^4+c^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(bx+a)}{c+\frac{d}{\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(arctan(b*x + a)/(c + d/sqrt(x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{cx\arctan(bx+a)-d\sqrt{x}\arctan(bx+a)}{c^2x-d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x^(1/2)),x, algorithm="fricas")
```

```
[Out] integral((c*x*arctan(b*x + a) - d*sqrt(x)*arctan(b*x + a))/(c^2*x - d^2), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(c+d/x**(1/2)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(bx+a)}{c+\frac{d}{\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(arctan(b*x + a)/(c + d/sqrt(x)), x)
```

3.60 $\int \frac{\tan^{-1}(a+bx)}{1+x^2}\,dx$

Optimal. Leaf size=274

$$-\frac{1}{4}\text{PolyLog}\left(2,-\frac{-a-bx+i}{a-i(1-b)}\right)+\frac{1}{4}\text{PolyLog}\left(2,-\frac{-a-bx+i}{a-i(b+1)}\right)-\frac{1}{4}\text{PolyLog}\left(2,\frac{a+bx+i}{a-ib+i}\right)+\frac{1}{4}\text{PolyLog}\left(2,\frac{a+bx+}{a+i(b+}\right.$$

```
[Out] (Log[(b*(I - x))/(a + I*(1 + b))]*Log[1 - I*a - I*b*x])/4 - (Log[-((b*(I +
x))/(a + I*(1 - b)))]*Log[1 - I*a - I*b*x])/4 - (Log[(b*(I - x))/(a - I*(1
- b))]*Log[1 + I*a + I*b*x])/4 + (Log[-((b*(I + x))/(a - I*(1 + b)))]*Log[1
 + I*a + I*b*x])/4 - PolyLog[2, -((I - a - b*x)/(a - I*(1 - b)))]/4 + PolyL
og[2, -((I - a - b*x)/(a - I*(1 + b)))]/4 - PolyLog[2, (I + a + b*x)/(I + a
 - I*b)]/4 + PolyLog[2, (I + a + b*x)/(a + I*(1 + b))]/4
```

Rubi [A] time = 0.270686, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5051, 2409, 2394, 2393, 2391}

$$-\frac{1}{4}\text{PolyLog}\left(2,-\frac{-a-bx+i}{a-i(1-b)}\right)+\frac{1}{4}\text{PolyLog}\left(2,-\frac{-a-bx+i}{a-i(b+1)}\right)-\frac{1}{4}\text{PolyLog}\left(2,\frac{a+bx+i}{a-ib+i}\right)+\frac{1}{4}\text{PolyLog}\left(2,\frac{a+bx+}{a+i(b+}\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/(1 + x^2),x]
```


```
[Out] (Log[(b*(I - x))/(a + I*(1 + b))]*Log[1 - I*a - I*b*x])/4 - (Log[-((b*(I +
x))/(a + I*(1 - b)))]*Log[1 - I*a - I*b*x])/4 - (Log[(b*(I - x))/(a - I*(1
- b))]*Log[1 + I*a + I*b*x])/4 + (Log[-((b*(I + x))/(a - I*(1 + b)))]*Log[1
 + I*a + I*b*x])/4 - PolyLog[2, -((I - a - b*x)/(a - I*(1 - b)))]/4 + PolyL
og[2, -((I - a - b*x)/(a - I*(1 + b)))]/4 - PolyLog[2, (I + a + b*x)/(I + a
 - I*b)]/4 + PolyLog[2, (I + a + b*x)/(a + I*(1 + b))]/4
```


Rule 5051

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
```

```
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(a+bx)}{1+x^2}\,dx &= \frac{1}{2}i\int \frac{\log(1-ia-ibx)}{1+x^2}\,dx - \frac{1}{2}i\int \frac{\log(1+ia+ibx)}{1+x^2}\,dx\\
&= \frac{1}{2}i\int \left(\frac{i\log(1-ia-ibx)}{2(i-x)} + \frac{i\log(1-ia-ibx)}{2(i+x)}\right)dx - \frac{1}{2}i\int \left(\frac{i\log(1+ia+ibx)}{2(i-x)} + \frac{i\log(1+ia+ibx}{2(i+x)}\right.\\
&= -\left(\frac{1}{4}\int \frac{\log(1-ia-ibx)}{i-x}\,dx\right) - \frac{1}{4}\int \frac{\log(1-ia-ibx)}{i+x}\,dx + \frac{1}{4}\int \frac{\log(1+ia+ibx)}{i-x}\,dx + \frac{1}{4}\int \frac{\log(1}{}\\
&= \frac{1}{4}\log\left(\frac{b(i-x)}{a+i(1+b)}\right)\log(1-ia-ibx) - \frac{1}{4}\log\left(-\frac{b(i+x)}{a+i(1-b)}\right)\log(1-ia-ibx) - \frac{1}{4}\log\left(\frac{b(i-x)}{a-i(1-b}\right.\\
&= \frac{1}{4}\log\left(\frac{b(i-x)}{a+i(1+b)}\right)\log(1-ia-ibx) - \frac{1}{4}\log\left(-\frac{b(i+x)}{a+i(1-b)}\right)\log(1-ia-ibx) - \frac{1}{4}\log\left(\frac{b(i-x)}{a-i(1-b}\right.\\
&= \frac{1}{4}\log\left(\frac{b(i-x)}{a+i(1+b)}\right)\log(1-ia-ibx) - \frac{1}{4}\log\left(-\frac{b(i+x)}{a+i(1-b)}\right)\log(1-ia-ibx) - \frac{1}{4}\log\left(\frac{b(i-x)}{a-i(1-b}\right.\end{aligned}$$

Mathematica [A] time = 0.05279, size = 283, normalized size = 1.03

$$-\frac{1}{4}\text{PolyLog}\left(2,\frac{-ia-ibx+1}{-ia-b+1}\right)+\frac{1}{4}\text{PolyLog}\left(2,\frac{-ia-ibx+1}{-ia+b+1}\right)-\frac{1}{4}\text{PolyLog}\left(2,\frac{ia+ibx+1}{ia-b+1}\right)+\frac{1}{4}\text{PolyLog}\left(2,\frac{ia+ibx}{ia+b+}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/(1 + x^2),x]
```

```
[Out] (Log[(b*(I - x))/(a + I*(1 + b))]*Log[1 - I*a - I*b*x])/4 - (Log[-((b*(I +
x))/(a + I*(1 - b)))]*Log[1 - I*a - I*b*x])/4 - (Log[(b*(I - x))/(a - I*(1
- b))]*Log[1 + I*a + I*b*x])/4 + (Log[-((b*(I + x))/(a - I*(1 + b)))]*Log[1
 + I*a + I*b*x])/4 - PolyLog[2, (1 - I*a - I*b*x)/(1 - I*a - b)]/4 + PolyLo
g[2, (1 - I*a - I*b*x)/(1 - I*a + b)]/4 - PolyLog[2, (1 + I*a + I*b*x)/(1 +
 I*a - b)]/4 + PolyLog[2, (1 + I*a + I*b*x)/(1 + I*a + b)]/4
```

Maple [B] time = 0.444, size = 833, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(x^2+1),x)
```

```
[Out] arctan(x)*arctan(b*x+a)+1/2*I*arctan((b*x+a)/b-a/b)*ln(1-(-I*b+a-I)*(1+I*((
b*x+a)/b-a/b))^2/(((b*x+a)/b-a/b)^2+1)/(-I*b+I-a))+1/2*arctan((b*x+a)/b-a/b
)^2+1/4*polylog(2,(-I*b+a-I)*(1+I*((b*x+a)/b-a/b))^2/(((b*x+a)/b-a/b)^2+1)/
(-I*b+I-a))+1/2*b/(I*b+I+a)*ln(1-(I+a-I*b)*(1+I*((b*x+a)/b-a/b))^2/(((b*x+a
)/b-a/b)^2+1)/(-I*b-I-a))*arctan((b*x+a)/b-a/b)+1/2/(I*b+I+a)*ln(1-(I+a-I*b
)*(1+I*((b*x+a)/b-a/b))^2/(((b*x+a)/b-a/b)^2+1)/(-I*b-I-a))*arctan((b*x+a)/
b-a/b)-1/2*I/(I*b+I+a)*ln(1-(I+a-I*b)*(1+I*((b*x+a)/b-a/b))^2/(((b*x+a)/b-a
/b)^2+1)/(-I*b-I-a))*arctan((b*x+a)/b-a/b)*a-1/2*I*b/(I*b+I+a)*arctan((b*x+
a)/b-a/b)^2-1/4*I*b/(I*b+I+a)*polylog(2,(I+a-I*b)*(1+I*((b*x+a)/b-a/b))^2/(
((b*x+a)/b-a/b)^2+1)/(-I*b-I-a))-1/2*I/(I*b+I+a)*arctan((b*x+a)/b-a/b)^2-1/
2/(I*b+I+a)*arctan((b*x+a)/b-a/b)^2*a-1/4*I/(I*b+I+a)*polylog(2,(I+a-I*b)*(
1+I*((b*x+a)/b-a/b))^2/(((b*x+a)/b-a/b)^2+1)/(-I*b-I-a))-1/4/(I*b+I+a)*poly
log(2,(I+a-I*b)*(1+I*((b*x+a)/b-a/b))^2/(((b*x+a)/b-a/b)^2+1)/(-I*b-I-a))*a
```

Maxima [A] time = 1.8351, size = 443, normalized size = 1.62

$$\frac{1}{8}\,b\left(\frac{8\,\arctan\left(x\right)\arctan\left(\frac{b^2x+ab}{b}\right)}{b}-\frac{4\,\arctan\left(x\right)\arctan\left(\frac{ab+\left(b^2+b\right)x}{a^2+b^2+2\,b+1},\,\frac{abx+a^2+b+1}{a^2+b^2+2\,b+1}\right)-4\,\arctan\left(x\right)\arctan\left(\frac{ab+\left(b^2-b\right)x}{a^2+b^2-2\,b+1},\,\ldots\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/8*b*(8*arctan(x)*arctan((b^2*x + a*b)/b)/b - (4*arctan(x)*arctan2((a*b +
(b^2 + b)*x)/(a^2 + b^2 + 2*b + 1), (a*b*x + a^2 + b + 1)/(a^2 + b^2 + 2*b
+ 1)) - 4*arctan(x)*arctan2((a*b + (b^2 - b)*x)/(a^2 + b^2 - 2*b + 1), (a*b
*x + a^2 - b + 1)/(a^2 + b^2 - 2*b + 1)) + log(x^2 + 1)*log((b^2*x^2 + 2*a*
b*x + a^2 + 1)/(a^2 + b^2 + 2*b + 1)) - log(x^2 + 1)*log((b^2*x^2 + 2*a*b*x
 + a^2 + 1)/(a^2 + b^2 - 2*b + 1)) + 2*dilog(-(I*b*x - b)/(I*a + b + 1)) -
2*dilog(-(I*b*x - b)/(I*a + b - 1)) + 2*dilog((I*b*x + b)/(-I*a + b + 1)) -
 2*dilog((I*b*x + b)/(-I*a + b - 1)))/b) + arctan(b*x + a)*arctan(x) - arct
an(x)*arctan((b^2*x + a*b)/b)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan(bx+a)}{x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(x^2+1),x, algorithm="fricas")
```

```
[Out] integral(arctan(b*x + a)/(x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}(a+bx)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(x**2+1),x)
```

```
[Out] Integral(atan(a + b*x)/(x**2 + 1), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(bx+a)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate(arctan(b*x + a)/(x^2 + 1), x)
```

3.61 $\int \frac{\tan^{-1}(d+ex)}{a+bx^2}\,dx$

Optimal. Leaf size=543

$$-\frac{i\text{PolyLog}\left(2,\frac{\sqrt{b}(-d-ex+i)}{-\sqrt{-a}e+\sqrt{b}(-d+i)}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{i\text{PolyLog}\left(2,\frac{\sqrt{b}(-d-ex+i)}{\sqrt{-a}e+\sqrt{b}(-d+i)}\right)}{4\sqrt{-a}\sqrt{b}}-\frac{i\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex+i)}{-\sqrt{-a}e+\sqrt{b}(d+i)}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{i\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex+i)}{\sqrt{-a}e+\sqrt{b}(d+i}\right.}{4\sqrt{-a}\sqrt{b}}$$

```
[Out] ((I/4)*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*(I + d) + Sqrt[-a]*e)]*Log[1
 - I*d - I*e*x])/(Sqrt[-a]*Sqrt[b]) - ((I/4)*Log[-((e*(Sqrt[-a] + Sqrt[b]*x
))/(Sqrt[b]*(I + d) - Sqrt[-a]*e))]*Log[1 - I*d - I*e*x])/(Sqrt[-a]*Sqrt[b]
) - ((I/4)*Log[-((e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*(I - d) - Sqrt[-a]*e))
]*Log[1 + I*d + I*e*x])/(Sqrt[-a]*Sqrt[b]) + ((I/4)*Log[(e*(Sqrt[-a] + Sqrt
[b]*x))/(Sqrt[b]*(I - d) + Sqrt[-a]*e)]*Log[1 + I*d + I*e*x])/(Sqrt[-a]*Sqr
t[b]) - ((I/4)*PolyLog[2, (Sqrt[b]*(I - d - e*x))/(Sqrt[b]*(I - d) - Sqrt[-
a]*e)])/(Sqrt[-a]*Sqrt[b]) + ((I/4)*PolyLog[2, (Sqrt[b]*(I - d - e*x))/(Sqr
t[b]*(I - d) + Sqrt[-a]*e)])/(Sqrt[-a]*Sqrt[b]) - ((I/4)*PolyLog[2, (Sqrt[b
]*(I + d + e*x))/(Sqrt[b]*(I + d) - Sqrt[-a]*e)])/(Sqrt[-a]*Sqrt[b]) + ((I/
4)*PolyLog[2, (Sqrt[b]*(I + d + e*x))/(Sqrt[b]*(I + d) + Sqrt[-a]*e)])/(Sqr
t[-a]*Sqrt[b])
```

Rubi [A] time = 0.600126, antiderivative size = 543, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5051, 2409, 2394, 2393, 2391}

$$-\frac{i\text{PolyLog}\left(2,\frac{\sqrt{b}(-d-ex+i)}{-\sqrt{-a}e+\sqrt{b}(-d+i)}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{i\text{PolyLog}\left(2,\frac{\sqrt{b}(-d-ex+i)}{\sqrt{-a}e+\sqrt{b}(-d+i)}\right)}{4\sqrt{-a}\sqrt{b}}-\frac{i\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex+i)}{-\sqrt{-a}e+\sqrt{b}(d+i)}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{i\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex+i)}{\sqrt{-a}e+\sqrt{b}(d+i}\right.}{4\sqrt{-a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[d + e*x]/(a + b*x^2),x]
```

```
[Out] ((I/4)*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*(I + d) + Sqrt[-a]*e)]*Log[1
 - I*d - I*e*x])/(Sqrt[-a]*Sqrt[b]) - ((I/4)*Log[-((e*(Sqrt[-a] + Sqrt[b]*x
))/(Sqrt[b]*(I + d) - Sqrt[-a]*e))]*Log[1 - I*d - I*e*x])/(Sqrt[-a]*Sqrt[b]
) - ((I/4)*Log[-((e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*(I - d) - Sqrt[-a]*e))
]*Log[1 + I*d + I*e*x])/(Sqrt[-a]*Sqrt[b]) + ((I/4)*Log[(e*(Sqrt[-a] + Sqrt
[b]*x))/(Sqrt[b]*(I - d) + Sqrt[-a]*e)]*Log[1 + I*d + I*e*x])/(Sqrt[-a]*Sqr
t[b]) - ((I/4)*PolyLog[2, (Sqrt[b]*(I - d - e*x))/(Sqrt[b]*(I - d) - Sqrt[-
a]*e)])/(Sqrt[-a]*Sqrt[b]) + ((I/4)*PolyLog[2, (Sqrt[b]*(I - d - e*x))/(Sqr
```

```
t[b]*(I - d) + Sqrt[-a]*e)])/(Sqrt[-a]*Sqrt[b]) - ((I/4)*PolyLog[2, (Sqrt[b
]*(I + d + e*x))/(Sqrt[b]*(I + d) - Sqrt[-a]*e)])/(Sqrt[-a]*Sqrt[b]) + ((I/
4)*PolyLog[2, (Sqrt[b]*(I + d + e*x))/(Sqrt[b]*(I + d) + Sqrt[-a]*e)])/(Sqr
t[-a]*Sqrt[b])
```

Rule 5051

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\tan^{-1}(d+ex)}{a+bx^2}\,dx = \frac{1}{2}i\int \frac{\log(1-id-iex)}{a+bx^2}\,dx - \frac{1}{2}i\int \frac{\log(1+id+iex)}{a+bx^2}\,dx$$
$$= \frac{1}{2}i\int \left(\frac{\sqrt{-a}\log(1-id-iex)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\log(1-id-iex)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx - \frac{1}{2}i\int \left(\frac{\sqrt{-a}\log(1+id+iex)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\,\mathrm{l}}{2a\,(}\right.$$
$$= -\frac{i\int \frac{\log(1-id-iex)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4\sqrt{-a}} - \frac{i\int \frac{\log(1-id-iex)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4\sqrt{-a}} + \frac{i\int \frac{\log(1+id+iex)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4\sqrt{-a}} + \frac{i\int \frac{\log(1+id+iex)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4\sqrt{-a}}$$
$$= \frac{i\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}(i+d)+\sqrt{-a}e}\right)\log(1-id-iex)}{4\sqrt{-a}\sqrt{b}} - \frac{i\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}(i+d)-\sqrt{-a}e}\right)\log(1-id-iex)}{4\sqrt{-a}\sqrt{b}} - \frac{i\log\left(-\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}(i-d)-\sqrt{-ae}}\right.}{4\sqrt{-}}$$

$$= \frac{i\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}(i+d)+\sqrt{-a}e}\right)\log(1-id-iex)}{4\sqrt{-a}\sqrt{b}} - \frac{i\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}(i+d)-\sqrt{-a}e}\right)\log(1-id-iex)}{4\sqrt{-a}\sqrt{b}} - \frac{i\log\left(-\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}(i-d)-\sqrt{-ae}}\right.}{4\sqrt{-}}$$
$$= \frac{i\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}(i+d)+\sqrt{-a}e}\right)\log(1-id-iex)}{4\sqrt{-a}\sqrt{b}} - \frac{i\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}(i+d)-\sqrt{-a}e}\right)\log(1-id-iex)}{4\sqrt{-a}\sqrt{b}} - \frac{i\log\left(-\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}(i-d)-\sqrt{-ae}}\right.}{4\sqrt{-}}$$

Mathematica [A] time = 0.354393, size = 409, normalized size = 0.75

$$\frac{i\left(\mathrm{PolyLog}\left(2,\frac{\sqrt{b}(d+ex-i)}{-\sqrt{-a}e+\sqrt{b}(d-i)}\right) - \mathrm{PolyLog}\left(2,\frac{\sqrt{b}(d+ex-i)}{\sqrt{-a}e+\sqrt{b}(d-i)}\right) - \mathrm{PolyLog}\left(2,\frac{\sqrt{b}(d+ex+i)}{-\sqrt{-a}e+\sqrt{b}(d+i)}\right) + \mathrm{PolyLog}\left(2,\frac{\sqrt{b}(d+ex+i)}{\sqrt{-a}e+\sqrt{b}(d+i)}\right) + \mathrm{lo}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[d + e*x]/(a + b*x^2),x]
```

```
[Out] ((I/4)*(-(Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*(-I + d) + Sqrt[-a]*e)]*L
og[1 + I*d + I*e*x]) + Log[(e*(Sqrt[-a] + Sqrt[b]*x))/(-(Sqrt[b]*(-I + d))
+ Sqrt[-a]*e)]*Log[1 + I*d + I*e*x] + Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[
b]*(I + d) + Sqrt[-a]*e)]*Log[(-I)*(I + d + e*x)] - Log[(e*(Sqrt[-a] + Sqrt
[b]*x))/(-(Sqrt[b]*(I + d)) + Sqrt[-a]*e)]*Log[(-I)*(I + d + e*x)] + PolyLo
g[2, (Sqrt[b]*(-I + d + e*x))/(Sqrt[b]*(-I + d) - Sqrt[-a]*e)] - PolyLog[2,
 (Sqrt[b]*(-I + d + e*x))/(Sqrt[b]*(-I + d) + Sqrt[-a]*e)] - PolyLog[2, (Sq
rt[b]*(I + d + e*x))/(Sqrt[b]*(I + d) - Sqrt[-a]*e)] + PolyLog[2, (Sqrt[b]*
(I + d + e*x))/(Sqrt[b]*(I + d) + Sqrt[-a]*e)]))/(Sqrt[-a]*Sqrt[b])
```

Maple [B] time = 0.702, size = 2192, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(e*x+d)/(b*x^2+a),x)
```

```
[Out] 1/2*I*e/b*(a*b*e^2)^(1/2)/(a*e^2+b*d^2+2*(a*b*e^2)^(1/2)+b)*ln(1-(2*I*b*d+a
*e^2+b*d^2-b)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2-b*d^2-2*(a*b*e^2)^(1/2)
-b))*arctan(e*x+d)+1/2*I/e*(a*b*e^2)^(1/2)/a/(a*e^2+b*d^2+2*(a*b*e^2)^(1/2)
+b)*ln(1-(2*I*b*d+a*e^2+b*d^2-b)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2-b*d^
2-2*(a*b*e^2)^(1/2)-b))*arctan(e*x+d)*d^2-1/2*I*e/b*(a*b*e^2)^(1/2)/(a*e^2+
b*d^2-2*(a*b*e^2)^(1/2)+b)*ln(1-(2*I*b*d+a*e^2+b*d^2-b)*(1+I*(e*x+d))^2/((e
*x+d)^2+1)/(-a*e^2-b*d^2+2*(a*b*e^2)^(1/2)-b))*arctan(e*x+d)+I*e/(a*e^2+b*d
^2-2*(a*b*e^2)^(1/2)+b)*ln(1-(2*I*b*d+a*e^2+b*d^2-b)*(1+I*(e*x+d))^2/((e*x+
d)^2+1)/(-a*e^2-b*d^2+2*(a*b*e^2)^(1/2)-b))*arctan(e*x+d)-1/2*e/b*(a*b*e^2)
^(1/2)/(a*e^2+b*d^2-2*(a*b*e^2)^(1/2)+b)*arctan(e*x+d)^2+e/(a*e^2+b*d^2-2*(
a*b*e^2)^(1/2)+b)*arctan(e*x+d)^2-1/2/e*(a*b*e^2)^(1/2)/a/(a*e^2+b*d^2-2*(a
*b*e^2)^(1/2)+b)*arctan(e*x+d)^2-1/2/e*(a*b*e^2)^(1/2)/a/(a*e^2+b*d^2-2*(a*
b*e^2)^(1/2)+b)*arctan(e*x+d)^2*d^2-1/4*e/b*(a*b*e^2)^(1/2)/(a*e^2+b*d^2-2*
(a*b*e^2)^(1/2)+b)*polylog(2,(2*I*b*d+a*e^2+b*d^2-b)*(1+I*(e*x+d))^2/((e*x+
d)^2+1)/(-a*e^2-b*d^2+2*(a*b*e^2)^(1/2)-b))+1/2*e/(a*e^2+b*d^2-2*(a*b*e^2)^
(1/2)+b)*polylog(2,(2*I*b*d+a*e^2+b*d^2-b)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-
a*e^2-b*d^2+2*(a*b*e^2)^(1/2)-b))-1/4/e*(a*b*e^2)^(1/2)/a/(a*e^2+b*d^2-2*(a
*b*e^2)^(1/2)+b)*polylog(2,(2*I*b*d+a*e^2+b*d^2-b)*(1+I*(e*x+d))^2/((e*x+d)
^2+1)/(-a*e^2-b*d^2+2*(a*b*e^2)^(1/2)-b))-1/4/e*(a*b*e^2)^(1/2)/a/(a*e^2+b*
d^2-2*(a*b*e^2)^(1/2)+b)*polylog(2,(2*I*b*d+a*e^2+b*d^2-b)*(1+I*(e*x+d))^2/
((e*x+d)^2+1)/(-a*e^2-b*d^2+2*(a*b*e^2)^(1/2)-b))*d^2-1/2*I/e*(a*b*e^2)^(1/
2)/a/(a*e^2+b*d^2-2*(a*b*e^2)^(1/2)+b)*ln(1-(2*I*b*d+a*e^2+b*d^2-b)*(1+I*(e
*x+d))^2/((e*x+d)^2+1)/(-a*e^2-b*d^2+2*(a*b*e^2)^(1/2)-b))*arctan(e*x+d)*d^
2+1/2*I/e*(a*b*e^2)^(1/2)/a/(a*e^2+b*d^2+2*(a*b*e^2)^(1/2)+b)*ln(1-(2*I*b*d
+a*e^2+b*d^2-b)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2-b*d^2-2*(a*b*e^2)^(1/
2)-b))*arctan(e*x+d)+I*e/(a*e^2+b*d^2+2*(a*b*e^2)^(1/2)+b)*ln(1-(2*I*b*d+a*
e^2+b*d^2-b)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2-b*d^2-2*(a*b*e^2)^(1/2)-
b))*arctan(e*x+d)-1/2*I/e*(a*b*e^2)^(1/2)/a/(a*e^2+b*d^2-2*(a*b*e^2)^(1/2)+
b)*ln(1-(2*I*b*d+a*e^2+b*d^2-b)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2-b*d^2
+2*(a*b*e^2)^(1/2)-b))*arctan(e*x+d)+1/2*e/b*(a*b*e^2)^(1/2)/(a*e^2+b*d^2+2
*(a*b*e^2)^(1/2)+b)*arctan(e*x+d)^2+e/(a*e^2+b*d^2+2*(a*b*e^2)^(1/2)+b)*arc
tan(e*x+d)^2+1/2/e*(a*b*e^2)^(1/2)/a/(a*e^2+b*d^2+2*(a*b*e^2)^(1/2)+b)*arct
an(e*x+d)^2+1/2/e*(a*b*e^2)^(1/2)/a/(a*e^2+b*d^2+2*(a*b*e^2)^(1/2)+b)*arcta
n(e*x+d)^2*d^2+1/4*e/b*(a*b*e^2)^(1/2)/(a*e^2+b*d^2+2*(a*b*e^2)^(1/2)+b)*po
lylog(2,(2*I*b*d+a*e^2+b*d^2-b)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2-b*d^2
```

```
-2*(a*b*e^2)^(1/2)-b))+1/2*e/(a*e^2+b*d^2+2*(a*b*e^2)^(1/2)+b)*polylog(2,(2
*I*b*d+a*e^2+b*d^2-b)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2-b*d^2-2*(a*b*e^
2)^(1/2)-b))+1/4/e*(a*b*e^2)^(1/2)/a/(a*e^2+b*d^2+2*(a*b*e^2)^(1/2)+b)*poly
log(2,(2*I*b*d+a*e^2+b*d^2-b)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2-b*d^2-2
*(a*b*e^2)^(1/2)-b))+1/4/e*(a*b*e^2)^(1/2)/a/(a*e^2+b*d^2+2*(a*b*e^2)^(1/2)
+b)*polylog(2,(2*I*b*d+a*e^2+b*d^2-b)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2
-b*d^2-2*(a*b*e^2)^(1/2)-b))*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x+d)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan(ex+d)}{bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x+d)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral(arctan(e*x + d)/(b*x^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(e*x+d)/(b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ex+d)}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x+d)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(arctan(e*x + d)/(b*x^2 + a), x)
```

3.62 $\int \frac{\tan^{-1}(d+ex)}{a+bx+cx^2}\, dx$

Optimal. Leaf size=367

$$-\frac{i\text{PolyLog}\left(2,1+\frac{2\left(-e\left(b-\sqrt{b^2-4ac}\right)-2c(d+ex)+2cd\right)}{(1-i(d+ex))\left(-e\sqrt{b^2-4ac}+be-2cd+2ic\right)}\right)}{2\sqrt{b^2-4ac}}+\frac{i\text{PolyLog}\left(2,1+\frac{2\left(-e\left(\sqrt{b^2-4ac}+b\right)-2c(d+ex)+2cd\right)}{(1-i(d+ex))\left(e\left(\sqrt{b^2-4ac}+b\right)+2c(-d+i)\right)}\right)}{2\sqrt{b^2-4ac}}+\frac{\tan^{-1}(d+ex)\log}{}$$

```
[Out] (ArcTan[d + e*x]*Log[(2*e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*(I - d) +
(b - Sqrt[b^2 - 4*a*c])*e)*(1 - I*(d + e*x)))]])/Sqrt[b^2 - 4*a*c] - (ArcTan
[d + e*x]*Log[(2*e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*(I - d) + (b + Sq
rt[b^2 - 4*a*c])*e)*(1 - I*(d + e*x)))]])/Sqrt[b^2 - 4*a*c] - ((I/2)*PolyLog
[2, 1 + (2*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e - 2*c*(d + e*x)))/(((2*I)*c -
 2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e)*(1 - I*(d + e*x)))]])/Sqrt[b^2 - 4*a*c]
+ ((I/2)*PolyLog[2, 1 + (2*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e - 2*c*(d + e*
x)))/((2*c*(I - d) + (b + Sqrt[b^2 - 4*a*c])*e)*(1 - I*(d + e*x)))]])/Sqrt[b
^2 - 4*a*c]
```

Rubi [A] time = 0.672732, antiderivative size = 367, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {618, 206, 6728, 5047, 4856, 2402, 2315, 2447}

$$-\frac{i\text{PolyLog}\left(2,1+\frac{2\left(-e\left(b-\sqrt{b^2-4ac}\right)-2c(d+ex)+2cd\right)}{(1-i(d+ex))\left(-e\sqrt{b^2-4ac}+be-2cd+2ic\right)}\right)}{2\sqrt{b^2-4ac}}+\frac{i\text{PolyLog}\left(2,1+\frac{2\left(-e\left(\sqrt{b^2-4ac}+b\right)-2c(d+ex)+2cd\right)}{(1-i(d+ex))\left(e\left(\sqrt{b^2-4ac}+b\right)+2c(-d+i)\right)}\right)}{2\sqrt{b^2-4ac}}+\frac{\tan^{-1}(d+ex)\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[d + e*x]/(a + b*x + c*x^2),x]
```

```
[Out] (ArcTan[d + e*x]*Log[(2*e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*(I - d) +
(b - Sqrt[b^2 - 4*a*c])*e)*(1 - I*(d + e*x)))]])/Sqrt[b^2 - 4*a*c] - (ArcTan
[d + e*x]*Log[(2*e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*(I - d) + (b + Sq
rt[b^2 - 4*a*c])*e)*(1 - I*(d + e*x)))]])/Sqrt[b^2 - 4*a*c] - ((I/2)*PolyLog
[2, 1 + (2*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e - 2*c*(d + e*x)))/(((2*I)*c -
 2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e)*(1 - I*(d + e*x)))]])/Sqrt[b^2 - 4*a*c]
+ ((I/2)*PolyLog[2, 1 + (2*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e - 2*c*(d + e*
x)))/((2*c*(I - d) + (b + Sqrt[b^2 - 4*a*c])*e)*(1 - I*(d + e*x)))]])/Sqrt[b
^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\tan^{-1}(d+ex)}{a+bx+cx^2}\,dx = \int \left(\frac{2c\tan^{-1}(d+ex)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx\right)} - \frac{2c\tan^{-1}(d+ex)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}+2cx\right)}\right)dx$$

$$= \frac{(2c)\int \frac{\tan^{-1}(d+ex)}{b-\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{\tan^{-1}(d+ex)}{b+\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{(2c)\,\mathrm{Subst}\left(\int \frac{\tan^{-1}(x)}{\frac{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}{e}+\frac{2cx}{e}}\,dx, x, d+ex\right)}{\sqrt{b^2-4ac}e} - \frac{(2c)\,\mathrm{Subst}\left(\int \frac{\tan^{-1}(x)}{\frac{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}{e}+\frac{2cx}{e}}\,dx, x, d+ex\right)}{\sqrt{b^2-4ac}e}$$

$$= \frac{\tan^{-1}(d+ex)\log\left(\frac{2e\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(2ic-2cd+be-\sqrt{b^2-4ac}e\right)(1-i(d+ex))}\right)}{\sqrt{b^2-4ac}} - \frac{\tan^{-1}(d+ex)\log\left(\frac{2e\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2c(i-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(1-i(d+ex)}\right.}{\sqrt{b^2-4ac}}$$

$$= \frac{\tan^{-1}(d+ex)\log\left(\frac{2e\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(2ic-2cd+be-\sqrt{b^2-4ac}e\right)(1-i(d+ex))}\right)}{\sqrt{b^2-4ac}} - \frac{\tan^{-1}(d+ex)\log\left(\frac{2e\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2c(i-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(1-i(d+ex)}\right.}{\sqrt{b^2-4ac}}$$

Mathematica [A] time = 0.412683, size = 443, normalized size = 1.21

$$i\left(-\mathrm{PolyLog}\left(2, \frac{2c(d+ex-i)}{e\left(\sqrt{b^2-4ac}-b\right)+2c(d-i)}\right) + \mathrm{PolyLog}\left(2, \frac{2c(d+ex-i)}{-e\left(\sqrt{b^2-4ac}+b\right)+2c(d-i)}\right) + \mathrm{PolyLog}\left(2, \frac{2c(d+ex+i)}{e\left(\sqrt{b^2-4ac}-b\right)+2c(d+i)}\right) - \mathrm{PolyLog}\left(\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[d + e*x]/(a + b*x + c*x^2),x]
```

```
[Out] ((I/2)*(Log[(e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*(I + d) + (-b + Sqrt[
b^2 - 4*a*c])*e)]*Log[1 - I*(d + e*x)] - Log[(e*(b + Sqrt[b^2 - 4*a*c] + 2*
c*x))/(-2*c*(I + d) + (b + Sqrt[b^2 - 4*a*c])*e)]*Log[1 - I*(d + e*x)] - Lo
g[(e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*(-I + d) + (-b + Sqrt[b^2 - 4*a
*c])*e)]*Log[1 + I*(d + e*x)] + Log[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2
*c*(-I + d) + (b + Sqrt[b^2 - 4*a*c])*e)]*Log[1 + I*(d + e*x)] - PolyLog[2,
 (2*c*(-I + d + e*x))/(2*c*(-I + d) + (-b + Sqrt[b^2 - 4*a*c])*e)] + PolyLo
g[2, (2*c*(-I + d + e*x))/(2*c*(-I + d) - (b + Sqrt[b^2 - 4*a*c])*e)] + Pol
yLog[2, (2*c*(I + d + e*x))/(2*c*(I + d) + (-b + Sqrt[b^2 - 4*a*c])*e)] - P
olyLog[2, (2*c*(I + d + e*x))/(2*c*(I + d) - (b + Sqrt[b^2 - 4*a*c])*e)]))/
Sqrt[b^2 - 4*a*c]
```

Maple [B] time = 1.036, size = 4743, normalized size = 12.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(e*x+d)/(c*x^2+b*x+a),x)
```

```
[Out] 1/e*(e^2*(4*a*c-b^2))^(1/2)*c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^
2))^(1/2)+c)*arctan(e*x+d)^2+1/2/e*(e^2*(4*a*c-b^2))^(1/2)*c/(4*a*c-b^2)/(a
*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(-I*b*e+2*I*d*c+a*e^2
-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2-(e^2*(4*a
*c-b^2))^(1/2)-c))-1/e*(e^2*(4*a*c-b^2))^(1/2)*c/(4*a*c-b^2)/(a*e^2-b*e*d+c
*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*arctan(e*x+d)^2-1/2/e*(e^2*(4*a*c-b^2))^(1/
2)*c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(-
I*b*e+2*I*d*c+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*
e*d-c*d^2+(e^2*(4*a*c-b^2))^(1/2)-c))+(e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)/(
a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*arctan(e*x+d)^2*b*d-1/2*(e^2*(
4*a*c-b^2))^(1/2)/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)
*polylog(2,(-I*b*e+2*I*d*c+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+
1)/(-a*e^2+b*e*d-c*d^2-(e^2*(4*a*c-b^2))^(1/2)-c))*b*d+1/2*(e^2*(4*a*c-b^2)
)^(1/2)/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2
,(-I*b*e+2*I*d*c+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2
+b*e*d-c*d^2+(e^2*(4*a*c-b^2))^(1/2)-c))*b*d-(e^2*(4*a*c-b^2))^(1/2)/(4*a*c
-b^2)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*arctan(e*x+d)^2*b*d+I/e
*(e^2*(4*a*c-b^2))^(1/2)*c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))
^(1/2)+c)*ln(1-(-I*b*e+2*I*d*c+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d
)^2+1)/(-a*e^2+b*e*d-c*d^2-(e^2*(4*a*c-b^2))^(1/2)-c))*arctan(e*x+d)+I*(e^2
```

```
*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+
c)*ln(1-(-I*b*e+2*I*d*c+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/
(-a*e^2+b*e*d-c*d^2+(e^2*(4*a*c-b^2))^(1/2)-c))*arctan(e*x+d)*b*d-I/e*(e^2*
(4*a*c-b^2))^(1/2)*c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)
+c)*ln(1-(-I*b*e+2*I*d*c+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)
/(-a*e^2+b*e*d-c*d^2+(e^2*(4*a*c-b^2))^(1/2)-c))*arctan(e*x+d)-I*(e^2*(4*a*
c-b^2))^(1/2)/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*ln(
1-(-I*b*e+2*I*d*c+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^
2+b*e*d-c*d^2-(e^2*(4*a*c-b^2))^(1/2)-c))*arctan(e*x+d)*b*d+1/2*e/(a*e^2-b*
e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(-I*b*e+2*I*d*c+a*e^2-b*e*d+
c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2-(e^2*(4*a*c-b^2)
)^(1/2)-c))+e/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*arctan(e*x+d)^2
+1/2*e/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(-I*b*e+2*I*
d*c+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2+
(e^2*(4*a*c-b^2))^(1/2)-c))+e/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)
*arctan(e*x+d)^2+I*e/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*ln(1-(-I
*b*e+2*I*d*c+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e
*d-c*d^2+(e^2*(4*a*c-b^2))^(1/2)-c))*arctan(e*x+d)-1/2/e*(e^2*(4*a*c-b^2))^
(1/2)*c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2
,(-I*b*e+2*I*d*c+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2
+b*e*d-c*d^2+(e^2*(4*a*c-b^2))^(1/2)-c))*d^2+1/4*I*e*(e^2*(4*a*c-b^2))^(1/2
)/c/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*ln(1-(-I*b*e+2*I*d*c+a*e^
2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2-(e^2*(4*
a*c-b^2))^(1/2)-c))*arctan(e*x+d)-1/4*I*e*(e^2*(4*a*c-b^2))^(1/2)/c/(a*e^2-
b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*ln(1-(-I*b*e+2*I*d*c+a*e^2-b*e*d+c*d
^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2+(e^2*(4*a*c-b^2))^(
1/2)-c))*arctan(e*x+d)+I/e*(e^2*(4*a*c-b^2))^(1/2)*c/(4*a*c-b^2)/(a*e^2-b*e
*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*ln(1-(-I*b*e+2*I*d*c+a*e^2-b*e*d+c*d^2-
c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2-(e^2*(4*a*c-b^2))^(1/2
)-c))*arctan(e*x+d)*d^2-I/e*(e^2*(4*a*c-b^2))^(1/2)*c/(4*a*c-b^2)/(a*e^2-b*
e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*ln(1-(-I*b*e+2*I*d*c+a*e^2-b*e*d+c*d^2
-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2+(e^2*(4*a*c-b^2))^(1/
2)-c))*arctan(e*x+d)*d^2+1/4*I*e*(e^2*(4*a*c-b^2))^(1/2)/c/(4*a*c-b^2)/(a*e
^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*ln(1-(-I*b*e+2*I*d*c+a*e^2-b*e*d+
c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2-(e^2*(4*a*c-b^2)
)^(1/2)-c))*arctan(e*x+d)*b^2-1/4*I*e*(e^2*(4*a*c-b^2))^(1/2)/c/(4*a*c-b^2)
/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*ln(1-(-I*b*e+2*I*d*c+a*e^2-b
*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2+(e^2*(4*a*c
-b^2))^(1/2)-c))*arctan(e*x+d)*b^2-1/8*e*(e^2*(4*a*c-b^2))^(1/2)/c/(4*a*c-b
^2)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(-I*b*e+2*I*d*c
+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2+(e^
2*(4*a*c-b^2))^(1/2)-c))*b^2-1/4*e*(e^2*(4*a*c-b^2))^(1/2)/c/(4*a*c-b^2)/(a
*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*arctan(e*x+d)^2*b^2+1/8*e*(e^2*
(4*a*c-b^2))^(1/2)/c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)
+c)*polylog(2,(-I*b*e+2*I*d*c+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)
```

```
^2+1)/(-a*e^2+b*e*d-c*d^2-(e^2*(4*a*c-b^2))^(1/2)-c))*b^2+1/4*e*(e^2*(4*a*c
-b^2))^(1/2)/c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*ar
ctan(e*x+d)^2*b^2+1/e*(e^2*(4*a*c-b^2))^(1/2)*c/(4*a*c-b^2)/(a*e^2-b*e*d+c*
d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*arctan(e*x+d)^2*d^2-1/e*(e^2*(4*a*c-b^2))^(1
/2)*c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*arctan(e*x+
d)^2*d^2+1/2/e*(e^2*(4*a*c-b^2))^(1/2)*c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2+(e^
2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(-I*b*e+2*I*d*c+a*e^2-b*e*d+c*d^2-c)*(1+I
*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2-(e^2*(4*a*c-b^2))^(1/2)-c))*d
^2+I*e/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*ln(1-(-I*b*e+2*I*d*c+a
*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2-(e^2*
(4*a*c-b^2))^(1/2)-c))*arctan(e*x+d)+1/8*e*(e^2*(4*a*c-b^2))^(1/2)/c/(a*e^2
-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(-I*b*e+2*I*d*c+a*e^2-b*e
*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2-(e^2*(4*a*c-b
^2))^(1/2)-c))-1/4*e*(e^2*(4*a*c-b^2))^(1/2)/c/(a*e^2-b*e*d+c*d^2-(e^2*(4*a
*c-b^2))^(1/2)+c)*arctan(e*x+d)^2-1/8*e*(e^2*(4*a*c-b^2))^(1/2)/c/(a*e^2-b*
e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(-I*b*e+2*I*d*c+a*e^2-b*e*d+
c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2+(e^2*(4*a*c-b^2)
)^(1/2)-c))+1/4*e*(e^2*(4*a*c-b^2))^(1/2)/c/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-
b^2))^(1/2)+c)*arctan(e*x+d)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x+d)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan\left(ex+d\right)}{cx^2+bx+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x+d)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral(arctan(e*x + d)/(c*x^2 + b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(e*x+d)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ex+d)}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x+d)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate(arctan(e*x + d)/(c*x^2 + b*x + a), x)
```

3.63 $\int \frac{\tan^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=132

$$\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}-\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}-\frac{2i\tan^{-1}(a+bx)\tan^{-1}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}$$

```
[Out] ((-2*I)*ArcTan[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)]]
)/b + (I*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/b
- (I*PolyLog[2, (I*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/b
```

Rubi [A] time = 0.0962101, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5055, 4886}

$$\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}-\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}-\frac{2i\tan^{-1}(a+bx)\tan^{-1}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ((-2*I)*ArcTan[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)]]
)/b + (I*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/b
- (I*PolyLog[2, (I*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/b
```

Rule 5055

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((A_.) + (B_.)*(x_) + (
C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)^
q*(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C, p
, q}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
```

```
GtQ[d, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\text{Subst}\left(\int \frac{\tan^{-1}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b}$$
$$= -\frac{2i\tan^{-1}(a+bx)\tan^{-1}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b} + \frac{i\text{Li}_2\left(-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b} - \frac{i\text{Li}_2\left(\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}$$

Mathematica [A] time = 0.101348, size = 97, normalized size = 0.73

$$\frac{i\text{PolyLog}\left(2,-ie^{i\tan^{-1}(a+bx)}\right) - i\text{PolyLog}\left(2,ie^{i\tan^{-1}(a+bx)}\right) + \tan^{-1}(a+bx)\left(\log\left(1-ie^{i\tan^{-1}(a+bx)}\right) - \log\left(1+ie^{i\tan^{-1}(a+b}\right.\right.}{b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a + b*x]/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] (ArcTan[a + b*x]*(Log[1 - I*E^(I*ArcTan[a + b*x])] - Log[1 + I*E^(I*ArcTan[
a + b*x])]) + I*PolyLog[2, (-I)*E^(I*ArcTan[a + b*x])] - I*PolyLog[2, I*E^(
I*ArcTan[a + b*x])])/b
```

Maple [A] time = 0.397, size = 143, normalized size = 1.1

$$-\frac{\arctan(bx+a)}{b}\ln\left(1+i(1+i(bx+a))\frac{1}{\sqrt{1+(bx+a)^2}}\right) + \frac{\arctan(bx+a)}{b}\ln\left(1-i(1+i(bx+a))\frac{1}{\sqrt{1+(bx+a)^2}}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x)
```

```
[Out] -1/b*arctan(b*x+a)*ln(1+I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))+1/b*arctan(b*x
+a)*ln(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))+I/b*dilog(1+I*(1+I*(b*x+a))/(
1+(b*x+a)^2)^(1/2))-I/b*dilog(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan\left(bx+a\right)}{\sqrt{b^2x^2+2\,abx+a^2+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(arctan(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}\left(a+bx\right)}{\sqrt{a^2+2abx+b^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Integral(atan(a + b*x)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(bx+a)}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

3.64 $\int \frac{\tan^{-1}(a+bx)}{\sqrt{(1+a^2)c+2abcx+b^2cx^2}}\,dx$

Optimal. Leaf size=216

$$\frac{i\sqrt{(a+bx)^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c(a+bx)^2+c}}-\frac{i\sqrt{(a+bx)^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c(a+bx)^2+c}}-\frac{2i\sqrt{(a+bx)^2+1}\tan^{-1}(a+bx)}{b\sqrt{c(a+bx)^2+}}$$

```
[Out] ((-2*I)*Sqrt[1 + (a + b*x)^2]*ArcTan[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)]/
Sqrt[1 - I*(a + b*x)]])/(b*Sqrt[c + c*(a + b*x)^2]) + (I*Sqrt[1 + (a + b*x)
^2]*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/(b*Sqrt
[c + c*(a + b*x)^2]) - (I*Sqrt[1 + (a + b*x)^2]*PolyLog[2, (I*Sqrt[1 + I*(a
 + b*x)])/Sqrt[1 - I*(a + b*x)]])/(b*Sqrt[c + c*(a + b*x)^2])
```

Rubi [A] time = 0.162288, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5055, 4890, 4886}

$$\frac{i\sqrt{(a+bx)^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c(a+bx)^2+c}}-\frac{i\sqrt{(a+bx)^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c(a+bx)^2+c}}-\frac{2i\sqrt{(a+bx)^2+1}\tan^{-1}(a+bx)}{b\sqrt{c(a+bx)^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2],x]
```

```
[Out] ((-2*I)*Sqrt[1 + (a + b*x)^2]*ArcTan[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)]/
Sqrt[1 - I*(a + b*x)]])/(b*Sqrt[c + c*(a + b*x)^2]) + (I*Sqrt[1 + (a + b*x)
^2]*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/(b*Sqrt
[c + c*(a + b*x)^2]) - (I*Sqrt[1 + (a + b*x)^2]*PolyLog[2, (I*Sqrt[1 + I*(a
 + b*x)])/Sqrt[1 - I*(a + b*x)]])/(b*Sqrt[c + c*(a + b*x)^2])
```

Rule 5055

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((A_.) + (B_.)*(x_) + (
C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)^
q*(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C, p
, q}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \frac{\text{Subst}\left(\int \frac{\tan^{-1}(x)}{\sqrt{c+cx^2}}\,dx, x, a+bx\right)}{b}$$

$$= \frac{\sqrt{1+(a+bx)^2}\,\text{Subst}\left(\int \frac{\tan^{-1}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b\sqrt{c+c(a+bx)^2}}$$

$$= -\frac{2i\sqrt{1+(a+bx)^2}\tan^{-1}(a+bx)\tan^{-1}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}} + \frac{i\sqrt{1+(a+bx)^2}\,\text{Li}_2\left(-\frac{i\sqrt{1+i(a}}{\sqrt{1-i(a}}\right.}{b\sqrt{c+c(a+bx)^2}}$$

Mathematica [A] time = 0.0636544, size = 125, normalized size = 0.58

$$\frac{\sqrt{(a+bx)^2+1}\left(i\text{PolyLog}\left(2,-ie^{i\tan^{-1}(a+bx)}\right)-i\text{PolyLog}\left(2,ie^{i\tan^{-1}(a+bx)}\right)+\tan^{-1}(a+bx)\left(\log\left(1-ie^{i\tan^{-1}(a+bx)}\right)-\log\right.\right.}{b\sqrt{c\left((a+bx)^2+1\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a + b*x]/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2],x]
```

```
[Out] (Sqrt[1 + (a + b*x)^2]*(ArcTan[a + b*x]*(Log[1 - I*E^(I*ArcTan[a + b*x])] -
 Log[1 + I*E^(I*ArcTan[a + b*x])]) + I*PolyLog[2, (-I)*E^(I*ArcTan[a + b*x]
)] - I*PolyLog[2, I*E^(I*ArcTan[a + b*x])]))/(b*Sqrt[c*(1 + (a + b*x)^2)])
```

Maple [A] time = 0.439, size = 176, normalized size = 0.8

$$\frac{i}{bc}\left(i\arctan(bx+a)\ln\left(1+i(1+i(bx+a))\frac{1}{\sqrt{1+(bx+a)^2}}\right)-i\arctan(bx+a)\ln\left(1-i(1+i(bx+a))\frac{1}{\sqrt{1+(bx+a)}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x)
```

```
[Out] I*(I*arctan(b*x+a)*ln(1+I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))-I*arctan(b*x+a
)*ln(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))+dilog(1+I*(1+I*(b*x+a))/(1+(b*x
+a)^2)^(1/2))-dilog(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)))*(c*(-I+a+b*x)*(
I+a+b*x))^(1/2)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+c*x^2*b^2)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan(bx+a)}{\sqrt{b^2cx^2+2\,abcx+\left(a^2+1\right)c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+c*x^2*b^2)^(1/2),x, algorithm=
"fricas")
```

```
[Out] integral(arctan(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}(a+bx)}{\sqrt{c(a^2+2abx+b^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/((a**2+1)*c+2*a*b*c*x+c*x**2*b**2)**(1/2),x)
```

```
[Out] Integral(atan(a + b*x)/sqrt(c*(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(bx+a)}{\sqrt{b^2cx^2+2\,abcx+(a^2+1)c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+c*x^2*b^2)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate(arctan(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c), x)
```

3.65 $\int \frac{\tan^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{\tan^{-1}(a+bx)}{\sqrt[3]{(a+bx)^2+1}},x\right)$$

```
[Out] Unintegrable[ArcTan[a + b*x]/(1 + (a + b*x)^2)^(1/3), x]
```

Rubi [A] time = 0.0379871, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a + b*x]/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3),x]
```

```
[Out] Defer[Subst][Defer[Int][ArcTan[x]/(1 + x^2)^(1/3), x], x, a + b*x]/b
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx=\frac{\text{Subst}\left(\int \frac{\tan^{-1}(x)}{\sqrt[3]{1+x^2}}\,dx,x,a+bx\right)}{b}$$

Mathematica [A] time = 0.363002, size = 163, normalized size = 7.41

$$\frac{\frac{5\sqrt[3]{2}\sqrt{\pi}\text{Gamma}\left(\frac{5}{3}\right)\text{HypergeometricPFQ}\left(\left\{1,\frac{4}{3},\frac{4}{3}\right\},\left\{\frac{11}{6},\frac{7}{3}\right\},\frac{1}{(a+bx)^2+1}\right)}{(a+bx)^2+1}+6\text{Gamma}\left(\frac{11}{6}\right)\text{Gamma}\left(\frac{7}{3}\right)\left(\frac{4(a+bx)\tan^{-1}(a+bx)\text{Hypergeometric2F}}{(a+bx)^2+1}\right.}{20b\text{Gamma}\left(\frac{11}{6}\right)\text{Gamma}\left(\frac{7}{3}\right)\sqrt[3]{a^2+2abx+b^2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a + b*x]/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3),x]
```

```
[Out] (6*Gamma[11/6]*Gamma[7/3]*(15 + 10*(a + b*x)*ArcTan[a + b*x] + (4*(a + b*x)
*ArcTan[a + b*x]*Hypergeometric2F1[1, 4/3, 11/6, (1 + (a + b*x)^2)^(-1)])/(
1 + (a + b*x)^2)) + (5*2^(1/3)*Sqrt[Pi]*Gamma[5/3]*HypergeometricPFQ[{1, 4/
3, 4/3}, {11/6, 7/3}, (1 + (a + b*x)^2)^(-1)])/(1 + (a + b*x)^2))/(20*b*(1
+ a^2 + 2*a*b*x + b^2*x^2)^(1/3)*Gamma[11/6]*Gamma[7/3])
```

Maple [A] time = 1.044, size = 0, normalized size = 0.

$$\int \arctan \left(bx+a \right) {\frac {1}{\sqrt [3]{{b}^{2}{x}^{2}+2\,xab+{a}^{2}+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

```
[Out] int(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left(bx + a\right)}{{\left(b^{2} x^{2} + 2 \, abx + a^{2} + 1\right)}^{\frac{1}{3}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorithm="maxima"
)
```

```
[Out] integrate(arctan(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{\arctan\left(bx + a\right)}{{\left(b^{2} x^{2} + 2 \, abx + a^{2} + 1\right)}^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorithm="fricas"
)
```

```
[Out] integral(arctan(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}(a+bx)}{\sqrt[3]{a^2+2abx+b^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/3),x)
```

```
[Out] Integral(atan(a + b*x)/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(1/3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(bx+a)}{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(arctan(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3), x)
```

3.66 $\int \frac{\tan^{-1}(a+bx)}{\sqrt[3]{(1+a^2)c+2abcx+b^2cx^2}}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\tan^{-1}(a+bx)}{\sqrt[3]{c(a+bx)^2+c}},x\right)$$

```
[Out] Unintegrable[ArcTan[a + b*x]/(c + c*(a + b*x)^2)^(1/3), x]
```

Rubi [A] time = 0.0505649, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(a+bx)}{\sqrt[3]{\left(1+a^2\right)c+2abcx+b^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a + b*x]/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2)^(1/3),x]
```

```
[Out] Defer[Subst][Defer[Int][ArcTan[x]/(c + c*x^2)^(1/3), x], x, a + b*x]/b
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{\sqrt[3]{\left(1+a^2\right)c+2abcx+b^2cx^2}}\,dx=\frac{\text{Subst}\left(\int \frac{\tan^{-1}(x)}{\sqrt[3]{c+cx^2}}\,dx,x,a+bx\right)}{b}$$

Mathematica [A] time = 0.0728054, size = 165, normalized size = 6.88

$$\frac{\frac{5\sqrt[3]{2}\sqrt{\pi}\text{Gamma}\left(\frac{5}{3}\right)\text{HypergeometricPFQ}\left(\left\{1,\frac{4}{3},\frac{4}{3}\right\},\left\{\frac{11}{6},\frac{7}{3}\right\},\frac{1}{(a+bx)^2+1}\right)}{(a+bx)^2+1}+6\text{Gamma}\left(\frac{11}{6}\right)\text{Gamma}\left(\frac{7}{3}\right)\left(\frac{4(a+bx)\tan^{-1}(a+bx)\text{Hypergeometric2F1}(}{(a+bx)^2+1}\right.}{20b\,\text{Gamma}\left(\frac{11}{6}\right)\text{Gamma}\left(\frac{7}{3}\right)\sqrt[3]{c\left(a^2+2abx+b^2x^2+1\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a + b*x]/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2)^(1/3),x]
```

```
[Out] (6*Gamma[11/6]*Gamma[7/3]*(15 + 10*(a + b*x)*ArcTan[a + b*x] + (4*(a + b*x)
*ArcTan[a + b*x]*Hypergeometric2F1[1, 4/3, 11/6, (1 + (a + b*x)^2)^(-1)])/(
1 + (a + b*x)^2)) + (5*2^(1/3)*Sqrt[Pi]*Gamma[5/3]*HypergeometricPFQ[{1, 4/
3, 4/3}, {11/6, 7/3}, (1 + (a + b*x)^2)^(-1)])/(1 + (a + b*x)^2))/(20*b*(c*
(1 + a^2 + 2*a*b*x + b^2*x^2))^(1/3)*Gamma[11/6]*Gamma[7/3])
```

Maple [A] time = 0.996, size = 0, normalized size = 0.

$$\int \arctan\left(bx+a\right)\frac{1}{\sqrt[3]{\left(a^2+1\right)c+2\,abcx+b^2cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

```
[Out] int(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left(bx+a\right)}{\left(b^2cx^2+2\,abcx+\left(a^2+1\right)c\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+c*x^2*b^2)^(1/3),x, algorithm=
"maxima")
```

```
[Out] integrate(arctan(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)^(1/3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\arctan\left(bx+a\right)}{\left(b^2cx^2+2\,abcx+\left(a^2+1\right)c\right)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+c*x^2*b^2)^(1/3),x, algorithm=
"fricas")
```

```
[Out] integral(arctan(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)^(1/3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}(a+bx)}{\sqrt[3]{c\left(a^2+2abx+b^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/((a**2+1)*c+2*a*b*c*x+c*x**2*b**2)**(1/3),x)
```

```
[Out] Integral(atan(a + b*x)/(c*(a**2 + 2*a*b*x + b**2*x**2 + 1))**(1/3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(bx+a)}{\left(b^2cx^2+2\,abcx+\left(a^2+1\right)c\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+c*x^2*b^2)^(1/3),x, algorithm=
"giac")
```

```
[Out] integrate(arctan(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)^(1/3), x)
```

3.67 $\int \frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}} \, dx$

Optimal. Leaf size=187

$$-\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b}+\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b}-\frac{\sqrt{(a+bx)^2+1}}{2b}+\frac{i\tan^{-1}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)\tan^{-1}(a+bx)}{b}+\frac{(a+$$

```
[Out] -Sqrt[1 + (a + b*x)^2]/(2*b) + ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcTan[a +
b*x])/(2*b) + (I*ArcTan[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a
 + b*x)]])/b - ((I/2)*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a
 + b*x)]])/b + ((I/2)*PolyLog[2, (I*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a +
b*x)]])/b
```

Rubi [A] time = 0.215784, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {5057, 4952, 261, 4886}

$$-\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b}+\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b}-\frac{\sqrt{(a+bx)^2+1}}{2b}+\frac{i\tan^{-1}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)\tan^{-1}(a+bx)}{b}+\frac{(a+$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*ArcTan[a + b*x])/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] -Sqrt[1 + (a + b*x)^2]/(2*b) + ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcTan[a +
b*x])/(2*b) + (I*ArcTan[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a
 + b*x)]])/b - ((I/2)*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a
 + b*x)]])/b + ((I/2)*PolyLog[2, (I*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a +
b*x)]])/b
```

Rule 5057

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst
[Int[((d*e - c*f)/d + (f*x)/d)^m*(C/d^2 + (C*x^2)/d^2)^q*(a + b*ArcTan[x])^
p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &&
 EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 4952

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcTan[c*x])^p)/(c^2*d*m), x] + (-Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*(a
 + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(f^2*(m - 1))/(c^
2*m), Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^2], x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rubi steps

$$\int \frac{(a+bx)^2\tan^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\tan^{-1}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\sqrt{1+(a+bx)^2}\tan^{-1}(a+bx)}{2b} - \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{2b} - \frac{\text{Subst}\left(\int \frac{\tan^{-1}}{\sqrt{1+}}\right.}{2}$$

$$= -\frac{\sqrt{1+(a+bx)^2}}{2b} + \frac{(a+bx)\sqrt{1+(a+bx)^2}\tan^{-1}(a+bx)}{2b} + \frac{i\tan^{-1}(a+bx)\tan^{-1}\left(\frac{\sqrt{1+i(}}{\sqrt{1-i(}}\right.}{b}$$

Mathematica [A] time = 0.657008, size = 145, normalized size = 0.78

$$\frac{-i\text{PolyLog}\left(2,-ie^{i\tan^{-1}(a+bx)}\right)+i\text{PolyLog}\left(2,ie^{i\tan^{-1}(a+bx)}\right)-\sqrt{(a+bx)^2+1}+(a+bx)\sqrt{(a+bx)^2+1}\tan^{-1}(a+bx)+}{2b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x)^2*ArcTan[a + b*x])/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x
]
```

```
[Out] (-Sqrt[1 + (a + b*x)^2] + (a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcTan[a + b*x] -
 ArcTan[a + b*x]*Log[1 - I*E^(I*ArcTan[a + b*x])] + ArcTan[a + b*x]*Log[1 +
 I*E^(I*ArcTan[a + b*x])] - I*PolyLog[2, (-I)*E^(I*ArcTan[a + b*x])] + I*Po
lyLog[2, I*E^(I*ArcTan[a + b*x])])/(2*b)
```

Maple [A] time = 0.619, size = 187, normalized size = 1.

$$\frac{\arctan(bx+a)\,xb+\arctan(bx+a)\,a-1}{2\,b}\sqrt{b^2x^2+2\,xab+a^2+1}+\frac{\arctan(bx+a)}{2\,b}\ln\left(1+i\left(1+i\left(bx+a\right)\right)\frac{1}{\sqrt{1+(bx}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x)
```

```
[Out] 1/2*(arctan(b*x+a)*x*b+arctan(b*x+a)*a-1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b+1
/2/b*arctan(b*x+a)*ln(1+I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))-1/2/b*arctan(b
*x+a)*ln(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))-1/2*I/b*dilog(1+I*(1+I*(b*x
+a))/(1+(b*x+a)^2)^(1/2))+1/2*I/b*dilog(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/
2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorith
m="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^{2}x^{2}+2\,abx+a^{2}\right)\arctan\left(bx+a\right)}{\sqrt{b^{2}x^{2}+2\,abx+a^{2}+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorith
m="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*arctan(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx\right)^{2}\operatorname{atan}\left(a+bx\right)}{\sqrt{a^{2}+2abx+b^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*atan(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Integral((a + b*x)**2*atan(a + b*x)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx+a\right)^{2}\arctan\left(bx+a\right)}{\sqrt{b^{2}x^{2}+2\,abx+a^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate((b*x + a)^2*arctan(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

3.68 $\int \frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt{(1+a^2)c+2abcx+b^2cx^2}} \, dx$

Optimal. Leaf size=281

$$-\frac{i\sqrt{(a+bx)^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c(a+bx)^2+c}}+\frac{i\sqrt{(a+bx)^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c(a+bx)^2+c}}-\frac{\sqrt{c(a+bx)^2+c}}{2bc}+\frac{i\sqrt{(a+bx)^2}}{}$$

```
[Out] -Sqrt[c + c*(a + b*x)^2]/(2*b*c) + ((a + b*x)*Sqrt[c + c*(a + b*x)^2]*ArcTa
n[a + b*x])/(2*b*c) + (I*Sqrt[1 + (a + b*x)^2]*ArcTan[a + b*x]*ArcTan[Sqrt[
1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)]])/(b*Sqrt[c + c*(a + b*x)^2]) - ((I/
2)*Sqrt[1 + (a + b*x)^2]*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I
*(a + b*x)]])/(b*Sqrt[c + c*(a + b*x)^2]) + ((I/2)*Sqrt[1 + (a + b*x)^2]*Po
lyLog[2, (I*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/(b*Sqrt[c + c*(a
 + b*x)^2])
```

Rubi [A] time = 0.331005, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5057, 4952, 261, 4890, 4886}

$$-\frac{i\sqrt{(a+bx)^2+1}\text{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c(a+bx)^2+c}}+\frac{i\sqrt{(a+bx)^2+1}\text{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c(a+bx)^2+c}}-\frac{\sqrt{c(a+bx)^2+c}}{2bc}+\frac{i\sqrt{(a+bx)^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*ArcTan[a + b*x])/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2]
,x]
```


```
[Out] -Sqrt[c + c*(a + b*x)^2]/(2*b*c) + ((a + b*x)*Sqrt[c + c*(a + b*x)^2]*ArcTa
n[a + b*x])/(2*b*c) + (I*Sqrt[1 + (a + b*x)^2]*ArcTan[a + b*x]*ArcTan[Sqrt[
1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)]])/(b*Sqrt[c + c*(a + b*x)^2]) - ((I/
2)*Sqrt[1 + (a + b*x)^2]*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I
*(a + b*x)]])/(b*Sqrt[c + c*(a + b*x)^2]) + ((I/2)*Sqrt[1 + (a + b*x)^2]*Po
lyLog[2, (I*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/(b*Sqrt[c + c*(a
 + b*x)^2])
```


Rule 5057

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst
```

```
[Int[((d*e - c*f)/d + (f*x)/d)^m*(C/d^2 + (C*x^2)/d^2)^q*(a + b*ArcTan[x])^
p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &&
 EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 4952

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcTan[c*x])^p)/(c^2*d*m), x] + (-Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*(a
 + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(f^2*(m - 1))/(c^
2*m), Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^2], x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rubi steps

$$\int \frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\tan^{-1}(x)}{\sqrt{c+cx^2}}\,dx,x,a+bx\right)}{b}$$

$$= \frac{(a+bx)\sqrt{c+c(a+bx)^2}\tan^{-1}(a+bx)}{2bc} - \frac{\text{Subst}\left(\int \frac{x}{\sqrt{c+cx^2}}\,dx,x,a+bx\right)}{2b} - \frac{\text{Subst}}{}$$

$$= -\frac{\sqrt{c+c(a+bx)^2}}{2bc} + \frac{(a+bx)\sqrt{c+c(a+bx)^2}\tan^{-1}(a+bx)}{2bc} - \frac{\sqrt{1+(a+bx)^2}\,\text{Sub}}{2b\sqrt{c}}$$

$$= -\frac{\sqrt{c+c(a+bx)^2}}{2bc} + \frac{(a+bx)\sqrt{c+c(a+bx)^2}\tan^{-1}(a+bx)}{2bc} + \frac{i\sqrt{1+(a+bx)^2}\tan}{b\sqrt{}}$$

Mathematica [A] time = 0.133035, size = 189, normalized size = 0.67

$$\frac{\sqrt{a^2+2abx+b^2x^2+1}\left(-i\text{PolyLog}\left(2,-ie^{i\tan^{-1}(a+bx)}\right)+i\text{PolyLog}\left(2,ie^{i\tan^{-1}(a+bx)}\right)-\sqrt{(a+bx)^2+1}+(a+bx)\sqrt{(a+}\right.}{2b\sqrt{c\left(a^2+2abx+b\right.}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x)^2*ArcTan[a + b*x])/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*
c*x^2],x]
```

```
[Out] (Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(-Sqrt[1 + (a + b*x)^2] + (a + b*x)*Sqrt
[1 + (a + b*x)^2]*ArcTan[a + b*x] - ArcTan[a + b*x]*Log[1 - I*E^(I*ArcTan[a
 + b*x])] + ArcTan[a + b*x]*Log[1 + I*E^(I*ArcTan[a + b*x])] - I*PolyLog[2,
 (-I)*E^(I*ArcTan[a + b*x])] + I*PolyLog[2, I*E^(I*ArcTan[a + b*x])]))/(2*b
*Sqrt[c*(1 + a^2 + 2*a*b*x + b^2*x^2)])
```

Maple [A] time = 1.125, size = 222, normalized size = 0.8

$$\frac{\arctan(bx+a)\,xb+\arctan(bx+a)\,a-1}{2\,bc}\sqrt{c\,(-i+a+bx)\,(i+a+bx)} - \frac{\frac{i}{2}}{bc}\left(i\arctan(bx+a)\ln\left(1+i\,(1+i\,(bx+a))\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x)
```

```
[Out] 1/2*(arctan(b*x+a)*x*b+arctan(b*x+a)*a-1)*(c*(-I+a+b*x)*(I+a+b*x))^(1/2)/b/
c-1/2*I*(I*arctan(b*x+a)*ln(1+I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))-I*arctan
(b*x+a)*ln(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))+dilog(1+I*(1+I*(b*x+a))/(
1+(b*x+a)^2)^(1/2))-dilog(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)))*(c*(-I+a+
b*x)*(I+a+b*x))^(1/2)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+c*x^2*b^2)^(1/2),x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^2+2\,abx+a^2\right)\arctan\left(bx+a\right)}{\sqrt{b^2cx^2+2\,abcx+\left(a^2+1\right)c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+c*x^2*b^2)^(1/2),x,
algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*arctan(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c
*x + (a^2 + 1)*c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*atan(b*x+a)/((a**2+1)*c+2*a*b*c*x+c*x**2*b**2)**(1/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^2 \arctan(bx+a)}{\sqrt{b^2cx^2+2\,abcx+(a^2+1)c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+c*x^2*b^2)^(1/2),x,
algorithm="giac")
```

```
[Out] integrate((b*x + a)^2*arctan(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1
)*c), x)
```

3.69 $\int \frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}} \, dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt[3]{(a+bx)^2+1}},x\right)$$

```
[Out] Unintegrable[((a + b*x)^2*ArcTan[a + b*x])/(1 + (a + b*x)^2)^(1/3), x]
```

Rubi [A] time = 0.131172, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[((a + b*x)^2*ArcTan[a + b*x])/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3),x]
```

```
[Out] Defer[Subst][Defer[Int][(x^2*ArcTan[x])/(1 + x^2)^(1/3), x], x, a + b*x]/b
```

Rubi steps

$$\int \frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}} \, dx = \frac{\text{Subst}\left(\int \frac{x^2 \tan^{-1}(x)}{\sqrt[3]{1+x^2}} \, dx, x, a+bx\right)}{b}$$

Mathematica [A] time = 1.43275, size = 181, normalized size = 6.24

$$-\frac{3\left((a+bx)^2+1\right)^{2/3}\left(\frac{5\sqrt[3]{2}\sqrt{\pi}\text{Gamma}\left(\frac{5}{3}\right)\text{HypergeometricPFQ}\left(\left\{1,\frac{4}{3},\frac{4}{3}\right\},\left\{\frac{11}{6},\frac{7}{3}\right\},\frac{1}{(a+bx)^2+1}\right)}{\left((a+bx)^2+1\right)^2}+\text{Gamma}\left(\frac{11}{6}\right)\text{Gamma}\left(\frac{7}{3}\right)\left(\frac{24(a+bx)\tan^{-1}(\ldots}{\ldots}\right.\right.}{140b\text{Gamma}\left(\frac{11}{6}\right)\text{G}\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x)^2*ArcTan[a + b*x])/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3)
,x]
```

```
[Out] (-3*(1 + (a + b*x)^2)^(2/3)*((5*2^(1/3)*Sqrt[Pi]*Gamma[5/3]*HypergeometricP
FQ[{1, 4/3, 4/3}, {11/6, 7/3}, (1 + (a + b*x)^2)^(-1)])/(1 + (a + b*x)^2)^2
 + Gamma[11/6]*Gamma[7/3]*(15 + 90/(1 + (a + b*x)^2) + (24*(a + b*x)*ArcTan
[a + b*x]*Hypergeometric2F1[1, 4/3, 11/6, (1 + (a + b*x)^2)^(-1)])/(1 + (a
+ b*x)^2)^2 + 5*ArcTan[a + b*x]*(-4*(a + b*x) + 6*Sin[2*ArcTan[a + b*x]])))
)/(140*b*Gamma[11/6]*Gamma[7/3])
```

Maple [A] time = 1.329, size = 0, normalized size = 0.

$$\int (bx+a)^{2}\arctan(bx+a)\frac{1}{\sqrt[3]{b^{2}x^{2}+2\,xab+a^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

```
[Out] int((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{2}\arctan(bx+a)}{\left(b^{2}x^{2}+2\,abx+a^{2}+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorith
m="maxima")
```

```
[Out] integrate((b*x + a)^2*arctan(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3),
x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^2+2\,abx+a^2\right)\arctan\left(bx+a\right)}{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorith
m="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*arctan(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2
 + 1)^(1/3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx\right)^2\operatorname{atan}\left(a+bx\right)}{\sqrt[3]{a^2+2abx+b^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*atan(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/3),x)
```

```
[Out] Integral((a + b*x)**2*atan(a + b*x)/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(1/3)
, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx+a\right)^2\arctan\left(bx+a\right)}{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorith
m="giac")
```

```
[Out] integrate((b*x + a)^2*arctan(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3),
x)
```

3.70 $\int \frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt[3]{(1+a^2)c+2abcx+b^2cx^2}} dx$

Optimal. Leaf size=31

$$\text{Unintegrable}\left(\frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt[3]{c(a+bx)^2+c}}, x\right)$$

```
[Out] Unintegrable[((a + b*x)^2*ArcTan[a + b*x])/(c + c*(a + b*x)^2)^(1/3), x]
```

Rubi [A] time = 0.17908, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt[3]{(1+a^2)c+2abcx+b^2cx^2}} dx$$

Verification is Not applicable to the result.

```
[In] Int[((a + b*x)^2*ArcTan[a + b*x])/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2)^(1/
3),x]
```

```
[Out] Defer[Subst][Defer[Int][(x^2*ArcTan[x])/(c + c*x^2)^(1/3), x], x, a + b*x]/
b
```

Rubi steps

$$\int \frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt[3]{(1+a^2)c+2abcx+b^2cx^2}} dx = \frac{\text{Subst}\left(\int \frac{x^2 \tan^{-1}(x)}{\sqrt[3]{c+cx^2}} dx, x, a+bx\right)}{b}$$

Mathematica [A] time = 0.312256, size = 225, normalized size = 7.26

$$-\frac{3\sqrt[3]{a^2+2abx+b^2x^2+1}\left((a+bx)^2+1\right)^{2/3}\left(\frac{5\sqrt[3]{2}\sqrt{\pi}\text{Gamma}\left(\frac{5}{3}\right)\text{HypergeometricPFQ}\left(\left\{1,\frac{4}{3},\frac{4}{3}\right\},\left\{\frac{11}{6},\frac{7}{3}\right\},\frac{1}{(a+bx)^2+1}\right)}{\left((a+bx)^2+1\right)^2}+\text{Gamma}\left(\frac{11}{6}\right)\text{Ga}\right.}{140b\text{Gamma}\left(\frac{11}{6}\right)\text{G}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x)^2*ArcTan[a + b*x])/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^
2)^(1/3),x]
```

```
[Out] (-3*(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3)*(1 + (a + b*x)^2)^(2/3)*((5*2^(1/3)
*Sqrt[Pi]*Gamma[5/3]*HypergeometricPFQ[{1, 4/3, 4/3}, {11/6, 7/3}, (1 + (a
+ b*x)^2)^(-1)])/(1 + (a + b*x)^2)^2 + Gamma[11/6]*Gamma[7/3]*(15 + 90/(1 +
 (a + b*x)^2) + (24*(a + b*x)*ArcTan[a + b*x]*Hypergeometric2F1[1, 4/3, 11/
6, (1 + (a + b*x)^2)^(-1)])/(1 + (a + b*x)^2)^2 + 5*ArcTan[a + b*x]*(-4*(a
+ b*x) + 6*Sin[2*ArcTan[a + b*x]]))))/(140*b*(c*(1 + a^2 + 2*a*b*x + b^2*x^
2))^(1/3)*Gamma[11/6]*Gamma[7/3])
```

Maple [A] time = 1.3, size = 0, normalized size = 0.

$$\int (bx+a)^2 \arctan(bx+a) \frac{1}{\sqrt[3]{(a^2+1)c+2abcx+b^2cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

```
[Out] int((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^2 \arctan(bx+a)}{\left(b^2cx^2+2abcx+(a^2+1)c\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+c*x^2*b^2)^(1/3),x,
algorithm="maxima")
```

```
[Out] integrate((b*x + a)^2*arctan(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)
^(1/3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^2+2\,abx+a^2\right)\arctan\left(bx+a\right)}{\left(b^2cx^2+2\,abcx+\left(a^2+1\right)c\right)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+c*x^2*b^2)^(1/3),x,
algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*arctan(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x +
 (a^2 + 1)*c)^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*atan(b*x+a)/((a**2+1)*c+2*a*b*c*x+c*x**2*b**2)**(1/3),
x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx+a\right)^2\arctan\left(bx+a\right)}{\left(b^2cx^2+2\,abcx+\left(a^2+1\right)c\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+c*x^2*b^2)^(1/3),x,
algorithm="giac")
```

```
[Out] integrate((b*x + a)^2*arctan(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)
^(1/3), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal, debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=", ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
   flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0], Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1], Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0], Rational)
                return 1
```

```
      else:
        return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
      return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
  elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
  elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
  elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
  elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
  elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
def grade_antiderivative(result,optimal):
  debug = False;

  if debug: print ("Enter grade_antiderivative for sagemath")

  leaf_count_result  = leaf_count(result)
  leaf_count_optimal = leaf_count(optimal)

  if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)
```

```
    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```